VEHICLE COLLISION ANALYSIS

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About the Author

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■ *"Vehicle Collision Analysis", Chapter, Automotive Engineering and Litigation, Garland Law Publishing, New York, 1988.*

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I. INTRODUCTION

When two vehicles collide in a road accident, a series of complicated physical events occur. In addition, each vehicular accident contributes to significant engineering, medical, social, economic and often legal problems. Because of these problems, a body of scientific and engineering knowledge has been growing in the area of accident analysis. For example, injury severity is related statistically to each vehicle's ΔV (velocity change). Consequently, for socioeconomic purposes, there is a need for methods to calculate ΔV and its relationship to other accident factors such as initial speed, vehicle size, etc. Since an accident can be a tragic personal event to the involved parties, lawsuits are not uncommon. These often require estimates of vehicles' preimpact speeds based upon postimpact information, often referred to as accident reconstruction. Methods for reconstruction of accidents are available; the more sophisticated are computerized. For these methods and discussions of them see References 1 through 6.

An analysis of the physical events of a collision is always broken down into three phases: preimpact, impact, and postimpact. The impact phase is generally defined as the duration during which the vehicles are in contact. During postimpact motion, forces between a vehicle and the ground, which are of the order of magnitude of the vehicle's weight, determine its trajectory (path of motion). During impact, the intervehicular forces over their common contact surface control motion. These forces are ten or more times higher than ground friction and the duration of contact typically is 0.1 to 0.2 seconds. As a consequence, the engineering methods used to analyze impact and postimpact motions can differ greatly. This chapter deals only with the analysis of the impact phase.

The method presented here is based upon Newton's 2nd Law, stated in the form of impulse and momentum variables. Although Newton's Laws have been known and used for hundreds of years, the proper formulation and solution of the impact of two rigid bodies over a plane surface has not been published until recently [7,8]. This now allows the velocity changes of two colliding vehicles to be calculated in a general and relatively simple manner, provided, of course, certain information is available.

First, the method of analyzing impacts is presented for the case where little or no rotational motion exists before, during or immediately after the impact (for example, a "rear-end" collision). This is usually called a direct colinear impact. The equations for this case are simpler and their discussion allows introduction of some concepts directly transferable to the more general problem. Following this, the equations of the planar impact problem are presented. Several solution strategies are discussed. A computer program which solves these equations, written in Applesoft BASIC, is presented in an appendix. These equations contain certain coefficients whose numerical values are often needed. Typical values are presented, based upon information from experimental, or staged, collisions. Other results from these collisions is also presented such at energy loss levels. Finally, an empirical equation are presented which permits calculation of ΔV for each vehicle based upon the initial speeds, masses and collision energy loss.

II. LIST OF SYMBOLS

The symbols in parentheses are corresponding variable names used in the computer program listed in the Appendix

a,b subscripts used to indicate vehicle A or B, respectively

e coefficient of restitution (E)

e_m impact moment coefficient (EM)

d distance between the center of gravity and impact point in each vehicle (DA, DB)

I moment of inertia of a vehicle about a vertical axis through its center of gravity (IA, IB)

M impulse of the moment developed between two impacting vehicles

m mass of a vehicle (MA, MB)

P impulse component of the resultant force developed between two impacting vehicles

V velocity component of the mass center of each vehicle following impact (A (I,M+1), I=1,4)

v velocity component of the mass center of each vehicle before impact (UX, UY, VS, VY)

ΔV magnitude of the vector velocity change of a vehicle due to impact

Δ_T fraction of the total kinetic energy lost in a collision

μ equivalent coefficient of friction along the impact surface (MU)

θ heading angle of a vehicle relative to a fixed x axis (TA, TB)

Γ angle of impact surface relative to a fixed y axis (GA)

Ω angular velocity of each vehicle following impact (A(I, M+1), I=5,6)

ω angular velocity of each vehicle before impact (WA, WB)

ϕ angle between the length axis of a vehicle and a line between its center of gravity and the center of impact (PA, PB)

III. IMPULSE AND MOMENTUM

Typically, when two objects collide (such as pool balls) their paths and speeds change suddenly. Since speed and direction collectively comprise what is called velocity, we say that the objects undergo a velocity change, with a magnitude written symbolically as ΔV. During contact each object feels a force, equally and oppositely applied by one to the other. Figure 1-a shows a graph of this force as it typically occurs in an impact with duration t_f, seconds. The area under the force-time curve is given a specific name; it is called the "impulse" of the force. Figure 1-b shows the same impulse but determined by a constant, average force, F_A, over the same duration. This average force can often be estimated in vehicle collisions since contact time almost always is from 0.1 to 0.2 sec. [9,10]. Furthermore, the maximum force, F_M, is typically two to three times the average force and can also be grossly estimated. Just as the word impulse has a specific meaning, so does the term momentum. By "momentum" is meant the product of mass and velocity (of the center of gravity). Mass is weight divided by the acceleration of gravity, i.e., $m = W/g$. For a planar collision, Newton's 2nd Law says that the change in momentum of the center of gravity of an object (vehicle) is equal to the impluse applied to it in each of two mutually perpendicular directions. Figure 2 shows two vehicles, drawn separated, as they might appear during a collision. The n and t axes are chosen by criteria discussed later; P_n and P_t are the impulse components along the n and t axes, respectively. Then, from Newton's Law:*

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*Note that throughout this chapter, capital V's indicate velocity components at the end of contact (final velocities) and lower case, or small, v's indicate velocity components at the beginning of contact (initial velocities).

$$m_a(V_{an}-v_{an}) = P_n \quad (1)$$

$$m_a(V_{at}-v_{at}) = P_t \quad (2)$$

$$m_b(V_{bn}-v_{bn}) = -P_n \quad (3)$$

and

$$m_b(V_{bt}-v_{bt}) = -P_t \quad (4)$$

By adding Equation 1 to Equation 3 and Equation 2 to Equation 4, we get

$$m_a(V_{an}-v_{an}) + m_b(V_{bn}-v_{bn}) = 0 \quad (5)$$

and

$$m_a(V_{at}-v_{at}) + m_b(V_{bt}-v_{bt}) = 0 \quad (6)$$

Equations 5 and 6 are expressions of the "conservation of momentum" and are particularly useful since they do not contain the impulse components P_n and P_t.

These equations are not the whole story. Since much damage (crush) and friction (sliding) can occur during a collision, at the end of contact the vehicles do not separate as fast as they moved together at the beginning of contact. From the subject of mechanics, two coefficients can be used to model crush and sliding. A coefficient of restitution, e, can be defined:

$$e = \frac{\text{Relative Final Velocity in the Direction of Crushing}}{\text{Relative Initial Velocity in the Direction of Crushing}}$$

For the vehicles in Figure 2, this would be in the direction of the n axis and

$$e = -\frac{V_{an}-V_{bn}}{v_{an}-v_{bn}} \quad (7)$$

where $0 < e < 1$. The coefficient of restitution has some interesting properties. If $e = 0$, Equation 7 indicates that $V_{an} = V_{bn}$. This means that the vehicle's mating, crushed surfaces do not rebound from each other at the end of impact. If $e = 1$, Equation 7 indicates that $V_{an} - V_{bn} = -(v_{an}-v_{bn})$. This, in turn, means that the vehicles' common surfaces rebound with the same relative velocities (but opposite direction) at which they approached each other. Both of these extremes occur infrequently in road accidents; although some accident configurations can have a value of e at or near zero, as will be seen later. All information concerning energy lost in a direct colinear collision is controlled by e and the friction coefficient μ. If it is assumed that "sliding" takes place along the t axis, perpendicular to crushing, then another coefficient, μ, can be defined as:

$$\mu = \frac{\text{Impulse in the Direction of Sliding}}{\text{Impulse Normal to Sliding}}$$

Again, for Figure 2 this would be

$$\mu = P_t/P_n$$

Using Equations 2 and 3 this can be written as:

$$\mu m_b(V_{bn}-v_{bn}) + m_a(V_{at}-v_{at}) = 0 \qquad (8)$$

The quantity μ is not identical to the coefficient of dynamic friction commonly used in engineering, although of course, they are related. They differ if only because of possible momentary interconnection of crushed, broken parts during a vehicular collision. There are several other more subtle differences discussed elsewhere [11]. Consequently, μ will be called an equivalent coefficient of friction.

Several comments are in order here. The orientation of the n-t coordinate system must be chosen such that the n, or normal axis is in a direction predominantly perpendicular to the crushed surfaces. Correspondingly, the tangential, or t, axis is along the crush surface along which sliding has occurred. Choice of these axes are based upon observation and measurements of the damage profiles of the vehicles and judgement of the analyst. Secondly, the above equations are only valid for collisions with little or no rotational velocities for both vehicles. Another comment is that no impulses of external forces have been considered; P_n and P_t are due to intervehicular contact forces only. The impulse of forces such as aerodynamic drag and wheel/pavement friction during the duration of contact have been neglected. Except perhaps for very low-speed collisions this is usually a good assumption, since it is typical to find the average intervehicular collision force (F_A, Fig 1) is 5 to 10 times greater than the weight of each vehicle, whereas tire friction and aerodynamic forces are usually some fraction of a vehicle's weight.

For the present, assume that the initial velocity components, v_{1n}, v_{1t}, v_{2n} and v_{2t}, are all known. Equations 5, 6, 7 and 8 provide four linear algebraic equations from which the final velocity components can be solved. That is,

$$V_{an} = v_{an} + m_b(1+e)(v_{bn}-v_{an})/(m_a+m_b) \tag{9}$$

$$V_{at} = v_{at} + \mu m_b(1+e)(v_{bn}-v_{an})/(m_a+m_b) \tag{10}$$

$$V_{bn} = v_{bn} - m_a(1+e)(v_{bn}-v_{an})/(m_a+m_b) \tag{11}$$

$$V_{bt} = v_{bt} - \mu m_a(1+e)(v_{bn}-v_{an})/(m_a+m_b) \tag{12}$$

Obviously, if Equations 9 through 12 are used to model vehicle collisions, appropriate values for e and μ must be available. This is discussed later when experimentally obtained values are presented.

In reconstruction problems, the final velocity components are estimated by calculations; the initial velocity components are unknown. Equations 5, 6, 7 and 8 can be solved easily for the initial velocity components, to provide what can be called an inverse solution. Equations 9 through 12 can be used for the inverse solution with only two changes [8]. Replace e by 1/e and interchange the meaning of the V's and v's. That is, let upper case V's represent initial velocities and lower case v's represent final velocity components. (Note that if e = 0, an inverse solution cannot be obtained.)

IV. IMPACT EQUATIONS INCLUDING ROTATIONAL EFFECTS

Although the above equations are interesting, informative and in some cases (no rotational velocities) useful, they are not general enough for analyzing most vehicular collisions. Rotational effects must be accounted for. Figure 3 shows two isolated, colliding vehicles and two coordinate systems. First the coordinates and variables will be discussed; then a synopsis of the derivation of the impulse/momentum equations for the collision of Figure 3 will be presented. Following these, methods of solution of the equations will be addressed.

A. Physical Properties and Problem Variables

Dimensional and inertial properties of the vehicles enter into a collision analysis problem along with collision geometry, the coefficients of restitution and friction and initial and final velocity components. The x-y coordinates are usually chosen relative to the accident site. As illustrated in Figure 3, the crush surface on each vehicle is idealized as a flat vertical plane which makes an angle of Γ (positive counter clockwise) with the y axis. Actual crush surfaces are not flat and the areas in contact between vehicles vary over time during a collison. The flat, hypothetical surface represents an "average" contact plane, where the average is considered over both the time and geometry of contact. This plane defines the normal and tangential axes, n-t. The center of impact, point C, is a point coincident in both vehicles and can be concisely defined. It is the point at which resultant of the intervehicular surface forces act when averaged over the contact time. Obviously, its location must be approximated using judgement and observations and/or measurements of the vehicles' deformation. In the problem formulation, it is located by the angle ϕ and distance d in each vehicle. Fortunately, most results of a collision analysis are not critically sensitive to variations in the location of point C and the angle Γ [11]. The angles θ_1 and θ_2 define the vehicle orientations during the collision relative to the fixed x-y reference. Solution of the impact problem requires an assumption that these remain constant. More often than not, in a real collision, these angles can vary considerably during the impact phase. Test results seem to indicate [11] that orientation angle changes over about 45° (during contact) do affect solution accuracy. Lesser changes in θ_1 and θ_2 during the impact do not seem to be too important. Values of these angles at the beginning of contact are typically used, however, an average can be used if estimatable.

Vehicle weights (and masses) are usually easy to obtain. The yaw moment of inertia, I, about a vertical axis through each vehicle's center of gravity is quite the opposite however. Ordinarily accurate values are known only to the manufacturers. In accident studies, they are usually estimated by calculation [9].

In the planar impact problem including rotation, each vehicle has three initial velocity components such as v_{ax}, v_{ay} and ω_a for vehicle A. Each also has three final velocity compoents, such as V_{bx}, V_{by} and Ω_b, for vehicle B. As in the point-mass problem solved earlier, the final velocities can be found if the initial velocities and all other data are known; the inverse problem is also solvable. Coefficients of restitution and friction will again be defined. Results from experimental collisions will be presented.

B. Synopsis of the Impact Equations

The impulse components P_x and P_y are the components of the impulses of the forces due to the collision. An additional impulse, M, is displayed in Figure 3. This is the impulse of the moment which occurs over the crush surface during the collision. A moment impulse can occur physically but it is also affected by the choice of the point representing the center of the collision, point C (which is estimated from the amount and location of the vehicle's damage). An inaccurate choice will ficticiously create a moment and, consequently, a moment impulse. In any case, inclusion of a moment impulse M is necessary. A synopsis of the derivation of the impact equations follows; a more detailed derivation can be found in Reference 7. The equations are presented in final form with the impulse components, P_x, P_y and M eliminated from the equations. Conservation of momentum of the vehicles in a direction along the x axis is expressed by

$$m_a V_{ax} + m_b V_{bx} = m_a v_{ax} + m_b v_{bx} \tag{13}$$

Conservation of momentum in the y direction is

$$m_a V_{ay} + m_b V_{by} = m_a v_{ay} + m_b v_{by} \tag{14}$$

The relative final velocity normal to the crush surface following impact (rebound) must be some fraction of the relative intial velocity normal to the crush surface (approach). This involves the velocities of each vehicle at the contact point and the coefficient of restitution. The equation is:

$$\begin{aligned}(V_{ay}-d_4\Omega_a-V_{by}-d_2\Omega_b)\sin\Gamma + (V_{ax} + d_3\Omega_a-V_{bx}+d_1\Omega_b)\cos\Gamma \\ = -e[(v_{ay}-d_4\omega_a-v_{by}-d_2\omega_b)\sin\Gamma + (v_{ax}+d_3\omega_a-v_{bx}+d_1\omega_b)\cos\Gamma]\end{aligned} \tag{15}$$

In these equations Ω_a and Ω_b are the final angular velocities, ω_a and ω_b are the initial angular velocities and

$$d_1 = d_b \sin(\theta_b+\phi_b)$$

$$d_2 = d_b \cos(\theta_b+\phi_b)$$

$$d_3 = d_a \sin(\theta_a+\phi_\alpha)$$

$$d_4 = d_a \cos(\theta_a+\phi a)$$

Introduction of an equivalent friction coefficient μ as done prior to Equation 8 gives

$$\begin{aligned}m_a(V_{ay}-v_{ay})(\cos\Gamma - \mu\sin\Gamma) \\ +m_b(V_{bx}-v_{bx})(\sin\Gamma + \mu\cos\Gamma) = 0\end{aligned} \tag{16}$$

Conservation of angular momentum gives

$$I_b(\Omega_b-\omega_b) + I_a(\Omega_a-\omega_a) + m_b(d_1+d_3)(V_{bx}-v_{bx})$$
$$+m_a(d_2+d_4)(V_{ay}-v_{ay}) = 0 \qquad (17)$$

A sixth equation is obtained by defining a moment coefficient of restitution, e_m [7]. This coefficient relates the initial and final relative angular velocities in a manner similar to how e relates the normal velocity components at the crush surface. More discussion follows on these coefficients. The last equation is

$$(\Omega_b-\Omega_a)(1-e_m) = -e_m[(\Omega_a-\omega_a) - m_a d_3(V_{ax}-v_{ax})/I_a + m_a d_4(V_{ay}-v_{ay})/I_a$$
$$- (\Omega_b-\omega_b) - m_b d_1(V_{bx}-v_{bx})/I_b + m_b d_2(V_{by}-v_{by})/I_b] \qquad (18)$$

As defined, e_m is analogous to e except for sign. That is, $-1 < e_m < 0$, where $e_m = 0$ corresponds to the case where the angular impact is totally inelastic and $\Omega_a = \Omega_b$. For $e_m = -1$, the angular impact is totally elastic. In addition, the moment coefficient has the property that for $e_m = +1$, the moment impulse must be zero, i.e. M = 0.

C. Solution of the Impact Equations

Equations 13 through 18 form a set of 6 algebraic equations. If the final velocities are known, these equations are linear and can be solved for the initial velocities, as done in an accident reconstruction. As before, this inverse solution does not exist for e = 0 (see Equation 15). In this chapter, unless otherwise stated, it is assumed that the initial velocity components are known and the final velocities components are to be computed. That is, the unknowns are V_{ax}, V_{ay}, Ω_a, V_{bx}, V_{by} and Ω_b. Of course it is assumed that all of the vehicle and geometric quantities such as mass and vehicular orientations are known as well as the three coefficients. These equations can be solved analytically, but they are long and complicated. For most applications, several or more solutions for different values of some parameters are required and computers are used. For this reason numerical solutions of the equations are as convenient. A computer program for this purpose is provided in an appendix to this paper. It is written in Applesoft BASIC. Included are two sample runs for the physical variable values included in the program listing. If all angular velocites are set to zero and $d_a = d_b = 0$, the first four of Equations 13 through 18 reduce to the direct colinear case covered earlier. The computer program contains this feature as a solution option.

Another use for these equations involves collisions of vehicles with barriers. The above equations can be simplified for barrier impact studies. Since momentum of a single vehicle is not conserved in a barrier impact, Equations 13, 14 and 17 no longer apply. In the remaining three equations, let all velocity components of one vehicle approach zero and its mass and moment of inertia become infinitely large. This results in three equations for the three unknown velocity components of a vehicle striking a rigid barrier.

The physical quantities which enter into the planar impact equations can be broken down into 4 catagories. These correspond to the vehicles, collision, coefficients and velocities. In an application to an actual collision, finding the values of all of these presents problems of varying degrees of difficulty. The vehicle data such as masses, moments of inertia and dimensions is usually the easiest to obtain or estimate. The collision information such as the angular orientation of the vehicles during contact may require investigations and judgement. The same is generally true for a set of velocities. In an accident reconstruction, finding reasonably accurate values for those quantities presents a challenge to the investigator but in well documented collisions, a good job can be done. Determination or estimation of the restitution and friction coefficients however, can range from near impossible to a simple task. Sometimes the coefficients are not needed; an example is presented later.

To summarize, the six impact equations are extremely versatile. They can be reduced to the point-mass equations. They can be reduced to the rigid body, barrier impact problem. In addition, they can be used to solve for various combinations of six unknowns. The most common sets of unknown variables are:

1. Six final velocity components
2. Six initial velocity components
3. Three initial and three final velocity components of each vehicle respectively.
4. Three velocity components and three coefficients, e, e_m and μ.
5. Other combinations of six or less unknowns.

All other quantities are treated as known. In the first three cases above, the equations are linear. In the last case they may be nonlinear, but solvable analytically.

V. RESULTS OF STAGED COLLISIONS

A way of shedding light upon the values of e, e_m and μ is by means of experiments. Some information is currently available [11] based upon a group of staged collisions conducted for the National Highway Traffic Safety Administration [10]. Some of the more pertinent data from these collisions is summarized here with the intention of illustrating some typical values of the coefficients. Other important quantities such as energy loss and ΔV are also presented. Table 1 lists the experimental conditions under which eleven staged and fully-instrumented collisions were conducted. Both initial and final velocity components were measured; all initial angular velocities were zero. Four catagories of relative vehicle orientations were included in the experiments. See Figure 4. A reasonably good mix of vehicle sizes and initial speeds exist within these catagories. It should be noted that the highest speed was only about 40 mph (64 kph).

In order to obtain information about the coefficients, the method of least squares was used to fit the six, planar impact equations [11]. The measured final velocity components were treated as experimentally measured values. Minimization was of the sum of squares of deviations of these measured velocities from their corresponding values which satisfied the impact equations. This procedure provides values of the three coefficients e, e_m and μ which minimize the sum of squares. These coefficients, the percent energy loss and velocity changes for the staged collisions are listed in Table 2.

Although the resulting coefficients are certainly not equal constants within each collision catagory, there exists enough uniformity to permit some generalizations. First of all, the largest coefficient of restitution e for all of the collisions is 0.258; the smallest was zero. At least for the conditions of these staged collisions, this indicates that e typically is small. The moment coefficient as defined must lie between -1 and 0. For the front-to-front and front-to-rear collisions e_m remains consistently near -0.5. For the front-to-side catagories its magnitude is greater, and varies more. Analysis has shown [11] that the ΔV of each vehicle and the energy loss in the collision are not very sensitive to changes in e_m, however. The equivalent coefficient of friction μ (the ratio of the tangential to normal impulse components) seems to vary fairly consistently with the initial relative tangential velocity of the vehicles. The last 5 collisions are nearly direct (head-on and rear end); consequently the coefficient μ is near zero. With significant relative sliding, as in the first 6 collisions, the corresponding value of μ ranges roughly between 0.5 and 1. The relative sliding during collision and the resulting tangential impulse, P_t, can be influenced by momentary interference of structural members (such as between a bumper and door pillar). These effects, when present, must be considered if an estimate of μ is to be made.

Of course, when reconstructing a collision, reasonable ranges of these variables should be used and the resulting velocity changes can be used to provide upper and lower bounds on vehicle speed.

VI. DIRECT CALCULATION OF ΔV

Based upon the analysis of the above experimental collisions, an empirical relationship appears to exist between each vehicle's ΔV and the initial momentum and energy loss of the collision [11]. The following equation furnishes an estimate of ΔV for each vehicle for all of the staged collisions.

$$\Delta V_i = (\Delta_T)^{1/2} [(m_a v_a)^2 + (m_b v_b)^2]^{1/2}/m_i \qquad (19)$$

The subscript i represents either a (for vehicle A) or b (for vehicle B) and Δ_T is the fractional energy loss in the collision. The last three columns of Table 2 contain the values of each vehicle's velocity change from experimenal measurements, the least square fitting of the impact equations and the value predicted by Equation 19, respectively. Figure 5 illustrates this data. This graph indicates that Equation 19 does an excellent job of predicting the ΔV of each vehicle providing an error of a few ft/s is permissible.

In addition to its simplicity, a remarkable feature of Equation 19 is that it appears to apply equally well to all of the above collision conditions. With the exception of Δ_T, the energy loss, ΔV from Equation 19 depends only upon the mass and initial speeds of each vehicle. Table 2 illustrates, that within each collision catagory, the energy loss appears relatively constant. For collisions of these types, the Δ_T from the table can be used as an estimate.

At least for the types of collisions discussed here, Equation 19 should provide a reasonably accurate means of calculating ΔV. The manner in which Equation 19 is used depends upon the nature of the problem. For socioeconomic studies of vehicle collisions, Equation 19 can furnish ΔV directly as a function of initial speed if information is available concerning Δ_T. For accident reconstruction studies, Equation 19 can be used to check and verify (or support) calculations of ΔV's.

A word of caution must be made concerning Equation 19. Although it seems to fit the 11 staged collisions rather well, Equation 19 has not yet been compared to any other set of reliable collision data.

VII EXAMPLE CALCULATIONS

To supplement the two solutions in the Appendix, consider a hypothetical example in a reconstruction setting. Suppose a collision occurs which is a 90° front-to-side intersection collision. A full set of data in the form of a solution to the six equations is in Table 3. All of this information is not known to the analyst, however. Assume the following circumstances exist:

1. all physical properties of the vehicles are known including the extent and geometry of all damage.
2. neither driver performed any evasive maneuvers prior to the collision
3. postimpact skid paths and distances are well known and documented.

On the basis of item 2, it is known that $v_{ay} = v_{bx} = 0$. This is also a consequence of choosing the x-y coordinate system along the initial vehicle paths. From item 3, further assume that all postimpact velocity components for both vehicles can be estimated by trajectory calculations (not covered here). Suppose these calculations provide the following estimates: $V_{ax} = -10$ ft/s $V_{ay} = 20$ ft/s, $V_{bx} = -20$ ft/s and $V_{by} = 45$ ft/s. Being an accident reconstruction, the initial speeds of both vehicles are desired. These, of course, are v_{ax} and v_{by}.

The above velocity components can be substituted into Equations 13 and 14 to provide the unknown initial velocities. These give $v_a = v_{ax} = -40.21$ ft/s and $v_b = v_{by} = 58.24$ ft/s. The corresponding velocity changes are:

$$\Delta V_a = [(V_{ax}-v_{ax})^2 + (V_{ay}-v_{ay})^2]^{1/2} = 36.2 \text{ ft/s} \quad (20)$$

$$\Delta V_b = [(V_{bx}-v_{bx})^2 + (V_{by}-v_{by})^2]^{1/2} = 24.0 \text{ ft/s} \quad (21)$$

Remember that these are estimates (the true values calculated from Table 3 are $\Delta V_a = 39.1$ and $\Delta V_b = 25.9$ ft/s).

Equation 19 can be used to provide an independent set of estimates for these ΔV's as well as for the initial velocities themselves. Equation 19 requires an estimate for the collision's energy loss. As an approximation, a value of $\Delta_T = 0.4$ will be used which follows from Table 2 for this type of collision. Equation 19 gives $\Delta V_a = 39.1$ ft/s and $\Delta V_b = 25.9$ ft/s. Apparently a coincidence, these are exactly the same as the true ΔV's. They do not yield the exact initial velocities, however, since the estimated final velocities must be used. If these ΔV's and the final velocity estimates are placed into Equations 20 and 21, another set of initial velocity estimates results; that is $v_a = v_{ax} = -43.6$ ft/s and $v_b = v_{by} = 61.5$ ft/s.

Assumption 2 above, concerning no evasive maneuvers implies $\omega_a = \omega_b = 0$. Frequently, estimates can be obtained from post impact motion for the final angular velocities, Ω_a and Ω_b. Using all estimates and Equations 15, 16 and 18 provides estimates for e, e_m and μ. Generally, these estimates (in contrast to the least square estimates) are not very accurate.

Table 1. Experimental Collision/Vehicle Data (U.S. Units)

Collision Number/ Vehicle	Collision Category	Vehicle Mass $lb\text{-}s^2/ft$	Moment of Inertia $ft\text{-}lb\text{-}s^2$	Distance d feet	Angle ϕ deg	Angle of Veh ϕ deg	Crush Angle Γ deg	Initial Velocity ft/s v_x/v_y	Final Angular Velocity rad/s	Final Velocity ft/s V_x/V_y
1/A	60° FRONT TO SIDE	143.5	3728	7.59	-19.8	0	-30	-29.04/ 0.0	-1.57	-12.33/ 7.91
1/B		95.8	1961	3.44	-38.7	60	-30	14.52/25.15	0.0	- 6.80/16.97
6/A		133.6	3469	8.41	-17.9	0	-30	-31.53/ 0.0	-0.52	-18.68/ 4.12
6/B		81.5	1669	2.00	-90.0	60	-30	15.29/27.31	-3.14	- 4.20/18.02
7/A		115.0	2985	8.41	-17.9	0	-30	-42.68/ 0.0	-0.52	-25.41/ 4.85
7/B		81.1	1082	2.00	-90.0	60	-30	21.34/36.96	-3.35	- 7.63/28.34
8/A	90° FRONT TO SIDE	139.1	3614	7.90	0.0	0	0	-30.51/ 0.0	-1.99	-10.24/10.72
8/B		146.3	3800	2.77	-68.8	90	0	0.0 /30.51	-0.31	-12.02/19.72
9/A		70.1	976	4.80	6.0	0	0	-31.09/ 0.0	-3.14	- 2.81/14.83
9/B		152.2	3953	5.20	-29.7	90	0	0.0 /31.9	0.79	- 9.90/24.20
10/A		71.6	998	3.47	0.0	0	0	-48.84/ 0.0	-5.24	- 5.08/28.18
10/B		146.6	3008	5.29	-29.2	90	0	0.0/ 48.84	1.26	-14.57/36.55
11/A	10° FRONT TO SIDE	94.4	1935	6.14	9.4	0	0	-29.92/ 0.0	0.52	5.81/ 2.02
11/B		150.6	3913	7.66	11.3	-10	0	29.47/-5.2	0.0	6.42/-4.11
12/A		97.2	1992	5.90	9.6	0	0	-46.20/ 0.0	1.57	14.05/-1.61
12/B		140.2	3640	7.28	10.3	-10	0	45.50/-8.02	1.05	6.32/-9.65
3/A	10° FRONT TO REAR	153.7	3992	8.83	-17.0	0	-10	-31.09/ 0.0	-0.26	-17.15/ 0.24
3/B		97.0	1986	7.63	171.4	170	-10	0.0 / 0.0	0.0	-22.87/ 3.74
4/A		154.7	4018	8.02	-18.2	0	-10	-56.76/ 0.0	-0.65	-29.33/-1.43
4/B		99.1	2032	6.94	171.7	170	-10	0.0/ 0.0	-0.52	-32.54/ 1.38
5/A		142.9	3711	8.08	-20.7	0	-10	-58.23/ 0.0	-0.21	-34.31/ 0.57
5/B		78.6	1095	5.75	168.0	170	-10	0.0/ 0.0	-1.22	-37.15/ 2.77

Table 2, Results of Least Square Impact Analysis

COLLISION NUMBER	ENERGY LOSS, PERCENT	COEFFICIENT OF RESTITUTION, e	MOMENT COEFFICIENT, e_m	EQUIVALENT FRICTION COEFFICIENT, μ	SPEED CHANGE, ΔV EXPERIMENTAL, ft/s	SPEED CHANGE, ΔV COMPUTED, ft/s	SPEED CHANGE, ΔV PREDICTED, ft/s
1	57.5	.005	-.715	.911	18.5	16.1	17.6
					22.8	24.2	26.4
6	48.4	.000	-.430	.797	13.5	14.7	17.1
					21.6	24.1	28.1
7	47.9	.003	-.505	.656	17.9	20.4	24.1
					30.2	28.9	34.2
8	38.9	.043	-.705	.553	22.9	19.3	18.4
					16.2	18.4	17.5
9	40.0	.245	-.914	.714	31.9	32.0	31.3
					12.1	14.7	14.4
10	41.5	.258	-.914	.822	52.0	51.6	47.8
					19.1	25.2	23.3
11	92.0	.008	-.504	.049	35.8	36.6	36.0
					23.1	22.9	22.6
12	92.9	.112	-.499	-.009	60.3	59.3	52.1
					39.2	41.4	36.1
3	34.0	.221	-.489	-.069	13.9	14.1	12.1
					23.2	22.4	19.2
4	36.1	.071	-.489	-.008	27.5	22.4	22.7
					32.6	34.9	35.5
5	32.1	.075	-.475	-.034	23.9	20.5	22.0
					37.3	37.3	40.0

TABLE 3. Hypothetical Collision For Example Calculations.

```
ENTER THE INITIAL VELOCITY COMPONENTS OF MA: Vax,Vay,Wa ?-36.667,Ø,Ø

ENTER THE INITIAL VELOCITY COMPONENTS OF MB: Vbx,Vby,Wb ?Ø,58.667,Ø


MA= 96 MB= 145   IA= 197Ø  IB= 377Ø

DA= 4.5          DB= 4

  PA       PB       TA       TB       GA
  Ø       -4Ø       Ø        9Ø       Ø

UX= -36.667  UY= Ø  WA= Ø  XA= -36.67  YA= Ø

VX= Ø  VY= 58.667  WB= Ø  XB= Ø  YB= 58.66

 E= .25  MU= .89  EM= -.9

INITIAL KE= 314Ø66.231  KE LOSS= 37.4 %

* * * * * * * * * * * * * * * * * * *

VAX= -7.46  VAY= 25.994  (XA=-7.46  YA= 39.Ø5)

VBX= -19.338  VBY= 41.456  (XB=-16.63  YB= 39.18)

 WA= -2.9Ø27   WB= -.885

* * * * * * * * * * * * * * * * * * *
```

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APPENDIX

COMPUTER SOLUTION OF PLANAR IMPACT EQUATIONS

A listing of a computer program follows which gives the solution of Equations 13 through 18. These equations are treated as a set of six simultaneous linear algebraic equations. It is assumed that all of the physical data for the vehicles are known (such as weight, dimensions, inertia, etc.). It is also assumed that the collision geometry and the coefficients are known. The program computes the final velocity components of both vehicles for any set of initial velocity components.

The program is written in the BASIC language for use on an Apple II Plus microcomputer with 48K memory. It is a self contained program and includes a subroutine for solving linear simultaneous algebraic equations using pivotal elimination as discussed in Todd [12]. Two example solutions are presented following the program listing for the data contained in the listing. These examples correspond to collision No. 1 in Tables 1 and 2.

(a) User's Information

Variable names used in the program are listed with the corresponding symbols in the list of symbols earlier in the paper. Other variables and information follow.

XA, YA, XB, YB. These represent the velocity components of each vehicle at the contact point (Figure 3) and are listed in the output. That is, XA represents the x component of the velocity of point C, vehicle A, etc. Two values are printed by the program for each variable, corresponding to the beginning of the impact (initial velocities) and the end of impact (final velocities).

Direction of the Frictional Impulse. The direction of the frictional force across the impact surface depends upon the direction of the initial relative velocities. The program automatically chooses the proper sign for the impulse P_t based upon initial velocities.

Velocity Constant. Equation 16 is used for whatever value of friction coefficient is read into the program. If the user wishes to use a condition of no sliding at separation along the tangent, a value of -10 should be used for the friction coefficient, μ.

Kinetic Energy. The total kinetic energy of the system (vehicle A plus vehicle B) is computed for both the initial velocities and the final velocities. Output is in the form of the initial kinetic energy and the percentage loss.

Units. The program is independent of the units of the input data. A consistent set of units should be followed. An exception is the units of the angles which define the collision geometry. All angles are read in and printed out in degrees. The program converts them to radians where appropriate. Note that angular velocities always have units of radians/second.

Point Mass Solution. The program can furnish the solution for the impact problem of point masses. If this is desired, two actions should be taken by the user. Variable M in statement 350 should be set equal to 4 (M = 4) and the vehicle parameters d_a (DA) and d_b (DB) must be input as zeroes. The second sample run following the program is an example of this option.

```
100  REM  THIS PROGRAM CALCULATES THE FINAL VELOCITIES OF AN IMPACT BETWEEN TWO RIGID BODIES
110  REM
120  REM  EQUATIONS SOLVED ARE THOSE IN SAE PAPER #770014, R. M. Brach
130  REM
140  REM   E,  COEFFICIENT OF RESTITUTION: 0 < E < 1
150  REM   EM, MOMENT COEFFICIENT OF RESTITUTION: -1 < EM < 0
160  REM   MU, EQUIVALENT FRICTION COEFFICIENT
170  REM
180  REM  IN NEXT LINE "0" IS OUTPUT SLOT NUMBER (1 FOR PRINTER, 0 FOR MONITOR, ETC.)
190 P$ = "PR#" + "0"
200  PRINT  CHR$ (140): PRINT "ENTER THE INITIAL VELOCITY COMPONENTS OF MA :Vax,Vay,Wa ";
210  INPUT UX,UY,WA
220  PRINT : PRINT "ENTER THE INITIAL VELOCITY COMPONENTS OF MB :Vbx,Vby,Wb ";
230  INPUT VX,VY,WB
240  REM
250  REM  IF FRICTION COEFFICIENT IS SET EQUAL TO -10,
260  REM   THE CONTACT POINT IS TREATED AS A PIVOT
270  REM
280 E = 0.005:MU = 0.911:EM =  - 0.715: REM   ENTER COEFFICIENTS HERE
290 DA = 7.59:DB = 3.44:GA =  - 30.0:PA =  - 19.8:PB =  - 38.7:TA = 0.0:TB = 60.0
300 AG = 3.14159265 / 180:GA = GA * AG:PA = PA * AG:PB = PB * AG:TA = TA * AG:TB = TB * AG
310 MA = 143.5:MB = 95.8:IA = 3728:IB = 1961: REM  VEHICLE INERTIAL PROPERTIES
320 KI = (MA * (UX ^ 2 + UY ^ 2) + MB * (VX ^ 2 + VY ^ 2) + IA * (WA ^ 2) + IB * (WB ^ 2)) / 2
330 D1 = DB *  SIN (TB + PB):D2 = DB *  COS (TB + PB):D3 = DA *  SIN (TA + PA):D4 = DA *  COS (TA + PA)
340 XA = UX + D3 * WA:YA = UY - D4 * WA:XB = VX - D1 * WB:YB = VY + D2 * WB
350 N = 6:M = 6: REM  SETTING M = 4 WILL PROVIDE A POINT-MASS SOLUTION (NO ROTATIONAL EFFECTS)
360  DIM A(N,N + 1),B(N,N + 1),BR(6)
370 A(1,1) = MA:A(1,3) = MB:A(1,2) = 0:A(1,4) = 0:A(1,5) = 0:A(1,6) = 0
380 A(2,2) = MA:A(2,4) = MB:A(2,1) = 0:A(2,3) = 0:A(2,5) = 0:A(2,6) = 0
390  IF MU =  - 10 THEN 430
400  IF (XA *  SIN (GA) - YA *  COS (GA)) < (XB *  SIN (GA) - YB *  COS (GA)) THEN MU =  - MU
410 A(3,2) = MA * ( COS (GA) - MU *  SIN (GA)):A(3,3) = MB * ( SIN (GA) + MU *  COS (GA)):A(3,1) = 0:A(3,4) = 0:A(3,5) = 0:A(3
    ,6) = 0
420  GOTO 450
430 A(3,1) =  SIN (GA):A(3,2) =  -  COS (GA):A(3,3) =  - A(3,1):A(3,4) =  - A(3,2)
440 A(3,5) = DA *  COS (PA + TA - GA):A(3,6) = DB *  COS (PB + TB - GA)
450 A(4,1) =  COS (GA):A(4,2) =  SIN (GA):A(4,3) =  - A(4,1):A(4,4) =  - A(4,2)
460 A(4,5) = D3 *  COS (GA) - D4 *  SIN (GA):A(4,6) = D1 *  COS (GA) - D2 *  SIN (GA)
470 A(5,2) = MA * (D2 + D4):A(5,3) = MB * (D1 + D3):A(5,5) = IA:A(5,6) = IB:A(5,1) = 0:A(5,4) = 0
480 A(6,1) =  - EM * MA * D3 / IA:A(6,2) = EM * MA * D4 / IA:A(6,3) =  - EM * MB * D1 / IB:A(6,4) = EM * MB * D2 / IB
490 A(6,5) = 2 * EM - 1:A(6,6) = 1 - 2 * EM
500 A(1,M + 1) = MA * UX + MB * VX:A(2,M + 1) = MA * UY + MB * VY
510  IF MU >  - 10 THEN 530
520 A(3,M + 1) = 0: GOTO 540
530 A(3,M + 1) = MA * ( COS (GA) - MU *  SIN (GA)) * UY + MB * ( SIN (GA) + MU *  COS (GA)) * VX
540 A(4,M + 1) =  - E * ((UY - D4 * WA - VY - D2 * WB) *  SIN (GA) + (UX + D3 * WA - VX + D1 * WB) *  COS (GA))
550 A(5,M + 1) = MA * (D2 + D4) * UY + MB * (D1 + D3) * VX + IA * WA + IB * WB
560 A(6,M + 1) = EM * ( - MA * D3 * UX / IA + MA * D4 * UY / IA - MB * D1 * VX / IB + MB * D2 * VY / IB + WA - WB)
570  GOSUB 1000
580 KF = (MA * (A(1,M + 1) ^ 2 + A(2,M + 1) ^ 2) + MB * (A(3,M + 1) ^ 2 + A(4,M + 1) ^ 2)) / 2
590  IF N = M THEN KF = KF + (IA * A(5,M + 1) ^ 2 + IB * A(6,M + 1) ^ 2) / 2
600 TA = TA / AG:TB = TB / AG:PA = PA / AG:PB = PB / AG:GA = GA / AG
610  PRINT  CHR$ (140): PRINT  CHR$ (4);P$
620  PRINT : PRINT : PRINT "MA= ";MA;" MB= ";MB;"   IA= ";IA;"  IB= ";IB
630  PRINT : PRINT "DA= ";DA,"DB= ";DB
640  PRINT : PRINT "  PA      PB      TA      TB      GA"
```

```
5Ø  PRINT "  ";PA"      ";PB"      ";TA;"      ";TB"      ";GA
6Ø  PRINT : PRINT "UX= ";UX;"  UY= ";UY;"  WA= ";WA;"  XA= "; INT (1ØØ * XA) / 1ØØ;"  YA= "; INT (1ØØ * YA) / 1ØØ
7Ø  PRINT : PRINT "VX= ";VX;"  VY= ";VY;"  WB= ";WB;"  XB= "; INT (1ØØ * XB) / 1ØØ;"  YB= "; INT (1ØØ * YB) / 1ØØ
    PRINT : PRINT " E= ";E;"  MU= ";MU;"  EM= ";EM
9Ø  PRINT : PRINT "INITIAL KE= ";KI;"  KE LOSS= ";( INT (1E4 * (KI - KF) / KI)) / 1ØØ;" %"
ØØ XA = A(1,M + 1) + D3 * A(5,M + 1):YA = A(2,M + 1) - D4 * A(5,M + 1)
1Ø XB = A(3,M + 1) - D1 * A(6,M + 1):YB = A(4,M + 1) + D2 * A(6,M + 1)
2Ø  PRINT : PRINT "* * * * * * * * * * * * * * * * * * * * *"
3Ø  PRINT : PRINT "VAX= "; INT (1E3 * A(1,M + 1)) / 1E3;"  VAY= "; INT (1E3 * A(2,M + 1)) / 1E3;"  (XA="; INT (1ØØ * XA) / 1Ø
    Ø;"  YA= "; INT (1ØØ * YA) / 1ØØ;")"
4Ø  PRINT : PRINT "VBX= "; INT (1E3 * A(3,M + 1)) / 1E3;"  VBY= "; INT (1E3 * A(4,M + 1)) / 1E3;"  (XB="; INT (1ØØ * XB) / 1Ø
    Ø;"  YB= "; INT (1ØØ * YB) / 1ØØ;")"
5Ø  IF M <  > N THEN 77Ø
6Ø  PRINT : PRINT " WA= "; INT (1ØØØØ * A(5,M + 1)) / 1ØØØØ;"   WB= "; INT (1ØØØØ * A(6,M + 1)) / 1ØØØØ
7Ø  PRINT : PRINT "* * * * * * * * * * * * * * * * * * * * *"
8Ø  PRINT  CHR$ (4)"PR#Ø": END
ØØØ  REM  SUB TO SOLVE SIMULTANEOUS EQUATIONS BY PIVOTAL ELIMINATION (TODD, P234)
Ø1Ø  FOR J = 1 TO M:I = J: GOSUB 1Ø7Ø
Ø2Ø  FOR RO = 1 TO M: IF RO = I THEN 1Ø6Ø
Ø3Ø AR = A(RO,J)
Ø4Ø  FOR CO = J TO M + 1:A(RO,CO) = A(RO,CO) - A(I,CO) * AR
Ø5Ø  NEXT CO
Ø6Ø  NEXT RO: NEXT J: RETURN
Ø7Ø  REM  SUB TO FIND MAX PIVOT & CHANGE ROWS
Ø8Ø MX = 1E - 9:CO = J: FOR RO = I TO M
Ø9Ø  IF  ABS (MX) >  ABS (A(RO,CO)) AND RO * CO < (M * M) THEN 111Ø
1ØØ MX = A(RO,CO):RX = RO
11Ø  NEXT RO
 .Ø  IF  ABS (A(RX,J)) < 1E - 9 THEN 116Ø
13Ø AP = A(RX,J): FOR OL = J TO M + 1
14Ø AS = A(I,OL):A(I,OL) = A(RX,OL):A(RX,OL) = AS
15Ø A(I,OL) = A(I,OL) / AP: NEXT OL: RETURN
16Ø  PRINT : PRINT "ZERO DETERMINANT": END
17Ø  RETURN
```

```
ENTER THE INITIAL VELOCITY COMPONENTS OF MA: Vax,Vay,Wa ?-29.04,0,0

ENTER THE INITIAL VELOCITY COMPONENTS OF MB: Vbx,Vby,Wb ?14.52,25.15,0

MA= 143.5 MB= 95.8   IA= 3728  IB= 1961

DA= 7.59        DB= 3.44

  PA       PB       TA       TB       GA
  -19.8      -38.7      0       60       -30

UX= -29.04  UY= 0  WA= 0  XA= -29.04  YA= 0

VX= 14.52  VY= 25.15  WB= 0  XB= 14.51  YB= 25.14

 E= 5E-03  MU= .911  EM= -.715

INITIAL KE= 100904.929  KE LOSS= 57.47 %

* * * * * * * * * * * * * * * * * * *

VAX= -13.263  VAY= 3.449  (XA=-8.78  YA= 15.91)

VBX= -9.114  VBY= 19.982  (XB=-8.09  YB= 17.35)

 WA= -1.745   WB= -.8202

* * * * * * * * * * * * * * * * * * *
```

```
ENTER THE INITIAL VELOCITY COMPONENTS OF MA: Vax,Vay,Wa ?-29.04,0,0

ENTER THE INITIAL VELOCITY COMPONENTS OF MB: Vbx,Vby,Wb ?14.52,25.15,0


MA= 143.5 MB= 95.8   IA= 3728   IB= 1961

DA= 0              DB= 0

  PA        PB        TA        TB        GA
 -19.8       -38.7       0        60        -30

UX= -29.04  UY= 0  WA= 0  XA= -29.04  YA= 0

VX= 14.52  VY= 25.15  WB= 0  XB= 14.51  YB= 25.14

 E= 5E-03  MU= .911  EM= -.715

INITIAL KE= 100904.929  KE LOSS= 60.01 %

* * * * * * * * * * * * * * * * * * *

VAX= -15.669  VAY= 2.923  (XA=-15.67  YA= 2.92)

VBX= -5.51  VBY= 20.77  (XB=-5.51  YB= 20.77)

* * * * * * * * * * * * * * * * * * *
```

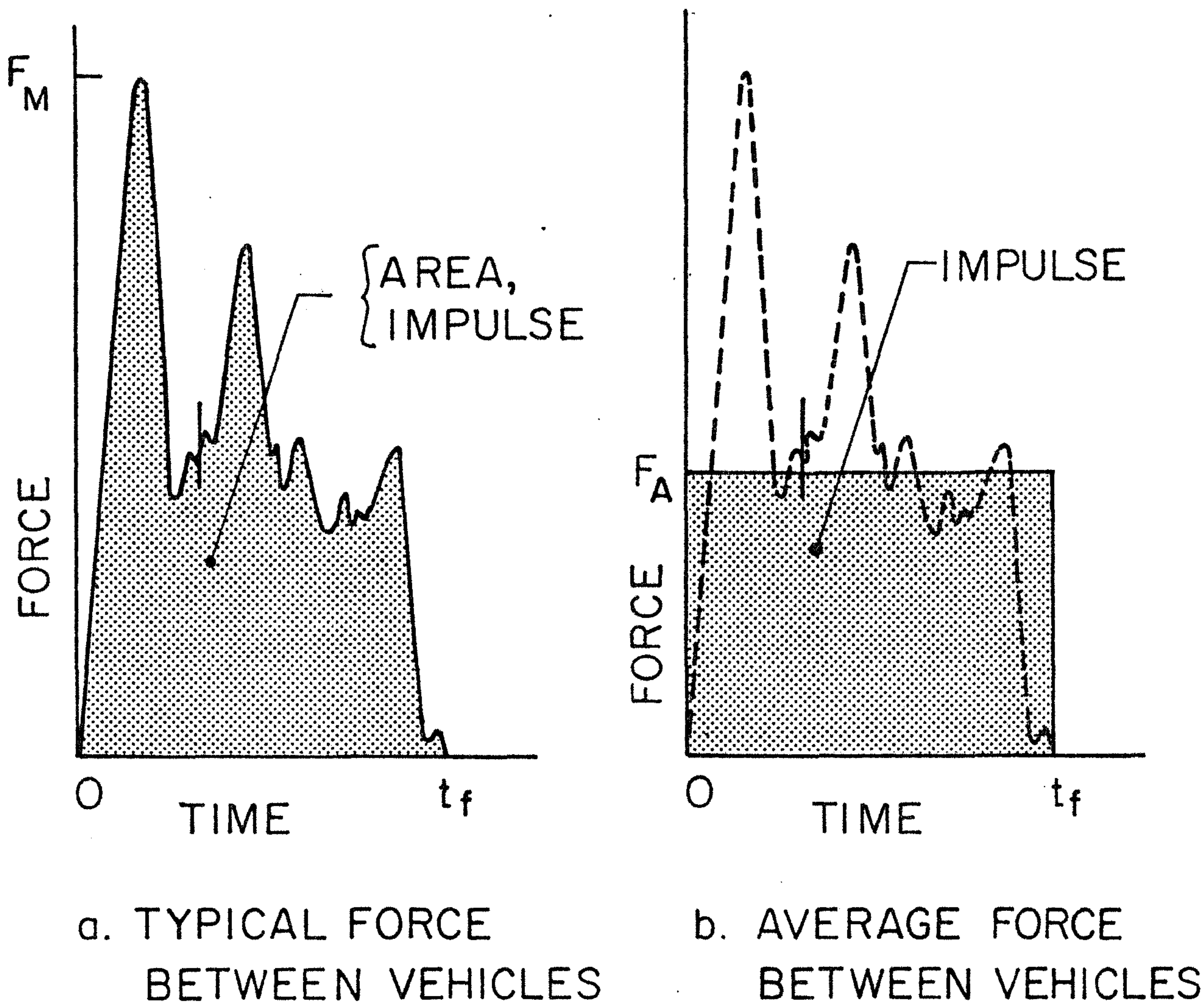

Figure 1. Intervehicular Forces and Impulses.

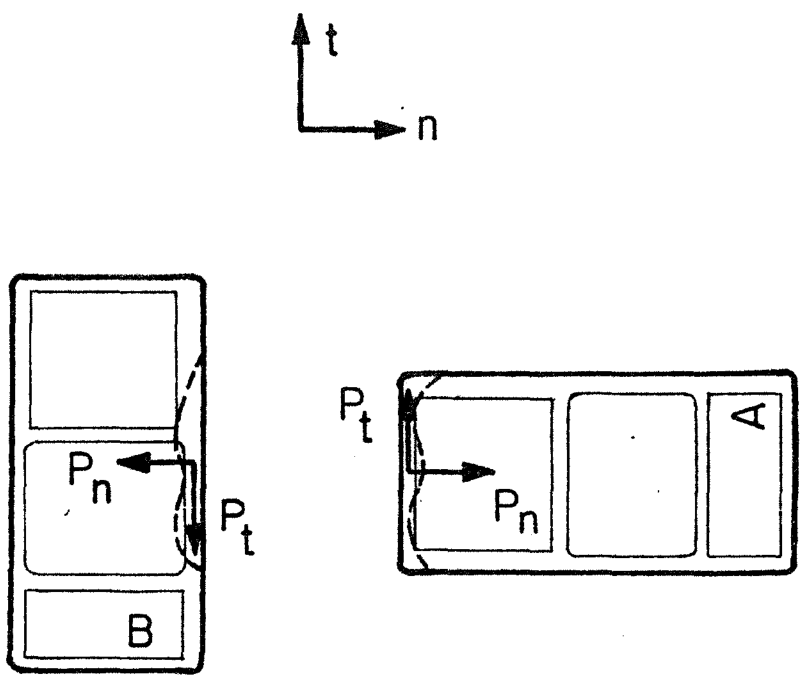

Figure 2. Free Body Diagrams of Vehicles
for Direct Colinear Impact.

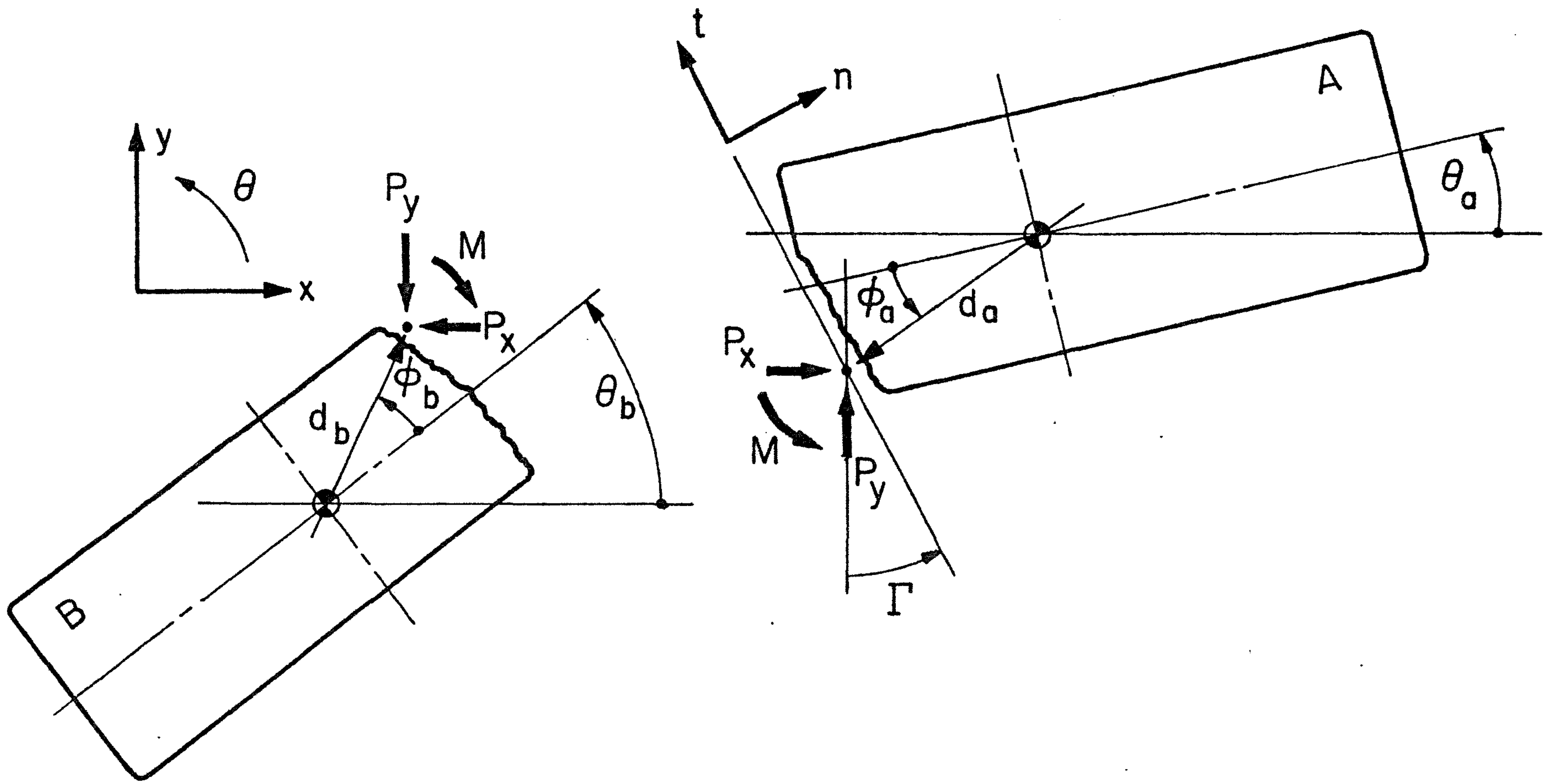

Figure 3. Free Body Diagrams for Planar Impact Model.

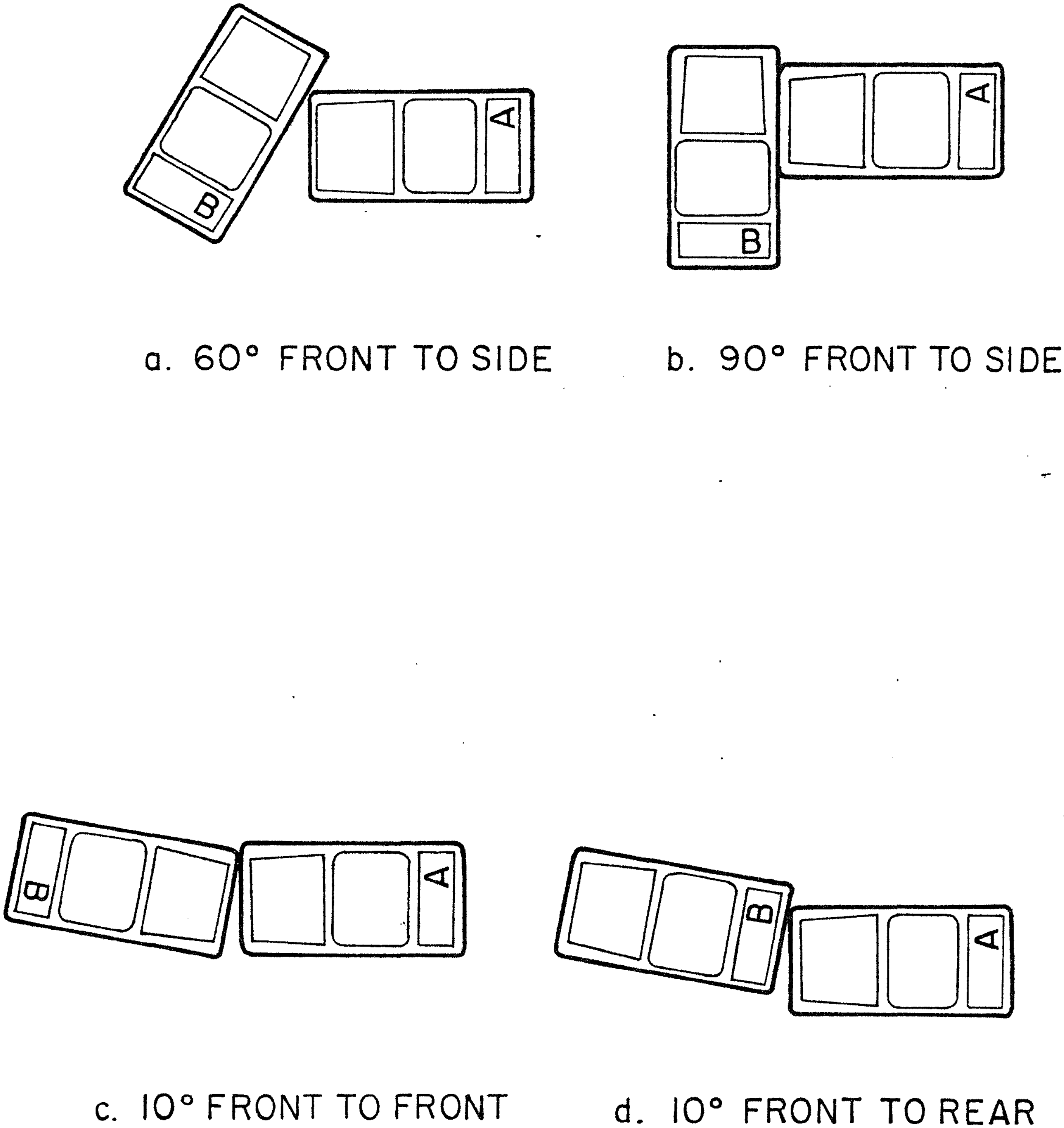

Figure 4. Experimental Collision Configurations.

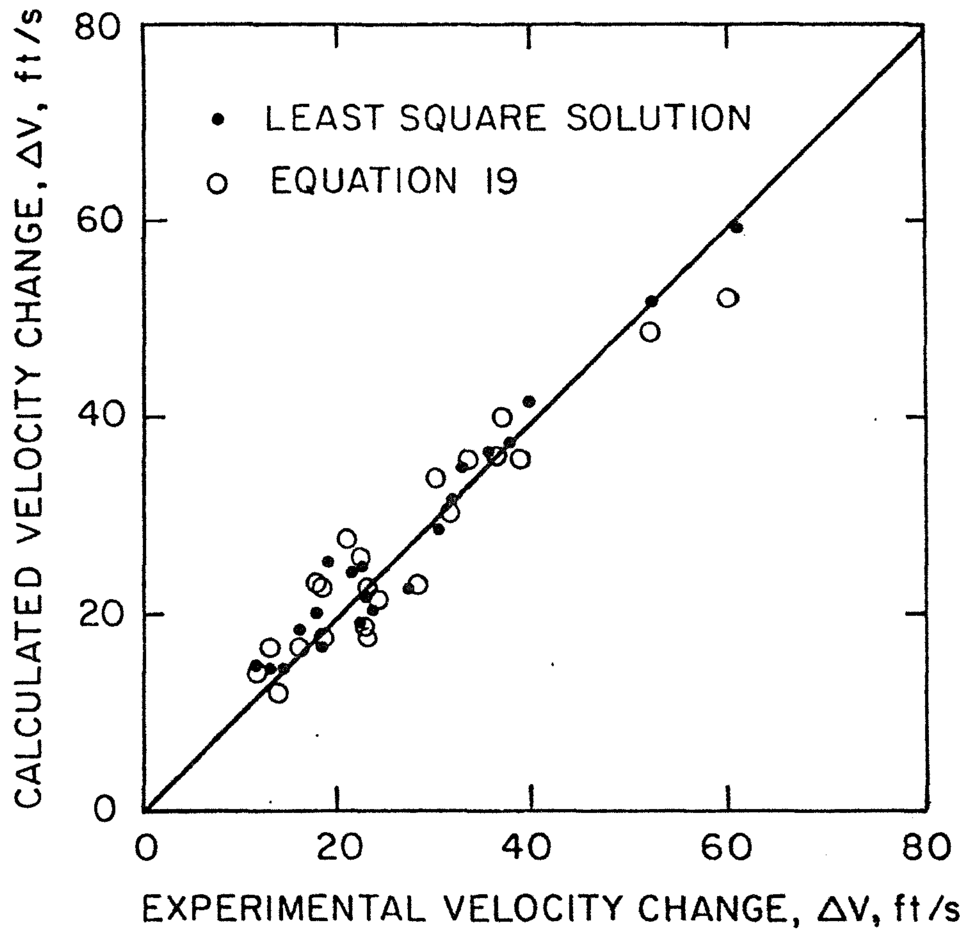

Figure 5. Comparison of Calculated Values of Velocity Change to Experimental Values.