Calculation of Crush Stiffness Coefficients from Two-Vehicle Collisions

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INTRODUCTION:

The CRASH3 (1982) method uses a *damage algorithm* that permits the energy loss to be estimated from the damage profiles of 2 vehicles involved in a collision. The estimation procedure uses *crush stiffness coefficients* for the appropriate region of crush (front, rear, side) for the specific vehicles. These crush stiffness coefficients are determined experimentally from crash tests. This paper discusses the calculation of the crush stiffness coefficients using data from controlled crash tests. In particular, it addresses a problem that arises when the crashes are of a vehicle-to-vehicle type.

Two formulations exist of the basic relationships between crush and energy. One is based on an assumed linear relationship between force and residual crush and is used in CRASH3. Another, developed by Prasad (1992) assumes a linear relationship between the square root of the energy loss and crush. The methods are equivalent, a feature discussed and exploited in this paper.

At least 3 types of collisions typically are used in the automotive industry and can provide data to determine the parameters of the relationship between energy loss and residual crush. These are a moving vehicle into a fixed, rigid barrier, a moving rigid barrier into a vehicle and a moving vehicle into a stationary vehicle. Deformable barriers are used also. The method developed in this paper assumes that experimental crush is direct contact damage, distributed fairly uniformly over the crush surface. So a single average value of crush is reasonably descriptive of the crush. This, of course, eliminates the use here of offset collision data. Actual measurement of crush is not discussed here. Readers should refer to the protocol discussed in the paper by Tumbas and Smith (1988).

NOTATION:

A	CRASH3 crush stiffness coefficient,
B	CRASH3 crush stiffness coefficient,
C	residual crush, measured $\perp$ and relative ,to the original undeformed vehicle surface
d_{0i}	crush stiffness coefficient, zero crush value, vehicle i,
d_{1i}	crush stiffness coefficient, slope value, vehicle i,
E	energy loss,
e	coefficient of restitution,
F	force,
G	CRASH3 crush stiffness coefficient,
g	acceleration of gravity, g = 32.2 ft/sec^2 = 9.81 m/s^2,
m_i	mass of vehicle i, $m_1 = W_i/g$,
$\bar{m}$	$m_1 m_2/(m_1 + m_2)$,
v	initial velocity,
W	weight,
w_i	crush width of vehicle i.

FIXED RIGID BARRIER TESTS, CRASH3 FORMULATION:

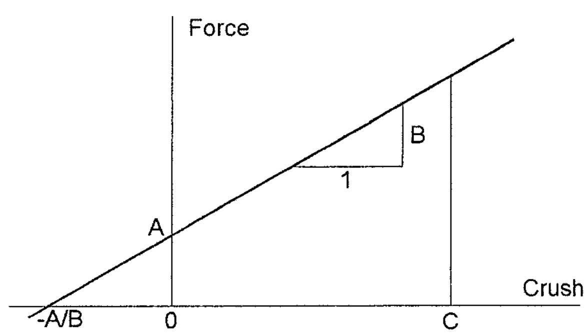

Figure 1. Relationship between the force and residual crush used by CRASH3 to determine the energy loss due to crush.

Theory: In the case of a vehicle moving into a fixed rigid barrier, the CRASH3 formulation of damage assumes that a linear force-crush relationship exists for the force between the vehicle and barrier such as illustrated in Fig 1. The equation is usually expressed as the force acting over a structural segment of width w, and is written as:

$$F/w = A + B\,C \qquad (1)$$

From algebra, A is the intercept and B is the slope of the line A + BC. The quantities A and B are the crush stiffness coefficients. It is important to note that Eq 1 does not represent the force that develops over the front of a vehicle during a test but rather represents the relationship between the force and the *residual* crush measured over a number of tests conducted at different impact speeds to produce different levels of crush. Such curves represent a structural property of a particular vehicle. Using mechanics and assuming that the force is fully plastic with no restitution (e = 0), the CRASH3 manual (1982) gives the energy absorbed per unit width in this process as:

$$E/w = AC + \tfrac{1}{2}BC^2 + G \qquad (2)$$

where E is the energy loss and $G = A^2/2B$. Notice that for zero crush, Eq 2 indicates that energy loss is not zero, but is $A^2/2B$. This is because of the existence of impact absorbing bumpers for front and rear collisions. It is typical to assume that the energy loss for C = 0 is that from a speed of 2½ to 10 mph with 4 to 5 mph frequently observed in practice. In some collisions and for some vehicles, the collision is not perfectly plastic and restitution can occur. This is not discussed here; see McHenry and McHenry (1997) and Neptune (1998).

Example: An example of the use of Eq 2 for the recovery of the coefficients A and B uses the data in Table 1 taken from the Accident Reconstruction Journal (1992) from a frontal barrier test of a 1990 Ford Mustang convertible:

Table 1, Barrier Test Data

Test Wt	Test v, mph	Avg Crush, in	NHTSA Report #
3865	35.0	17.8	79-17-N01-546

Zero crush at 7 mph and uniform crush over a full width of w = 69 inches is assumed. The energy loss at zero crush is the kinetic energy of the vehicle at this speed, $E = \tfrac{1}{2}mv^2 = 6{,}326$ ft-lb. Putting C = 0 and using this value of E allows Eq 2 to be solved for $G = A^2/2B = 1100$ lb. This, in turn, gives $B = A^2/2200$. This can be substituted into Eq 2 to provide an equation that involves A and A^2. Using the test data at 35 mph (51.3 ft/sec), E = 158,148 ft-lb and C = 17.8/12 = 1.48 ft, Eq 2 can be solved for A and gives a value of A = 5934 lb/ft. Since $B = A^2/2200$, then $B = 16002$ lb/ft^2.

FIXED RIGID BARRIER TESTS, DIRECT ENERGY FORMULATION:

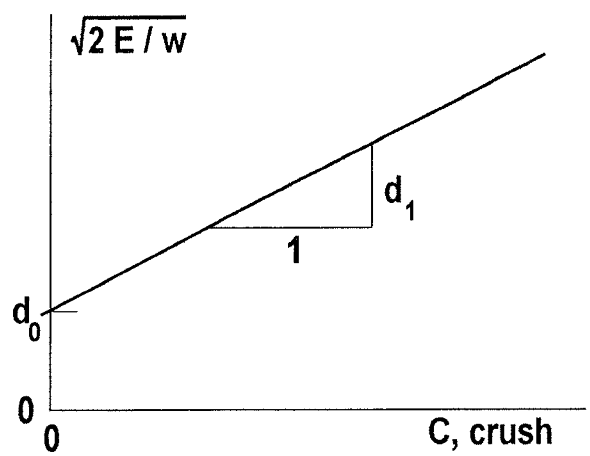

Figure 2. Relationship between energy loss, E, and residual crush, C, for vehicle collisions.

Theory: In this approach by Prasad (1992), it is assumed that the square root of the energy loss is proportional to residual crush as illustrated in Fig 2. Here, the equation for energy loss is written as:

$$\sqrt{2E/w} = d_0 + d_1 C \qquad (3)$$

where d_0, the intercept value and d_1, the slope are the crush stiffness coefficients of this approach. This formulation is entirely equivalent to that of

CRASH3 and there is a direct relationship between coefficients given by:

$$B = d_1^2 \text{ and } A = d_0 d_1 \quad (4a)$$

and

$$d_1 = \sqrt{B} \text{ and } d_0 = A/d_1 \quad (4b)$$

Example: The same example as above is worked using Eq 3. For a 7-mph zero crush condition © = 0), $d_0 = 46.91\ \sqrt{lb}$. Using 35 mph and C = 1.48 ft allows a direct solution for $d_1 = 126.5\ \sqrt{lb}/ft$. Use of Eq 4 shows that the results are equivalent to the values obtained from the previous example.

VEHICLE-TO-VEHICLE AND BARRIER-TO-VEHICLE TESTS:

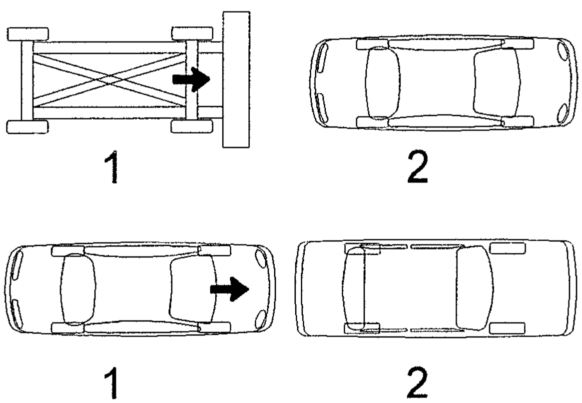

Figure 3. Types of collisions discussed in this paper showing the striking vehicle (veh 1) and the struck vehicle (veh 2) for barrier-to-vehicle collisions and vehicle-to-vehicle collisions.

Theory: Attention now is turned to controlled collisions between a moving rigid barrier or a moving vehicle (striking vehicle, veh 1) into a stationary vehicle (struck vehicle, veh 2). These are illustrated in Fig 3. A problem that arises with two vehicle tests is that the total kinetic energy loss of the collision can be determined easily but the crush energy for each vehicle is unknown. In fact, vehicle 2 ends up gaining kinetic energy as a result of the collision even though its crush contributes to the energy loss. For a direct collision, the energy loss is given by (Brach, 1991):

$$E = \tfrac{1}{2}\bar{m}(1 - e^2)(v_2 - v_1)^2 \quad (5)$$

where e is the coefficient of restitution and v_1 and v_2 are the initial speeds.

Let the crush energy loss of veh 1 and veh 2 be E_1 and E_2, respectively. From Eq 3,

$$\sqrt{2E_1/w_1} = d_{01} + d_{11}C_1 \quad (6a)$$

and

$$\sqrt{2E_2/w_2} = d_{02} + d_{12}C_2 \quad (6b)$$

Due to the nature of contact the widths must be the same so $w_1 = w_2$. Furthermore, from Newton's third law, it is known that the intervehicular contact forces must be equal and opposite. For the test values of crush C_1 (veh 1) and C_2 (veh 2), Eq 1 requires that:

$$A_1 + B_1C_1 = A_2 + B_2C_2 \quad (7)$$

Using Eq 4, this can be written as

$$d_{11}(d_{01} + d_{11}C_1) = d_{12}(d_{02} + d_{12}C_2) \quad (8)$$

Using Eq 3, an energy ratio can be developed such that

$$\frac{2E_1}{2E_2} = \left(\frac{d_{01} + d_{11}C_1}{d_{02} + d_{12}C_2}\right)^2 = \left(\frac{d_{12}}{d_{11}}\right)^2 \quad (9)$$

where Eq 8 was used. Since $E = E_1 + E_2$,

$$E_1 = \frac{E}{1 + (d_{11}/d_{12})^2} \quad (10a)$$

and

$$E_2 = \frac{E}{1 + (d_{12}/d_{11})^2} \quad (10b)$$

The overall objective is to find values of d_{01}, d_{02}, d_{11} and d_{12}. The first two, d_{01} and d_{02}, can be found from the zero crush speed using Eq 6. Equation 6 cannot be used to solve for d_{11} and d_{12} because E_1 and E_2 are not known. A combination of Eq 6 and 10 can be solved using an iteration process and can be done conveniently with a spreadsheet. The process is to (a), estimate the ratio d_{12}/d_{11} and calculate E_1 and E_2 using Eq 10; (b), calculate d_{11} and d_{12} from Eq 6; (c), compute the ratio d_{12}/d_{11} and go back to step (a) and repeat this process until the ratio stabilizes to a single value.

Example: Table 2 shows the results of a spreadsheet calculation for a front-to-rear, vehicle-to-vehicle collision. Quantities in italics are supplied as input information; all others are calculated. Zero crush speeds of 5 mph were used for both vehicles. The ratio of d_{12}/d_{11} stabilizes at a value of 2.197 and produces the values of d_{11} and d_{12} as listed at the bottom of the Table, shown with various units.

BARRIER-TO-VEHICLE AND VEHICLE-TO-VEHICLE CRUSH STIFFNESS COEFFICIENTS:

A question that often arises is whether barrier-to-vehicle and vehicle-to-vehicle tests provide the same values of crush stiffness coefficients. It may be expected that a flat rigid barrier may cause significantly different deformation than a striker vehicle, especially if the struck vehicle is impacted in the rear and is a station wagon, van or sport utility vehicle. The height of the direct damage differs. Data from 15 vehicle-to-vehicle tests and 8 barrier-to-vehicle tests are listed in Table 3 where the struck vehicles were station wagons from the late 80's produced by a single manufacturer. Figure 4 shows the data plotted for visual comparison. A great deal of scatter exists and the values from the different test types overlap. However, it is quite apparent that the levels of the crush stiffness coefficients obtained from the 2 type of tests differ. In fact a statistical analysis indicates that the type of test makes a significant difference.

Table 3, Values of the Crush Stiffness Coefficient, d_1, Determined from Experimental Data

Vehicle-to-Vehicle Collisions:					
60.03	42.29	36.95	70.88	52.10	64.37
45.39	40.47	51.65	43.79	46.73	47.67
45.78	40.95	40.34			
Barrier-to-Vehicle Collisions:					
67.16	62.27	68.34	97.19	78.19	59.56
80.21	75.06				

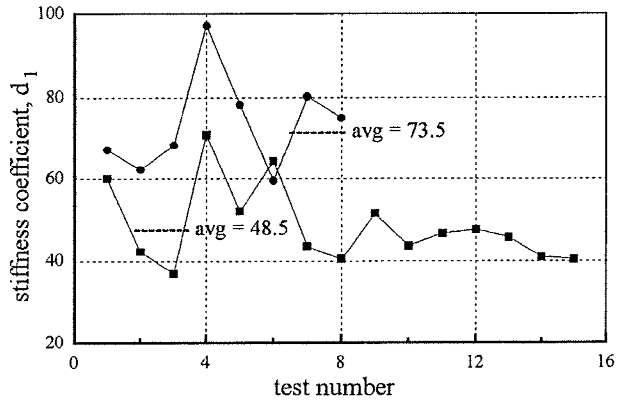

Figure 4. Crush stiffness coefficients, d_1 for station wagons; data are from front-to-rear, vehicle-to-vehicle collisions (squares) and barrier-to-rear, collisions (circles).

CONCLUSIONS:

It is possible to determine the crush stiffness coefficients from vehicle-to-vehicle barrier tests for use with CRASH3 using an iterative procedure. Values from rigid barrier tests can differ significantly from vehicle-to-vehicle tests, at least for certain model vehicles.

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Table 2. Spreadsheet Solution of Equations for Crush Coefficients d_{11} and d_{12}
(Quantities in italics indicate input information)

		MOMENTUM SOLUTION		ITERATION	
m_1:	109.9	***Coeff Restitution, e:***	0.00	***ratio, d11/d12:***	2.1968
m_2:	80.0	initial energy, ft-lb:	277801.1	E1, ft-lb:	20094.4
mbar:	46.3	final energy, ft-lb:	160732.8	E2, ft-lb:	96974.0
		energy loss, ft-lb:	117068.3	E = E1 + E2:	117068.3
				d11, √lb/ft:	91.2
STRIKER (VEH 1):				d12, √lb/ft:	41.5
Wt, Vehicle 1, lb:	3539.0	delta v, ft/s:	-30.0	new ratio, d11/d12:	2.1969
Initial speed, ft/s:	71.1	final speed, ft/s:	41.1		
Initial Speed, mph	48.5	final speed, mph:	28.0		
Crush 1, in:	8.3	energy loss, ft-lb:	184802.8	coefficient d0, √lb:	33.6
Crush 2, in:	5.9			E1, energy check	20094.4
Avg Crush, in :	7.1				
zero crush , mph:	5.0				
crush width, in:	63.0				
STRUCK (VEH 2):					
Wt, Vehicle 2, lb:	2577.6	delta v, ft/s:	41.1		
Initial speed, ft/s:	0.0	final speed, ft/s:	41.1		
Initial Speed, mph	0.0	final speed, mph:	28.0		
Crush 1, in:	47.0	energy loss, ft-lb:	-67734.5	coefficient d0, √lb:	28.6
Crush 2, in:	47.6			E2, energy check	96974.0
Avg Crush, in :	47.3				
zero crush , mph:	5.0				
crush width, in:	63.0				
√lb d_{01}:	**33.6**	**√lb d_{01}:**	**33.6**	**√N d_{01}:**	**70.8**
√lb/ft d_{11}:	**91.2**	**√lb/in d_{11}:**	**7.6**	**√N/m d_{11}:**	**630.8**
√lb d_{02}:	**28.6**	**√lb d_{02}:**	**28.6**	**√N d_{02}:**	**60.4**
√lb/ft d_{12}:	**41.5**	**√lb/in d_{12}:**	**3.5**	**√N/m d_{12}:**	**287.1**