Design of experiments and parametric sensitivity of planar impact mechanics

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Abstract

Design of Experiments (DOE) is a method for organizing and analyzing multivariable experiments in an efficient and rigorous manner. Applications of DOE are not restricted to physical tests, however. Results from computer "experiments" can also be analyzed to determine the sensitivity and significance of changes in input variables on the output (response) of any computerized process model. This work includes a brief coverage of some of the fundamentals of DOE, its terminology and some shortcuts in its application.

Examples from the field of accident reconstruction are presented that contain a large number of input variables. Under rather broad conditions and meeting proper assumptions, Planar Impact Mechanics (PIM) can be used to analyze collisions between two vehicles. Given each vehicle's physical properties, the collision configuration and information about the contact process (coefficient of restitution and tangential impact coefficient), final impact velocities, collision kinetic energy loss, ΔV values and other results can be calculated algebraically when the initial velocities are specified. DOE is applied to the planar impact collision model for two collision configurations as a method to examine the uncertainty of energy loss and ΔV of typical collisions relative to the input variables.

Introduction

This paper covers two topics. The first is a short introduction to a method referred to as the Design of Experiments (DOE). The second is the topic of Planar Impact Mechanics (PIM) as applied to vehicle accident reconstruction. These are combined through the application of DOE to investigate the uncertainty of the results of reconstructions related to changes of various input quantities. Another method, differential variations, can be used in a similar way. It has been applied recently to analyze the uncertainty of crash test data using PIM[1].

DOE[2,3,4,5] is a highly efficient method used to organize and analyze the effect of changes of the response of a process (or system) to the factors (input parameters) that control the process output. The method is known and described using different names[5] including the Design and Analysis of Experiments and Factorial Design. The acronym DOE is used throughout to indicate the Design of Experiments and Factorial Design. DOE is ordinarily intended to be used with experimental measurements. However, the method can be applied equally well to computerized experiments, that is, the use of computer simulations. This is the approach used here.

PIM[6,7,8,9,10,11,12,13] is an acronym representing the set of algebraic equations that results from the application of Newton's Second Law of motion in the form of impulse and momentum that can be used to analyze a collision (impact) of two rigid bodies in a plane. There are 13 input parameters to a collision analysis including impact coefficients, vehicle masses and inertias, impact center (damage area) coordinates and vehicle orientations. In addition, six initial velocity components are needed for a total of 13 input parameters for an analysis of each impact.

In this work the two methods, DOE and PIM, are combined and used in such a way as to examine two specific process responses. These are the loss, T_L, of the total collision kinetic energy, T, and the magnitude of the velocity change, ΔV, of one of the two colliding vehicles. The uncertainty of each of eight vehicle-crash parameters is found and ranked.

Design of Experiments with Two Factors

This section contains an introduction to the basic concepts of DOE using a simple example. Certain terminology and concepts are required in DOE (Factorial Design). Consider a process as illustrated in Figure 1 whose output or response, y, depends on k

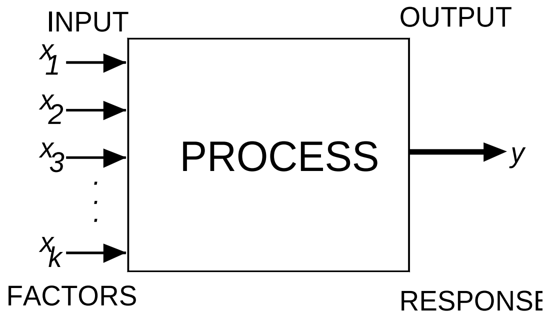

Figure 1. Schematic Diagram of a Process Treatable by the Design of Experiments.

parameters or factors, x_1, x_2, x_3, ..., x_k, The process is implemented or run repeatedly for different, preselected values or levels of the factors, where the *j*-th run or factor combination produces the *j*-th response value, y_j. The primary function of DOE is to determine a quantitative measure of the effect of each of the factor changes on the response. The response must be quantitative or measurable. The factors can be quantitative but also can be attribute variables (such as fast or slow, red or green, etc.). In the coverage here, each factor will be allowed to take on only two values: a low level (-), and a high level (+). The low and high levels of the factors are selected by the experimenter or analyst to represent a practical range of values (viewed from the application and context of the process) large enough to have an influence, yet small enough to determine the local behavior of the process. It is not uncommon to first carry out an *exploratory* DOE to establish ranges of variables followed by another DOE, based on the results of the first, for a more refined analysis. In the case of attribute variables the choice of low and high is arbitrary. In the case of quantitative variables the choice usually is intuitive, but still remains arbitrary since the results depend upon the choices.

Table 1. Experimental Layout for Two Factors Each at Two Levels

	x_1	x_1^+
x_2^-	y_1	y_2
x_2^+	y_3	y_4

Consider a simple case with two factors, k = 2: x_1, with low and high values x_1^- and x_1^+, and x_2, with low and high values x_2^- and x_2^+. There are $n = 2^k$ = 4 runs (possible combinations of response values, y_j), for the low and high values of the factors as shown in Table 1. Corresponding estimates of the main effect, *ME*, of changes in the response are made using the differences in the average response at each of the different levels. For factor x_1,

$$ME_{x_1} = \frac{1}{2}[(y_2 - y_1) + (y_4 - y_3)] \tag{1}$$

And for factor x2,

$$ME_{x_2} = \frac{1}{2}[(y_3 - y_1) + (y_4 - y_2)] \tag{2}$$

A measure of the interaction effect of the two factors can be found by taking the difference in the diagonal values:

$$ME_{x_1 x_2} = \frac{1}{2}[(y_1 + y_4) - (y_2 + y_3)] \tag{3}$$

The overall (average) process response is

$$ME_{avg} = \frac{1}{4}[y_1 + y_2 + y_3 + y_4] \tag{4}$$

Table 2. Standard Experimental Layout for Two Factors and Two Levels

run	x_1	x_2	x_1x_2	y
1	-	-	+	y_1
2	+	-	-	y_2
3	-	+	-	y_3
4	+	+	+	y_4

This analysis can be placed into a convenient, more standard, format as shown in Table 2. The main effect of each factor and interaction becomes the inner product of the sign in each factor column with the corresponding value in the response (*y*) column, each divided by 2^{k-1}. For example, it can be seen that for k = 2,

$$ME_{x_1} = \frac{1}{2^{k-1}}[-y_1 + y_2 - y_3 + y_4] \tag{5}$$

is the same as Eq 1.

Accident Reconstruction Example

The process of using DOE is examined here through the use of a simple skid-to-stop formula used to determine the initial speed, v_0, of a vehicle undergoing constant deceleration, *fg*, over a distance, *d*.

$$v_0 = \sqrt{2fgd} \tag{6}$$

where the initial speed v_0 becomes the response and the frictional drag coefficient, *f*, is the first factor ($x_1 = f$) and the distance of skid, *d*, is the second factor ($x_2 = d$). To investigate the uncertainty of the initial speed of a vehicle consider the following low and high values of the factors: $x_1^- = f^-$ = 0.6, $x_1^+ = f^+$

= 0.8, $x_2^- = d^- = 32$ m and $x_2^+ = d^+ = 34$ m. Table 3 shows the experimental layout with response values computed from Eq 6.

Table 3. Standard Experimental Layout for Example 1 with Response Values

run	x_1	x_2	x_1x_2	$y = v_0$
1	-	-	+	y_1 = 19.41 m/s
2	+	-	-	y_2 = 22.41 m/s
3	-	+	-	y_3 = 20.00 m/s
4	+	+	+	y_4 = 23.10 m/s

The main effects can be computed using the inner products as indicated by Eq 5. That is:

$$ME_{x_1} = \frac{1}{2}[-19.41 + 22.41 - 20.00 + 23.10] = 3.05\ m/s$$

$$ME_{x_2} = \frac{1}{2}[-19.41 - 22.41 + 20.00 + 23.10] = 0.64\ m/s$$

$$ME_{x_1x_2} = \frac{1}{2}[+19.41 - 22.41 - 20.00 + 23.10] = 0.05\ m/s$$

The main effect (ME) values are the uncertainties of the (skid-to-stop) process to the changes (variations) of the frictional drag coefficient values and the skid distance values. That is, the initial speed is 3.05/0.64 = 4.8 times more sensitive to the ± 0.10 variation in *f* than to the ± 2.0 m variation in *d*. The value of the interaction effect $ME_{x_1x_2} = 0.05$ is relatively small and so is insignificant. Because of the square root sign (Eq 6), the skid-to-stop process is nonlinear. This has no effect on the ease or difficulty of estimating sensitivities using DOE. Furthermore, the results from this example are virtually identical to using the method of differential variations[13] to estimate uncertainty of the calculation of initial speed. This is because both the DOE method and the method of differential variations linearize the process response.

Design of Experiments with *k* Factors and with *p* Half Fractions

The experimental layout from the above simple DOE process can be generalized to take *k* factors into account. The patterns for the plus and minus signs extend quite simply as is shown for 4 factors in Appendix B. Signs associated with interactions are the products of the signs associated with each of the individual factors. With two levels and *k* factors, there are 2^k total combinations, or runs.

The above process can also be modified to take into account *k* factors (each with two levels) but where not all possible 2^k factor combinations are considered. If $n = 2^k$ is large, the number of runs or experiments, (or calculations) can be reduced by *p* half fractions with little loss in effectiveness by taking into account that with many factors the effects of high order interactions are likely to be negligible. The number of runs becomes $n = 2^{k-p}$. This reduced DOE process[4,5] is called a fractional factorial design. In all cases, the Main Effects (ME) are estimated using

$$ME_{x_j} = \frac{1}{2^{k-p-1}}\left[\sum_{i=1}^{2^{k-p}} \pm y_i\right] \qquad (7)$$

where the ± multipliers of the response values are determined by the *j*-th factor column of the standard experimental layout matrix (for example, see Appendix B). The selection of which runs are included and which are dropped for a fractional factorial can affect which interactions are included (confounded) and which are not. This selection process requires forethought and planning[2,3,5] and is not discussed here.

Planar Impact Mechanics

The use of impact mechanics, in one form or another, to model the collision of two vehicles has been studied and used for many years. Only a summary of some of the main points is included here. Detailed discussions exist[13].

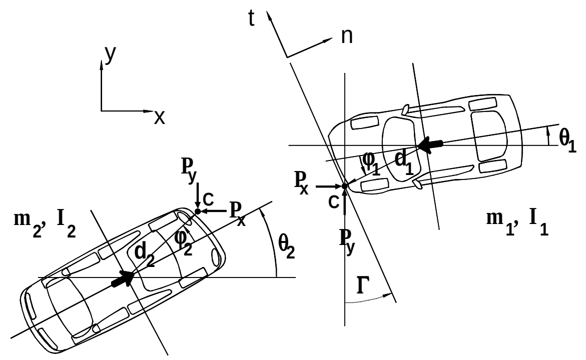

Figure 2. Free body diagrams and coordinates of impacting vehicles.

Figure 2 shows free body diagrams of two colliding vehicles. The *x-y* axes are fixed to the ground. The relative orientation of the normal and tangential, *n-t*, axes is through the angle Γ defined by a planar intervehicular crush surface. Following the usual assumptions (see Appendix A) for the use of impulse and momentum and from Newton's laws, the equations for the changes in the velocity components can be written in closed form and are given in Appendix A. Note that this problem is posed and solved as an initial value problem, that is, the initial velocities are known and the final velocities are calculated. The equations in Appendix A are the solution equations of the planar impact mechanics problem. These are the analytical solution of the equations of motion; a numerical solution is not necessary.

An important result of the analytical solution equations is the ability to obtain the expressions for the collision energy loss and the Δ*V* of each vehicle. If kinetic energy is given the symbol *T*, the energy loss is T_L; the velocity changes of vehicles 1 and 2 are ΔV_1 and ΔV_2. All of the variables that appear in these equations are known and are defined in Appendix A. Note that two impact coefficients, the coefficient of restitution, *e*, and the impulse ratio, *μ*, are part of the planar impact mechanics model. The definitions of both *e* and *μ* are given in Appendix A. The coefficient of restitution, *e*, is based on the normal (perpendicular) contact process over the intervehicular surface and *μ* is associated with the tangential contact process over the intervehicular surface. Two DOE response variables are used in this paper. They are the collision energy loss, T_L, and the velocity change, Δ*V* of Vehicle 2. The energy loss is given by Eq 8 and the velocity change by Eq 9. When modeling collisions, the value of *e* and the value of *μ* must be known. For all collisions other than sideswipes, the impulse ratio takes on its critical value[13], μ_0, (see also Appendix A). Recall that the critical impulse ratio, $\mu = \mu_0$ corresponds to the condition that the final relative tangential velocity at the impact center is zero, that is, sliding at the intervehicular surface ends at or before separation. Finally, it is important to note that the critical value of *μ* is dependent on the initial conditions (initial velocities) and the collision configuration.

$$T_L = \frac{1}{2}\bar{m}qv_{rn}^2(1+e)\left[2+2\mu r-(1+e)q\left(1+\mu^2+\frac{\bar{m}d_e^2}{m_1k_1^2}+\frac{\bar{m}d_f^2}{m_2k_2^2}\right)\right] \quad (8)$$

$$m_i\Delta V_i = \bar{m}q(1+e)\sqrt{1+\mu^2}\,[(v_{2n}-d_a\omega_2)+(v_{1n}+d_c\omega_1)],\ i=1,2 \quad (9)$$

Applications of Design of Experiments to Planar Impact Mechanics

Two collision configurations are used to illustrate the application of DOE to crash reconstruction. The first is a 60º front-to-side collision shown in Figure 3 where both vehicles have an initial velocity. The second is a 90º collision shown in Figure 4, where Veh 2 initially is stationary.

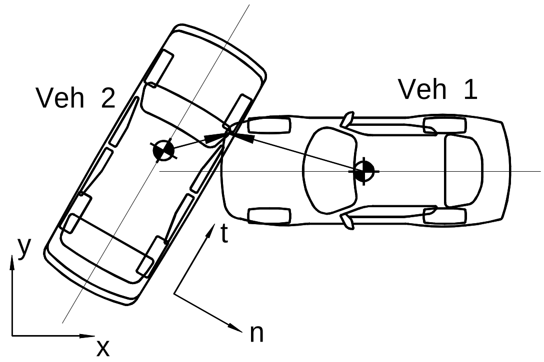

Figure 3. Vehicle configuration for 60º Front-to-Side Collision.

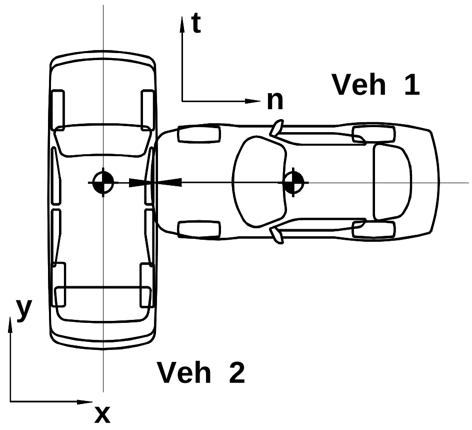

Figure 4. Vehicle Configuration for 90º Front-to-Side Collision.

For each of these collisions, two different DOE response variables are considered:

1. The first is the collision energy loss, $y = T_L$, as a fraction of the initial kinetic energy,
2. The second is the magnitude of the velocity change of Veh 2, $y = \Delta V_2$.

The factors, x_1, x_2, . . ., x_8 and their high and low values are:

1. mass, $x_1 = m_1$, of Veh 1, ± 5%,
2. yaw inertia, $x_2 = I_1$, of Veh 1, ± 5%,
3. distance, $x_3 = d_1$, from the mass center of center of Veh 1 to the impact center, ± 5%
4. angle, $x_4 = \varphi_1$, of the line from the mass center of Veh 1 to the impact center, ± 5º
5. orientation, $x_5 = \theta_1$, of Veh 1, ± 5º,
6. collision coefficient of restitution, $x_6 = e$, $e = 0.1 \pm 0.1$,
7. collision critical impulse ratio, $x_7 = \mu_0$, 90% μ_0 and 100% μ_0,
8. angle, $x_8 = \Gamma$, of the intervehicular crush surface ± 5º.

The geometric factors/quantities are illustrated in Figure 1. Their low and high values are (arbitrarily) chosen to illustrate their effect on a reconstruction calculation. The values of the variables for the 60° impact case, including initial conditions, are found in Table 4. The values of the variables for the 90° impact case, including initial conditions, are found in Table 5.

60º Front-to-Side Collision

With k = 8 factors, a full DOE would require $2^k = 2^8$ = 256 computed values of the response, one for each of the possible combinations of the factors. To conserve time and effort, p = 4 half fractions of the full design are used here. This allows estimation of all of the main effects of each factor and all first order factor interactions. It requires computation of $2^{k-p} = 2^4$ = 16 values of the response. It also requires a selection of which 16 combinations of the full 256 are chosen. The strategies of the selection process is beyond the scope of this paper and a choice is presented without comment. Consider first the collision energy loss as the response $y = T_L$. Table 6 shows the combination of low and high values of the factors and initial conditions (velocity components) for each of the 16 runs. These provide the response values, namely 16 values of T_L (listed as a percentage of the initial system kinetic energy). The corresponding Main Effects of the factors and their first order interactions are calculated using Eq 7 and are listed in Table 7. Main Effects (ME) are both negative and positive. A negative ME means that when the factor is increased, the response decreases. A positive ME means that when the factor is increased, the response increases. The ME with the largest negative value is for the distance, d_1, from the mass center of Veh 2 to the impact center, ME_d = -1.675. This indicates that energy loss is relatively sensitive to this factor. The factor with next highest negative ME is the mass, m_1, ME_m = -1.200. At the positive end, the factor with the largest ME is associated with the vehicle orientation angle, θ_1, ME_θ = 5.800. The next two highest main effects are ME_φ = 5.775 and ME_μ = 0.775. The interpretation is that the kinetic energy loss of a collision of this type is most sensitive to the location of the impact center, the vehicle mass and the vehicle orientation. All other ME values (including factor interactions) are relatively small.

A graphical means of displaying the significance of the factors exists exists[4] which is to plot the ordered ME values on normal probability paper. Figure 5 is such a plot. It is seen that the 4 highest ME values stand out from the remaining ones which appear to lie along a nearly straight line.

Table 4. Low and High Values of the Eight Factors, 60º Collision

	Vehicle 1 low (-)	Vehicle 1 nominal	Vehicle 1 high (+)	Vehicle 2	
m_1 (± 5%)	1506	1581	1660	m_2	1166 kg
I_1 (± 5%)	3100	3255	3418	I_2	1958 kg-m^2
d_1 (± 5%)	1.98	2.08	3.13	d_2	1.06 m
φ_1 (±5°)	-16.5	-11.5	-6.5	φ_2	-45°
θ_1 (±5°)	-5.0	0	5.0	θ_2	60°
e (±0.1)	0.0	0.1	0.2		
μ (±5% μ_0)	90%	95%	100%		
Γ (±5°)	-35	-30	-25		

Initial Conditions

v_{1x} = -13.41 m/s	v_{2x} = 6.71 m/s
v_{1y} = 0	v_{2y} = 11.62 m/s
ω_1 = 0	ω_2 = 0

Table 5. Low and High Values of the Eight Factors, 90º Collision

	Vehicle 1 low (-)	nominal	high (+)	Vehicle 2	
m_1 (± 5%)	1506	1581	1660	m_2	1166 kg
I_1 (± 5%)	3100	3255	3418	I_2	1958 kg-m^2
d_1 (± 5%)	1.88	1.98	2.08	d_2	0.76 m
φ_1 (±5°)	-5.0	0.0	5.0	φ_2	-45°
θ_1 (±5°)	-5.0	0	5.0	θ_2	90°
e (±0.1)	0.0	0.1	0.2		
μ (±5% μ_0)	90%	95%	100%		
Γ (±5°)	-5.0	0.0	5.0		

Initial Conditions

v_{1x} = -13.41 m/s	v_{2x} = 0 m/s
v_{1y} = 0	v_{2y} = 0 m/s
ω_1 = 0	ω_2 = 0

Table 6. Response Values for the 60º Front-to-Side Collision. Collision Energy Loss, $y = T_L$ expressed as a Percentage of the Total System Initial Kinetic Energy and $y = \Delta V_2$, m/s.

run	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Energy Loss Response	Velocity Change Response
1	-	-	-	-	-	-	-	-	y_1 = 52.3 %	y_1 = 8.87
2	+	-	-	-	+	+	+	-	y_2 = 62.1 %	y_2 = 10.79
3	-	+	-	-	+	+	-	+	y_3 = 57.4 %	y_3 = 10.67
4	+	+	-	-	-	-	+	+	y_4 = 58.2 %	y_4 = 12.07
5	-	-	+	-	+	-	+	+	y_5 = 55.5 %	y_5 = 10.30
6	+	-	+	-	-	+	-	+	y_6 = 56.7 %	y_6 = 11.77
7	-	+	+	-	-	+	+	-	y_7 = 63.4 %	y_7 = 12.25
8	+	+	+	-	+	-	-	-	y_8 = 49.2 %	y_8 = 10.70
9	-	-	-	+	-	+	+	+	y_9 = 64.1 %	y_9 = 10.79
10	+	-	-	+	+	-	-	+	y_{10} = 50.9 %	y_{10} = 10.36
11	-	+	-	+	+	-	+	-	y_{11} = 58.2 %	y_{10} = 10.76
12	+	+	-	+	-	+	-	-	y_{12} = 59.4 %	y_{12} = 11.77
13	-	-	+	+	+	+	-	-	y_{13} = 55.7 %	y_{13} = 9.81
14	+	-	+	+	-	-	+	-	y_{14} = 57.4 %	y_{14} = 10.88
15	-	+	+	+	-	-	-	+	y_{15} = 53.0 %	y_{15} = 10.36
16	+	+	+	+	+	+	+	+	y_{16} = 62.1 %	y_{16} = 13.47

from the remaining ones which appear to lie along a nearly straight line. A straight line on normal probability paper indicates normally (Gausian) distributed random values, implying that the remaining factors and interactions have a small random influence on the response, T_L.

Table 7 and Figure 6 show the ME values of the DOE where the response is the Δ*V* of Veh 2. These show that the velocity change of Veh 2 is most highly influenced by the two impact coefficients, e and μ, the impact orientation of the vehicle, θ_1, and, to a lesser extent, the location of the impact center (d_1 and φ_1).

Table 7. Main Effects, *ME*, for the 60º Front-to-Side Collision. Collision Energy Loss, $y = T_L$ expressed as a Percentage of the Total System Initial Kinetic Energy and $y = \Delta V_2$, m/s.

Factor	$y = T_L$ *ME*	$y = \Delta V_2$ *ME*
e	- 0.450	+0.998
μ	+0.775	+1.059
m	- 1.200	+0.434
I	+0.750	+0.099
d	- 1.675	- 0.236
φ	+5.775	+0.876
θ	+5.800	+0.876
Γ	+0.025	+0.495
e μ	- 0.325	- 0.008
e m	- 0.100	+0.023
e I	+0.150	+0.191
e d	- 0.175	- 0.053
e φ	+0.375	+0.069
e θ	+0.100	- 0.221
e Γ	- 0.075	+0.389
μ m	- 0.175	- 0.053
μ I	+0.375	+0.069
μ d	- 0.100	+0.023
μ φ	+0.150	+0.191
μ θ	- 0.075	+0.389
μ Γ	+0.100	- 0.221
m I	+0.100	- 0.221
m d	- 0.325	- 0.008
m φ	- 0.075	+0.389
m θ	+0.150	+0.191
m Γ	+0.375	+0.069
I d	- 0.075	+0.389
I φ	- 0.325	- 0.008
I θ	- 0.100	+0.023
I Γ	- 0.175	- 0.053
d φ	+0.100	- 0.221
d θ	+0.175	+0.069
d Γ	+0.150	+0.191
φ θ	- 0.175	- 0.053
φ Γ	- 0.100	+0.023
θ Γ	- 0.325	- 0.008

90º Front-to-Side Collision

The same process was carried out for the 90º front-to-side collision illustrated in Fig 4. Full results are given in Table 8 with graphical results displayed in Fig 7 and 8. For this collision configuration, the DOE analysis indicates that the kinetic energy loss is most significantly sensitive to the mass, m_1 and the coefficient of restitution, *e*. Similarly, the velocity change, ΔV_2, is most sensitive to the mass, m_1 and the coefficient of restitution, *e*. Note that an increase in the coefficient of restitution, *e*, decreases T_L but increases ΔV_2. The same trend exists for m_1. The implication for impact analyses is that to get the most accurate results for this type of collision, the values of m_1 and *e* should be as accurate as possible.

Table 8. Main Effects, *ME*, for the 90º Front-to-Side Collision. Collision Energy Loss, $y = T_L$ expressed as a Percentage of the Total System Initial Kinetic Energy and $y = \Delta V_2$, m/s.

Factor	$y = T_L$ *ME*	$y = \Delta V_2$ *ME*
e	-1.675	4.950
μ	-0.050	0.100
m	-2.400	1.250
I	0.025	0.000
d	-0.025	-0.100
φ	0.000	0.000
θ	0.000	0.000
Γ	0.025	0.000
e μ	-0.025	0.050
e m	0.025	0.200
e I	0.000	0.000
e d	-0.600	-0.350
e φ	-0.025	0.000
e θ	-0.025	0.000
e Γ	0.000	0.000
μ m	-0.600	-0.350
μ I	-0.025	0.000
μ d	0.025	0.200
μ φ	0.000	0.000
μ θ	0.000	0.000
μ Γ	-0.025	0.000
m I	-0.025	0.000
m d	-0.025	0.050
m φ	0.000	0.000
m θ	0.000	0.000
m Γ	-0.025	0.000
I d	0.000	0.000
I φ	-0.025	0.050
I θ	0.025	0.200
I Γ	-0.600	-0.350
d φ	-0.025	0.000
d θ	-0.025	0.000
d Γ	0.000	0.000
φ θ	-0.600	-0.350
φ Γ	0.025	0.200
θ Γ	-0.025	0.050

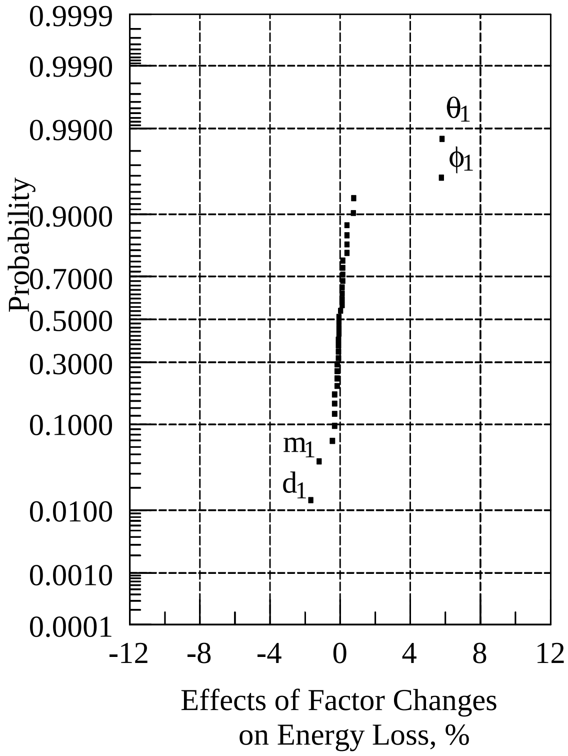

Figure 5. Normal Probability Plot of Main Effects, Kinetic Energy Loss, T_L, 60º Front-to-Side Collision.

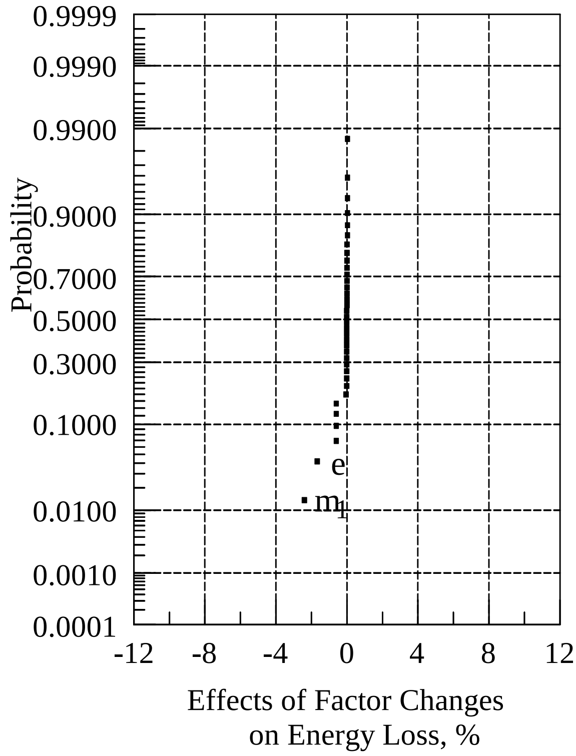

Figure 7. Normal Probability Plot of Main Effects, Kinetic Energy Loss, TL, 90º Front-to-Side Collision.

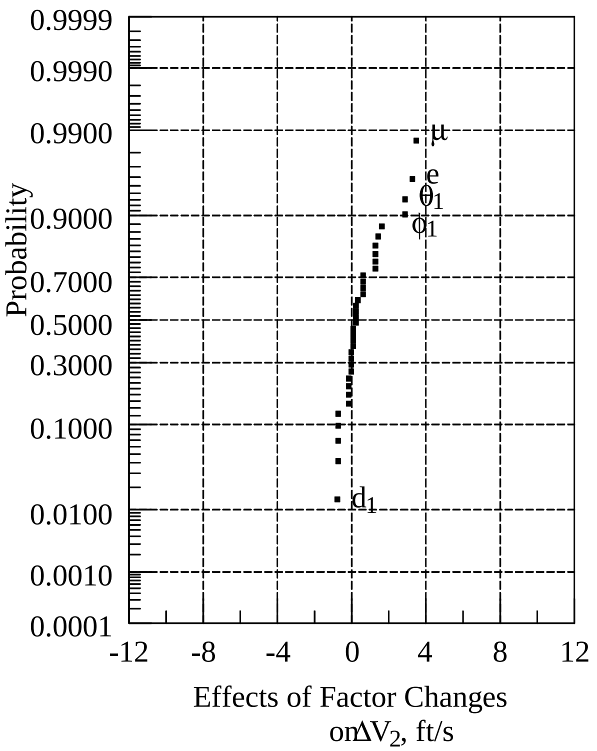

Figure 6. Normal Probability Plot of Main Effects, Velocity Change, ΔV_2, 60º Front-to-Side Collision.

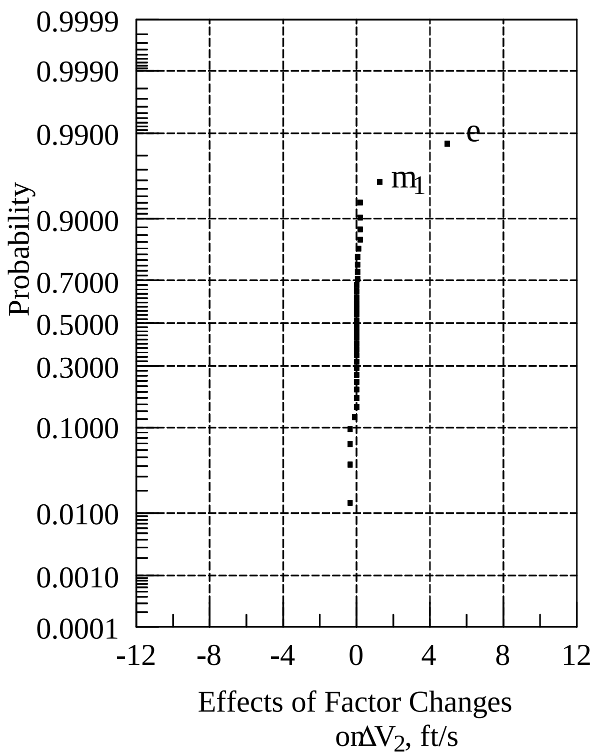

Figure 8. Normal Probability Plot of Main Effects, Velocity Change, ΔV_2, 90º Front-to-SIde Collision.

This collision is very close to being a central collision. For a central impact[13] with v_{2n} = 0 and $\omega_1 = \omega_2 = 0$,

$$T_L \approx \frac{1}{2}\bar{m}(1-e^2)v_{1n}^2 \tag{10}$$

and

$$m_2 \Delta V_2 \approx \bar{m}(1+e)v_{1n} \tag{11}$$

where, from Appendix A

$$\bar{m} = \frac{m_1 m_2}{m_1 + m_2} \tag{12}$$

Since m_2 and the initial velocities are not factors, it is clear from these equations that for this collision configuration the only factors that physically play a role in the kinetic energy loss and ΔV_2 are m_1 and e. This is in agreement between impact theory and the above DOE results.

A comparison to the above results can be made using the method of differential variations[13]. To correspond to the DOE results, the percentage of T_L relative to the initial kinetic energy, T_i, must first be found. This is

$$T_{L\%} = 100\frac{T_L}{T_i} = 100\frac{\bar{m}}{m_1}(1-e^2) \tag{13}$$

Uncertainty can be estimated using

$$\delta T_{L\%} = \frac{\partial T_{L\%}}{\partial e}\delta e + \frac{\partial T_{L\%}}{\partial m_1}\delta m_1 \tag{14}$$

After evaluation of the derivatives and substitution of data, the numerical quantities corresponding to the individual terms of Eq 14, for the 90° collision are

$$\delta T_{L\%} = -1.698 - 2.419$$

The corresponding ME values from Table 8 are ME_e = –1.675 and ME_{m1} = -2.400. Agreement is quite good.

Conclusions

The Design of Experiments (Factorial Design) is a well known and commonly used method for organizing and analyzing experimental measurements and to measure the sensitivity and uncertainty of a process to its input parameters. It is highly efficient and relatively simple method to use. When applied to a simulation it requires only the evaluation of the simulation (process) response for a limited number of cases.

Planar impact mechanics is commonly used for reconstructing vehicle collisions. In all reconstruction applications, uncertainty can occur. This work shows that DOE can be used to estimate the sensitivity and uncertainty of reconstruction calculations. Both the skid-to-stop example and the 90° front-to-side collision give the same results as more classical uncertainty techniques. One of the advantages of the use of DOE is that it requires only numerical evaluation of the process being examined. Another, important advantage of the use of DOE to evaluate uncertainty is that it automatically determines the effects of any significant interactions between individual factors.

Though not evaluated here through the DOE process, it is clear that the two different collision configurations used here as examples led to different significant factors. Each collision configuration must be treated as a different process.

References

1. Rose, N. A., G Beauchamp and W Bortles, “Quantifying the Uncertainty in the Coefficient of Restitution Obtained with Accelerometer Data from a Crash Test”, Paper 2007-01-0737, SAE International Congress, 2007.
2. Davies, O. L., Editor, The Design and Analysis of Industrial Experiments, 2nd Ed., Hafner, New York, 1956.
3. Guttman, I., Wilks, S. and Hunter, J., Introductory Engineering Statistics, 3rd Ed., John Wiley & Sons, New York, 1982.
4. Brach, R. M. and P. F. Dunn, Uncertainty Analysis for Forensic Science, Lawyers & Judges Publishers, March, 2004.
5. Montgomery, D. C., Design and Analysis of Experiments, 3rd Ed., John Wiley & Sons, New York, 1991.
6. Kudlich H., “Beitrag zur Mechanik des Kraftfahreug-Verkehrsunfalls”, Dissertation TU-Wien, 1966.
7. Slibar A., “Die mechanischen Grundsätze des Stoßvorganges freier und geführter Körper und ihre Anwendung auf den Stoßvorgang van Fahrzeugen” Archiv für Unfallforschung, 2. Jg., H. 1, 1966, 31ff.
8. Brach, R. M., "Impact Analysis of Two-Vehicle Collisions", Paper 830468, SAE International Congress and Exposition, Detroit, MI, 1983.
9. Brach, R. M., "Analysis of Planar Vehicle Collisions using Equations of Impulse and Momentum", J Accd Anal & Prev, Vol 15, No 2, April 1983.
10. Woolley, R, “ The IMPAC Program for Collision Analysis”, Paper 870046, SAE, Warrendale, PA, 1987.
11. Brach, R. M. Mechanical Impact Dynamics, Wiley Interscience, New York, 1991.
12. Steffan, H. and A. Moser, “The Collision and Trajectory Models of PC-Crash”, Paper 960886, SAE, Warrendale, PA, 1996.
13. Brach, Raymond and Matthew Brach, Vehicle Accident Analysis and Reconstruction Methods, SAE, Warrendale, PA, 2005.

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Versuchsplanung und Parameterempfindlichkeit in der ebenen Stoßmechanik

Zusammenfassung

Experimentenplanung (DOE) ist eine Methode zur Organisierung und Analyse verschiedener Experimente in einer effizienter und exakter Weise. Die Anwendung der DOE Methode ist aber nicht nur auf physikalische Experimente beschränkt. Auch Ergebnisse von „Komputerexperimenten” können analysiert werden, um die Empfindlichkeit und Bedeutung von Eingangsparameter für die Ergebnisse (den Output) bei beliebigen Experimenten zu bestimmen. Dieser Beitrag enthält eine kurze Darstellung einiger Grundlagen der DOE Methode, ihrer Terminologie und ihrer Anwendungsmöglichkeiten.

Es werden Beispiele von Unfallrekonstruktionen präsentiert, die eine große Menge von Eingangsparametern enthalten. Unter breit gefassten Bedingungen und Annahme von entsprechen Prämissen kann Planar Impact Mechanics (PIM, Ebenenkollisionsmodell, zweidimensionales Kollisionsmodell) zur Analyse von Zweifahrzeugkollisionen verwendet werden. Bei gegebenen physikalischen Eigenschaften der Fahrzeuge, gegebener Unfallkonfiguration und Information über den Kontaktprozess (Restitutionskoeffizient und Stoßreibungskoeffizient), Kollisionsgeschwindigkeiten der Fahrzeuge, Verlust der kinetischen Energie bei der Kollision, können ΔV Werte und andere Parameter algebraisch berechnet werden unter der Voraussetzung, dass die Fahrzeuggeschwindigkeiten spezifiziert sind. Die DOE Methode fand ihre Anwendung im Modell der Planar Impact Collision für Zweifahrzeugkollisionen als eine Methode zur Überprüfung der Unsicherheit des Energieverlustes und des ΔV Wertes bei typischen Kollisionen in Bezug auf die Eingangsparameter.

APPENDIX A: Notation and Solution Equations of Planar Impact Mechanics

Notation, Subscripts:

n, t	normal & tangential axes (Fig 2)
x, y	ground based axes (Fig 2)
r	relative
C	impact center
1, 2	vehicle number

Notation, Variables:

d_0, d_1	crush stiffness coefficients
d_c, d_d	distances, Appendix A
d_e, d_e	distances, Appendix A
e	coefficient of restitution
I	yaw moment of inertia
m	mass
P	impulse
r	velocity ratio
T	kinetic energy
v	initial velocity
V	final velocity
ΔV	velocity change
Γ	crush surface angle
ω	initial angular velocity
Ω	final angular velocity
μ	impulse ratio

Summary of assumptions for planar impact mechanics:

1. A single dynamic contact, taking place over a short duration.
2. Forces other than the contact force and impulses of forces other than the contact force are negligible.
3. Rotational motion of the masses can be significant.
4. Initial velocities are known and final velocities are unknown.
5. Deformation is localized and small compared to the size of the bodies.
6. During the contact duration, position and orientation changes are negligibly small, velocity changes are instantaneous and accelerations are large.
7. The effects of the normal (crush) and tangential (sliding, shearing, entanglement, crush, etc.) contact processes are known (through coeffi-cients).
8. A point (impact center), C, common to both vehicles and on the line of action of the contact impulse is known
9. A common crush plane defined by the angle Γ, is known.

Solution Equations of planar impact mechanics:

$$V_{1n} - v_{1n} = \overline{m}(1+e)v_{rn}q / m_1 \quad \text{(A1)}$$

$$V_{1t} - v_{1t} = \mu\overline{m}(1+e)v_{rn}q / m_1 \quad \text{(A2)}$$

$$V_{2n} - v_{2n} = -\overline{m}(1+e)v_{rn}q / m_2 \quad \text{(A3)}$$

$$V_{2t} - v_{2t} = -\mu\overline{m}(1+e)v_{rn}q / m_2 \quad \text{(A4)}$$

$$\Omega_1 - \omega_1 = \overline{m}(1+e)v_{rn}(d_c - \mu d_d)q/(m_1k_1^2) \quad \text{(A5)}$$

$$\Omega_2 - \omega_2 = \overline{m}(1+e)v_{rn}(d_a - \mu d_b)q/(m_2k_2^2) \quad \text{(A6)}$$

$$e = -V_{Crn} / v_{Crn} \quad \text{(A7)}$$

$$\mu = P_t / P_n \quad \text{(A8)}$$

$$I_1 = m_1k_1^2 \quad \text{(A9)}$$

$$I_2 = m_2k_2^2 \quad \text{(A10)}$$

$$v_{rn} = (v_{2n} - d_a\omega_2) - (v_{1n} - d_c\omega_1) \quad \text{(A11)}$$

$$V_{Crn} = V_{1n} + d_c\Omega_1 - V_{2n} + d_a\Omega_2 \quad \text{(A12)}$$

$$v_{Crn} = v_{1n} + d_c\omega_1 - v_{2n} + d_a\omega_2 \quad \text{(A13)}$$

$$\frac{1}{q} = 1 + \frac{\overline{m}d_a^2}{m_2k_2^2} + \frac{\overline{m}d_c^2}{m_1k_1^2} - \mu\left(\frac{\overline{m}d_cd_d}{m_1k_1^2} + \frac{\overline{m}d_ad_b}{m_2k_2^2}\right) \quad \text{(A14)}$$

$$d_a = d_2\sin(\theta_2 + \varphi_2 - \Gamma) \quad \text{(A15)}$$

$$d_b = d_1\sin(\theta_1 + \varphi_1 - \Gamma) \quad \text{(A16)}$$

$$d_c = d_1\cos(\theta_1 + \varphi_1 - \Gamma) \quad \text{(A17)}$$

$$d_d = d_1\cos(\theta_1 + \varphi_1 - \Gamma) \quad \text{(A18)}$$

$$d_e = d_c - \mu d_d \quad \text{(A19)}$$

$$d_f = d_a - \mu d_b \quad \text{(A20)}$$

$$r = \frac{(v_{2t} - d_b\omega_2) - (v_{1t} + d_d\omega_1)}{(v_{2n} - d_a\omega_2) - (v_{1n} + d_c\omega_1)} \quad \text{(A21)}$$

$$P_x = m_1(V_{1x} - v_{1x}) \quad \text{(A22)}$$

$$P_y = m_1(V_{1y} - v_{1y}) \quad \text{(A23)}$$

$$P_n = P_x\cos\Gamma + P_y\sin\Gamma \quad \text{(A24)}$$

$$P_t = -P_x\sin\Gamma + P_y\cos\Gamma \quad \text{(A25)}$$

$$\overline{m} = m_1m_2/(m_1 + m_2) \quad \text{(A26)}$$

$$\mu_0 = \frac{rA + (1+e)B}{(1+e)(1+C) + rB} \quad \text{(A27)}$$

$$A = 1 + \overline{m}(d_c^2/I_1 + d_a^2/I_2) \quad \text{(A28)}$$

$$B = \overline{m}(d_cd_d/I_1 + d_ad_b/I_2) \quad \text{(A29)}$$

$$C = \overline{m}(d_d^2/I_1 + d_b^2/I_2) \quad \text{(A30)}$$

Appendix B: Full Factorial, Experimental Layout for Four Factors and Two Levels

Design of Experiments layout and sign pattern for k = 4, 2^4 = 16 runs

Run	x_1	x_2	$x_1 x_2$	x_3	$x_1 x_3$	x_2x_3	$x_1x_3x_3$	x_4	x_1x_4	x_2x_4	$x_1x_2x_4$	x_3x_4	$x_1x_3x_4$	$x_2x_3x_4$	$x_1x_2x_3x_4$
1	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
2	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
3	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-
4	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
5	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-
6	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
7	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
8	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
9	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-
10	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
11	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
12	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
13	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
14	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
15	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+