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Identification of Vehicle and Collosion Impact Parameters From Crash Tests

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In the mid 1970's a group of 12 staged and instrumented automobile collisions was conducted for the National Highway Traffic Safety Administration. These were two-vehicle collisions with a variety of initial speeds, vehicle orientations, and vehicle size mixes. Initial speeds were controlled and velocity components including angular velocities at separation were measured.

At about the same time, development of the classic theory of impact of rigid bodies to planar vehicle collisions was taking place. Users of classic theory heretofore had neglected the existence of a moment between impacting bodies. Inclusion of a moment and introduction of a moment coefficient of restitution allows the formulation of a planar collision model consisting of six algebraic equations relating the six initial velocity components of the two vehicles to their six final velocity components. The model contains collision geometry, vehicle geometry, vehicle inertial properties, and three coefficients. These coefficients are the classic coefficient of restitution, a friction coefficient, and the newly defined moment coefficient.

This paper discusses the application of the theory of least squares to fit the experimentally determined velocity components to the six equations of the vehicle collision model. The usual approach using the theory of least squares is to set to zero the partial derivatives of the sum of squares taken with respect to the unknowns. The original model equations can be added as constraints through the use of Lagrange multipliers. A set of 15 nonlinear algebraic equations results. This approach was tried unsuccessfully. Direct numerical minimization of the sum of squares using gradient project techniques proved to be far superior. Solutions are obtained for the staged collisions.

Results provide insight into velocity changes and their relationships to energy dissipation, the coefficients of restitution and friction and other collision parameters. The capability of calculating velocity changes of colliding vehicles should prove complementary to detailed finite element studies of vehicle crush properties.

Introduction

The equations of impulse and momentum are applied in almost every dynamics textbook to the calculation of velocity changes due to the impact of particles. In some, the techniques are extended to rigid-body impacts. Until relatively recently, a general formulation of impact of rigid bodies for planar impacts had not been developed, however. In 1977 [1], a set of equations was presented to form a planar, vehicle collision model. Although classic impact theory had been applied to vehicle collisions prior to 1977 [2, 3], extensive use of simplifying assumptions was made including neglecting angular velocities and their changes. Since the formulation of the rigid-body impact equations [1] is reasonably general, they can be applied to any planar impact [4]. Since classic theory is used, energy loss is imbedded in three coefficients. These are the coefficient of restitution, the moment coefficient of restitution [1, 4], and an equivalent coefficient of friction.

A group of staged, or experimental, automobile collisions was conducted for the National Highway Traffic Safety Administration [5]. Vehicle and collision parameters were controlled and final velocity components were measured. Because of missing data only 11 of these collisions are available for analysis by the model discussed above. Application of the classic theory of least squares is used to fit the experimental data to the model. The direct result of fitting the model equations is the estimation of the three coefficients. Comparisons and analysis of the results from the eleven collisions lead to some general conclusions about vehicular collisions as well as the validity of the model.

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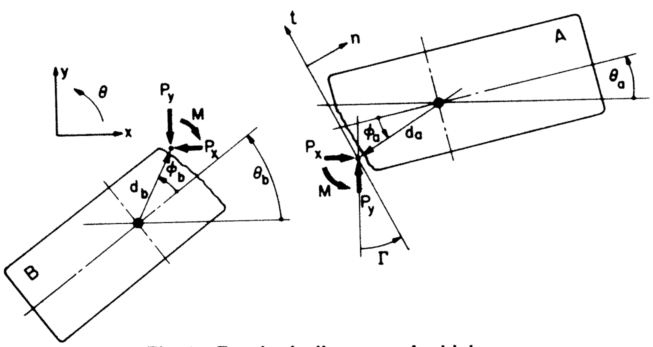

Fig. 1 Free body diagrams of vehicles

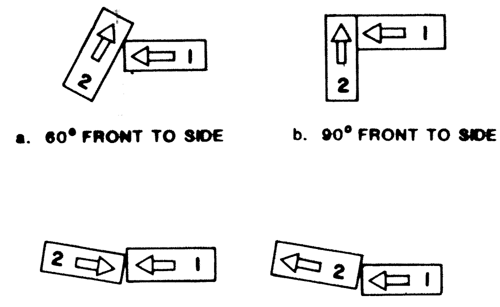

Fig. 2 Categories of vehicle orientations at beginning of impact

Impulse/Momentum Equations

A general model of impact of two rigid bodies (vehicles) in planar motion must relate both vehicles' three initial velocity components to their three final velocity components. Consequently, the model should have six equations. In addition, it should contain the geometrical variables of the impact and the physical parameters of the vehicles. A model with these features has been derived elsewhere [1]; the equations are provided in matrix form in an Appendix to this paper. A discussion of some of the more pertinent and unique features of these equations is presented here.

Figure 1 shows free body diagrams of the two vehicles. It illustrates a fixed $x-y-\theta$ reference coordinate system and shows impulse components P_x, P_y, and M. The initial velocity components of vehicle A are v_{ax}, v_{ay}, and ω_a; for vehicle B they are v_{bx}, v_{by}, and ω_b. Final velocity components are V_{ax}, V_{ay}, Ω_a, V_{bx}, V_{by}, and Ω_b, respectively.[1] The matrix form of the impulse/momentum equations listed in the Appendix is

$$f=AV-Cv=0 \tag{1}$$

A and C are 6 × 6 matrices; V and v are final and initial velocity vectors such that

$$V^T=\{V_1V_2V_3V_4V_5V_6\}=\{V_{ax}V_{ay}V_{bx}V_{by}\Omega_a\Omega_b\} \tag{2}$$

and

$$v^T=\{v_1v_2v_3v_4v_5v_6\}=\{v_{ax}v_{ay}v_{bx}v_{by}\omega_a\omega_b\} \tag{3}$$

The first, second, and fifth equations were derived by applying Newton's second law in the x and y directions and rotationally to each of the two free body diagrams of Fig. 1 (giving 6 equations) and then eliminating P_x, P_y, and M. The third equation uses a classical definition of the coefficient of restitution and the assumption that all permanent deformation (crush) takes place normal to the line making angle Γ relative to the y axis. This is

$$V_{an}-V_{bn}=-e(v_{an}-v_{bn}) \tag{4}$$

Note that these are velocities of the contact point, not mass center velocities. The fourth equation follows from a definition of an equivalent coefficient of friction, μ. This is done in the equation

$$P_t=\mu P_n \tag{5}$$

where P_t and P_n are the tangential and normal components of the impulse. Sliding may or may not actually occur; interconnection of deformed parts can prevent sliding. As a result, values of μ may be interpreted as equivalent coefficients of friction but they represent simply the ratio of P_t to P_n. The sixth and final equation in the model comes from the definition of a coefficient of moment restitution, e_m. This can be done with the equation

$$(\Omega_b-\Omega_a)(1-e_m)=e_m\left(\frac{M}{I_a}+\frac{M}{I_b}\right) \tag{6}$$

When defined by equation (6), e_m has the following properties. When $e_m = +1$, the moment impulse, M, is zero. Otherwise, $-1 \leq e_m \leq 0$. When $e_m = 0$, the two vehicles' final angular velocities are equal, a perfectly inelastic angular collision. Although not apparent from equation (6), $e_m = -1$ corresponds to a perfectly elastic rotational collision. If the moment impulse M is eliminated from equation (6), the last of the equations in equation (1) results.

A vector of coefficients is defined as

$$c^T=\{c_1c_2c_3\}=\{e \quad e_m \quad \mu\} \tag{7}$$

The equations given in the Appendix relate the initial and final velocity components of the impact to the vehicle properties, collision geometry, and the three coefficients.

Staged Collisions

Four collision configurations were used for the staged collisions. These are illustrated in Fig. 2. Within these four categories, various vehicle size mixes and initial velocity combinations were used. Table 1 lists all of the data from the experimental collisions used in this paper. It should be noted that the values of measured final velocities listed in Table 1 are corrected values. The velocity components listed in the original report [5] were not measured at the centers of gravity and had to be corrected using angular velocity and transducer locations. Further note that all initial angular velocities in the collisions were zero.

Twelve collisions were conducted but data were lost in Collision 2. All 11 remaining collisions are listed in Table 1 and will be analyzed. None of the collisions is identical to any other; at least one condition such as initial speed is different. Consequently no estimate of experimental error is available. The reports describing the experimental collisions include large amounts of data including post impact motion of the vehicles. Since this paper deals with the impact only, postimpact motion is not discussed.

Least Square Parameter Estimation

A typical approach [6] to the parameter estimation problem is to form and minimize a sum of squares of deviations of the model equations, that is

$$S=\sum_{i=1}^{6}\sum_{j=1}^{n}f_{ij}^2 \tag{8}$$

In S, f_{ij} represents the ith model equation and the sum over j is for each set of n experimental values. While S could have been used, an alternative was chosen which is a bit more general. A different sum, Q is defined where

[1] Throughout this paper, upper case symbols indicate final velocities and lower case symbols indicate initial velocities.

Table 1-A Collision/vehicle data (U.S. units)

Collision Number/ Vehicle	Collision Category	Vehicle Mass $lb\text{-}s^2/ft$	Moment of Inertia $ft\text{-}lb\text{-}s^2$	Distance, d feet	Angle, ϕ deg	Angle of Veh θ deg	Crush Angle Γ deg	Initial Velocity ft/s v_x/v_y	Final Angular Velocity Ω,rad/s	Final Velocity ft/s V_x/V_y
1/A		143.5	3728	7.59	-19.8	0	-30	-29.04/ 0.0	-1.57	-12.33/ 7.91
1/B	60°	95.8	1961	3.44	-38.7	60	-30	14.52/25.15	0.0	- 6.80/16.97
6/A	FRONT	133.6	3469	8.41	-17.9	0	-30	-31.53/ 0.0	-0.52	-18.68/ 4.12
6/B	TO	81.5	1669	2.00	-90.0	60	-30	15.29/27.31	-3.14	- 4.20/18.02
7/A	SIDE	115.0	2985	8.41	-17.9	0	-30	-42.68/ 0.0	-0.52	-25.41/ 4.85
7/B		81.1	1082	2.00	-90.0	60	-30	21.34/36.96	-3.35	- 7.63/28.34
8/A	90°	139.1	3614	7.90	0.0	0	0	-30.51/ 0.0	-1.99	-10.24/10.72
8/B		146.3	3800	2.77	-68.8	90	0	0.0 /30.51	-0.31	-12.02/19.72
9/A	FRONT	70.1	976	4.80	6.0	0	0	-31.09/ 0.0	-3.14	- 2.81/14.83
9/B	TO	152.2	3953	5.20	-29.7	90	0	0.0 /31.09	0.79	- 9.90/24.20
10/A		71.6	998	5.20	0.0	0	0	-48.84/ 0.0	-5.24	- 5.08/28.18
10/B	SIDE	146.6	3008	5.29	-29.2	90	0	0.0 /48.84	1.26	-14.57/36.55
11/A	10°	94.4	1935	6.14	9.4	0	0	-29.92/ 0.0	0.52	5.81/ 2.02
11/B	FRONT	150.6	3913	7.66	11.3	-10	0	29.47/-5.2	0.0	6.42/-4.11
12/A	TO	97.2	1992	5.90	9.6	0	0	-46.20/ 0.0	1.57	14.05/-1.61
12/B	FRONT	140.2	3640	7.28	10.3	-10	0	45.50/-8.02	1.05	6.32/-9.65
3/A	10°	153.7	3992	8.83	-17.0	0	-10	-31.09/ 0.0	-0.26	-17.15/ 0.24
3/B		97.0	1986	7.63	171.4	170	-10	0.0 / 0.0	0.0	-22.87/ 3.74
4/A	FRONT	154.7	4018	8.02	-18.2	0	-10	-56.76/ 0.0	-0.65	-29.33/-1.43
4/B		99.1	2032	6.94	171.7	170	-10	0.0/ 0.0	-0.52	-32.54/ 1.38
5/A	TO	142.9	3711	8.08	-20.7	0	-10	-58.23/ 0.0	-0.21	-34.31/ 0.57
5/B	REAR	78.6	1095	5.75	168.0	170	-10	0.0/ 0.0	-1.22	-37.15/ 2.77

Table 1-B Collision/vehicle date (S.I. units)

Collision Number/ Vehicle	Collision Category	Vehicle Mass kg	Moment of $kg\text{-}m^2$	Distance d m	Angle ϕ deg	Angle of Veh θ deg	Crush Angle Γ deg	Initial Velocity m/s v_x/v_y	Final Angular Velocity Ω,rad/s	Final Velocity m/s V_x/V_y
1/A	60°	2095	5055	2.31	-19.8	0	-30	- 8.95/ 0.0	-1.57	- 3.76/ 2.41
1/B	FRONT	1398	2659	1.05	-38.7	60	-30	4.43/ 7.67	0.0	- 2.07/ 5.17
6/A		1949	4704	2.56	-17.9	0	-30	- 9.61/ 0.0	-0.52	- 5.69/ 1.26
6/B	TO	1189	2263	0.61	-90.0	60	-30	4.66/ 8.32	-3.14	- 1.28/ 5.49
7/A		1678	4047	2.56	-17.9	0	-30	-13.01/ 0.0	-0.52	- 7.74/ 1.48
7/B	SIDE	1184	1467	0.61	-90.0	60	-30	6.50/11.27	-3.35	- 2.22/ 8.64
8/A	90°	2030	4899	2.41	0.0	0	0	- 9.30/ 0.0	-1.99	- 3.12/ 3.27
8/B		2135	5152	0.84	-68.8	90	0	0.0 / 9.30	-0.31	- 3.66/ 6.01
9/A	FRONT	1023	1323	1.46	6.0	0	0	- 9.48/ 0.0	-3.14	- 0.86/ 4.52
9/B	TO	2221	5360	1.58	-29.7	90	0	0.0 / 9.48	0.79	- 3.02/ 7.38
10/A		1046	1353	1.58	0.0	0	0	-14.89/ 0.0	-5.24	- 1.55/ 8.59
10/B	SIDE	2140	5163	1.61	-29.2	90	0	0.0 /14.89	1.26	- 4.44/11.14
11/A	10°	1378	2624	1.87	9.4	0	0	- 9.12/ 0.0	0.52	1.77/ 0.62
11/B	FRONT	2198	5305	2.33	11.3	-10	0	8.98/-1.58	0.0	1.96/-1.25
12/A	TO	1419	2701	1.80	9.6	0	0	-14.08/-4.28	1.57	4.28/-0.49
12/B	FRONT	2045	4936	2.22	10.3	-10	0	13.87/ 2.44	1.05	1.93/-2.94
3/A	10°	2243	5414	2.69	-17.0	0	-10	- 9.48/ 0.0	-0.26	- 5.23/ 0.07
3/B		1415	2692	2.33	171.4	170	-10	0.0 / 0.0	0.0	- 6.97/ 1.14
4/A	FRONT	2257	5448	2.44	-18.2	0	-10	-17.3 / 0.0	-0.65	- 8.94/-0.44
4/B	TO	1447	2755	2.12	171.7	170	-10	0.0 / 0.0	-0.52	- 9.92/ 0.42
5/A		2086	5032	2.46	-20.7	0	-10	-17.75/ 0.0	-0.21	-10.46/ 0.17
5/B	REAR	1147	1482	1.76	168.0	170	-10	0.0 / 0.0	-1.22	-11.32/ 0.84

$$Q=\sum_{i=1}^{6}\sum_{p=1}^{n_i} w_i(V_i-V_{ip})^2+\sum_{t=1}^{6}\lambda_t f_t(c_i,V_i) \qquad (9)$$

With Q, each velocity component, V_i, can have a different number of experimental values which is useful in other applications of the model. Each term of Q can be weighted with the weighting factors w_i. The λ's in equation (9) are Lagrange multipliers and the f_t's represent the model equations. The partial derivatives of Q can be taken with respect to the final velocities, V_i, and the coefficients, c_i, and set to zero. This will provide a set of nine nonlinear algebraic equations plus the six original model equations, all of whose solutions will provide the least-square solution of the model equation.

The numerical solution of 15 nonlinear equations was attempted with little success. An alternative was sought. Q was minimized directly using a gradient projection technique [7] where the Golden Section method was used to search along the projection line. This provided relatively rapid convergence in all cases with anywhere from 4 to about 20 iterations in order to reach a point where Q decreased by less than 0.5 percent.

A solution to the least-square problem for each collision provided a set of coefficients, e, e_m, and μ and a corresponding set of final velocity components, V_{ax}, V_{ay}, V_{bx}, V_{by}, Ω_a, and Ω_b. With these, velocity changes for each vehicle could be calculated along with the energy loss of the collision. These are listed in Table 2.

Choices had to be made for many quantities which were assumed to be known and remain constant during each collision. Examples are d and ϕ, Fig. 1, for each vehicle. These are discussed later. Another point to be mentioned involves the coefficients e and e_m. These coefficients cannot take on arbitrary values. The coefficient e is such that $0 \le e \le 1$. The coefficient e_m is such that $-1 \le e_m \le 0$ or $e_m = +1$. Prior to the gradient search, these variables were redefined such that $e = \sin^2 t$ and $e_m = -\sin^2 t_m$. This allowed an unconstrained search to be carried out. However, since e_m could also equal $+1$ (when the moment impulse, $M = 0$), a second set of constrained solutions was calculated, all for $e_m = +1$. Table 2 contains the set of results for each collision for the choice of e_m which provided the lower minimum sum of squares.

Results

Many interesting trends can be seen and comparisons made from such a wide variety of experimental collisions. Some of the more significant observations are summarized in the following comments.

Final Velocity Estimates. The values of individual final velocity components vary considerably from their corresponding experimental values, in some cases by more than 100 percent. However, the magnitude of each vehicle's total velocity change in all but a few cases varies by a few feet per second, or less, from the experimental value.

Kinetic Energy Loss. The change in total kinetic energy from before the collision to after varied over all the collisions from about 30 to 90 percent. Energy loss remained fairly consistent within each category, however.

Coefficient of Restitution. Estimates of coefficients of restitution for all collisions were about 0.4 or less. If the 90 deg Front-to-Side collision category is omitted, all values are about 0.2, or less. Estimates of e seem to correlate with collision category but wide variations are evident.

Moment Coefficient of Restitution. Moment coefficients also seem to correlate reasonably well with collision geometry (category), perhaps even a bit better than e. For the second category, 90 deg Front to Side, a value of $e_m = +1$ gave lower sums of squares. This indicates that no moment developed over the crush surface by the force distributions which actually occurred or that the choice of d and ϕ for each car did not create a fictitious moment during the analysis.

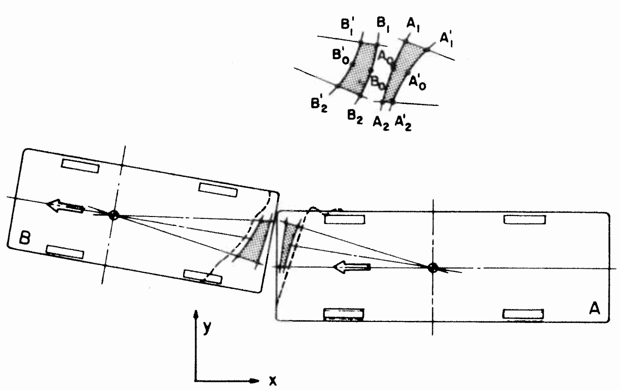

Fig. 3 Plan profile with final crush surface, Collision 4

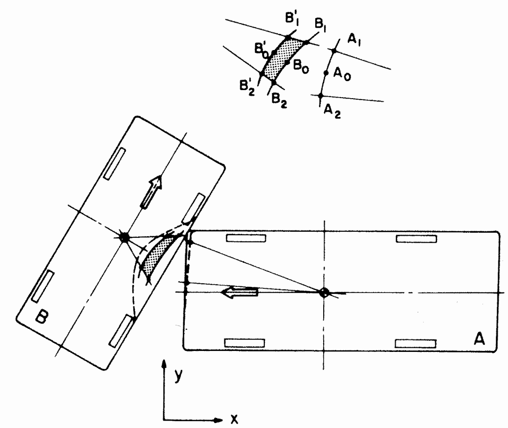

Fig. 4 Plan profile with final crush surface, Collision 6

Table 2 Results of least-square calculations

Collision Number/ Vehicle	Sum of Squares	Energy Loss %	Coeff of Restitution	Moment Coeff	Coeff of Friction	Final Velocity ft/s V_x/V_y	Final Velocity m/s V_x/V_y	Final Angular Velocity rad/s
1/A	52.8	57.5	.005	-.715	.911	-13.27/ 3.44	- 4.04/ 1.05	- 1.74
1/B						- 9.12/19.99	- 2.78/ 6.09	- 0.82
6/A	154.1	48.4	.000	-.430	.797	-16.99/ 2.19	- 5.18/ 0.67	- 1.96
6/B						- 8.54/23.73	- 2.60/ 7.23	- 1.81
7/A	182.0	47.9	.003	-.505	.656	-22.29/ 1.17	- 6.79/ 0.36	- 2.39
7/B						- 7.56/35.31	- 2.30/10.76	- 2.39
8/A	49.5	35.3	.085	+ 1	.510	-13.44/ 8.70	- 4.10/ 2.65	- 2.65
8/B						-16.23/22.24	- 4.95/ 6.78	- 0.20
9/A	29.8	28.7	.414	+ 1	.491	- 4.52/13.05	- 1.38/ 3.98	- 3.52
9/B						-12.24/25.08	- 3.73/ 7.64	1.53
10/A	97.2	31.0	.412	+ 1	.620	- 9.44/24.43	- 2.88/ 7.45	- 6.09
10/B						-19.25/36.91	- 5.87/11.25	2.24
11/A	6.3	92.0	.008	-.504	.049	6.53/ 1.79	1.99/ 0.55	0.54
11/B						6.61/-6.32	2.01/-1.93	0.18
12/A	10.3	92.9	.112	-.499	-.009	13.06/-0.55	3.98/-0.17	1.63
12/B						4.41/-7.64	1.34/-2.33	0.92
3/A	23.2	34.0	.221	-.489	-.069	-17.37/-3.40	- 5.29/-1.04	-0.36
3/B						-21.75/ 5.40	- 6.63/ 1.65	-0.47
4/A	83.4	36.1	.071	-.489	-.008	-34.75/-4.07	-10.59/-1.24	-1.10
4/B						-34.35/ 6.35	-10.47/ 1.94	-1.27
5/A	96.4	32.1	.075	-.475	-.034	-38.17/-4.24	-11.63/-1.29	-1.27
5/B						-36.46/ 7.72	-11.11/ 2.35	-1.68

Table 3 Measured and calculated results

Collision Number/ Vehicle	Final Angular Velocity rad/s	$\lvert\Delta V\rvert$, Vel Change Magnitude m/s	$\lvert\Delta V\rvert$, Vel Change Magnitude ft/s	$\lvert\Delta V\rvert$, Vel Change Magnitude m/s	$\lvert\Delta V\rvert$, Vel Change Magnitude ft/s	Normalized Momentum Change L'	Predicted Velocity Change m/s	Predicted Velocity Change ft/s
1/A	-1.57	5.64	18.5	4.91	16.1	.6084	5.36	17.6
1/B	0.00	6.95	22.8	7.38	24.2	.6101	8.05	26.4
6/A	-0.52	4.11	13.5	4.48	14.7	.5721	5.21	17.1
6/B	-3.14	6.58	21.6	7.35	24.1	.5722	8.26	28.1
7/A	-0.52	5.46	17.9	6.22	20.4	.5641	7.35	24.1
7/B	-3.35	9.20	30.2	8.81	28.9	.5641	10.42	34.2
8/A	-1.99	6.98	22.9	5.85	19.2	.7300	5.61	18.4
8/B	-0.31	4.94	16.2	5.55	18.2	.7278	5.33	17.5
9/A	-3.14	9.72	31.9	9.02	29.6	.7431	9.54	31.3
9/B	0.79	3.69	12.1	4.15	13.6	.7431	4.39	14.4
10/A	-5.24	15.85	52.0	14.14	46.4	.7487	14.57	47.8
10/B	1.26	5.82	19.1	6.92	22.7	.7497	7.10	23.3
11/A	0.52	10.91	35.8	11.16	36.6	.6776	10.97	36.0
11/B	0.00	7.04	23.1	6.98	22.9	.6760	6.89	22.6
12/A	1.57	18.38	60.3	18.07	59.3	.7587	15.88	52.1
12/B	1.05	11.95	39.2	12.62	41.4	.7640	11.00	36.1
3/A	-0.26	4.24	13.9	4.30	14.1	.7778	3.69	12.1
3/B	0.00	7.07	23.2	6.83	22.4	.7787	5.85	19.2
4/A	-0.65	8.38	27.5	6.83	22.4	.6567	6.92	22.7
4/B	-0.52	9.94	32.6	10.64	34.9	.6556	10.82	35.5
5/A	-0.21	7.28	23.9	6.25	20.5	.6214	6.71	22.0
5/B	-1.22	11.37	37.3	11.37	37.3	.6220	12.19	40.0

Equivalent Coefficient of Friction. The coefficient μ is a relative measure of the tangential and normal impulse components. Values ranged considerably from small negative values to greater than 0.9. A consistency does seem to appear however. "Head-on" collisions should produce small transverse impulses. Collisions 3, 4, 5, 11, and 12 are colinear type collisions and all have friction coefficient magnitudes less than 0.07. The front-to-side collisions all have initial transverse velocities and the coefficients are higher, about 0.5 and above.

Normalized Velocity Changes. Although final velocity estimates have been discussed, velocity changes are pursued a little more. Consider an arbitrary quantity, L'

$$L_i' = \frac{m_i\,|\Delta V_i|}{\sqrt{\Delta_T}[(m_1 v_1)^2 + (m_2 v_2)^2]^{1/2}} \tag{10}$$

where $|\Delta V_i|$ is the magnitude of the vector velocity change of vehicle of mass m_i in a given collision. The denominator of L_i' contains the magnitude of the initial momentum of the collision and ΔT is the fractional kinetic energy loss. Values of L_i' are listed in Table 3 for each collision under the heading Normalized Momentum change. Surprisingly, the values of L_i' remain relatively constant. If a value of 2/3 is used as an average for L_i' then equation (10) can be written as

$$|\Delta V_i| = 2\sqrt{\Delta_T}[(m_1 v_1)^2 + (m_2 v_2)^2]^{1/2}/3m_i \tag{11}$$

permitting the calculation of a vehicle's velocity change given the collision's initial momentum and energy loss. Values of these "predicted" velocity changes for each collision using equation (10) are also contained in Table 3. Values of ΔT for these calculations were taken from the least-square solutions. Note that the predicted ΔV's are fairly close to the measured and least-square values. In fact, the deviations of all three sets are quite similar.

Crush Geometry Variations

Within the plan outline of each vehicle, a single point called the "center of impact" is located with a vector from the mass center (see Fig. 1). This center of impact is assumed to have the following characteristics:

Table 4 Values of variables locating impact center for parameter variation study

Collision 6	Vehicle A			Vehicle B		
	A_1/A_1'	A_o/A_o'	A_2/A_2'	B_1/B_1'	B_o/B_o'	B_2/B_2'
Distance, d,	8.50/7.75	8.45/7.88	8.40/8.00	8.50/7.80	8.38/7.53	8.25/7.25
ft (m)	(2.59/2.36)	(2.58/2.40)	(2.56/2.44)	(2.59/2.38)	(2.55/2.30)	(2.51/2.21)
Angle, ϕ, deg	-16	-8	0	188	180	171
Crush Angle, Γ, deg	-10/-20					

Collision 4	Vehicle A			Vehicle B		
	A_1	A_o	A_2	B_1/B_1'	B_o/B_o'	B_2/B_2'
Distance, d,	8.40	8.00	7.60	3.75/2.25	2.50/1.75	2.75/2.00
ft (m)	(2.56)	(2.44)	(2.32)	(1.14/0.69)	(6.76/0.53)	(0.84/0.61)
Angle, ϕ, deg	-25	-15	-5	-60	-90	-120
Crush Angle, Γ, deg	-15/-30					

1. It is a single point common to both vehicles during the collision.
2. It is the point of application of the net resultant force impulse of the actual collision.
3. Although its precise location is unknown, it lies within the region bounded by the original vehicle's undeformed outline and its deformed outline.

One implication of item 3 is that if a location is chosen for analysis, which is remote from the true center of impact, a fictitious moment is created. This is one justification for the inclusion of a moment impulse on each free body diagram.

When the previous least-square solutions were computed, an impact center was chosen arbitrarily. This was done with the selection of d and ϕ for each vehicle. In this section of the paper, variations in the selection of this point will be examined in detail for two of the collisions. In addition, variations of the orientation of the "impact surface" as determined by selection of Γ will also be examined. Figures 3 and 4 show plan outlines of the vehicles involved in collisions 4 and 6, respectively. Their final deformed surfaces as illustrated in [5] are shown with dashed lines. Various combinations of impact center locations were analyzed from

Table 5 Summary of variations for Collision 4

Results Averaged Over Parameter Values	Sum of Squares	Energy Loss %	Coeff of Restitution e	Moment Coefficient e_m	Coeff of Friction μ	$\|\Delta V_1\|$ ft/s (m/s)	$\|\Delta V_2\|$ ft/s (m/s)
$A_1\ A_0\ A_2$ and $B_1\ B_0\ B_2$	41.3	38.0	.043	-.465	.175	22.9 (6.98)	35.7 (10.88)
$A_1'\ A_0'\ A_2'$ and $B_1'\ B_0'\ B_2'$	38.6	38.1	.043	-.452	.182	22.8 (6.95)	35.8 (10.91)
$A_1\ A_1'$ and $B_1\ B_1'$	35.0	38.0	.041	-.492	.192	23.0 (7.01)	35.8 (10.91)
$A_2\ A_2'$ and $B_2\ B_2'$	48.5	38.1	.049	-.392	.167	22.8 (6.95)	35.5 (10.82)
Γ_1	39.1	38.1	.043	-.443	.091	22.7 (6.92)	35.5 (10.82)
Γ_2	40.6	38.0	.044	-.474	.266	23.1 (7.04)	36.1 (11.00)

Table 6 Summary of variations for Collision 6

Results Averaged Over Parameter Values	Sum of Squares	Energy Loss %	Coeff of Restitution e	Moment Coefficient e_m	Coeff of Friction μ	$\|\Delta V_1\|$ ft/s (m/)	$\|\Delta V_2\|$ ft/s (m/s)
$A_1\ A_0\ A_2$ and $B_1\ B_0\ B_2$	135.9	49.3	0	-.405	.658	15.2 (4.63)	25.0 (7.62)
$A_1\ A_0\ A_2$ and $B_1'\ B_0'\ B_2'$	144.5	52.0	0	-.361	.688	15.7 (4.79)	25.8 (7.86)
$A_1\ B_1\ B_1'$	159.3	49.3	0	-.026	.636	15.6 (4.75)	25.5 (7.77)
$A_2\ B_2\ B_2'$	111.2	52.4	0	-.640	.715	15.4 (4.69)	25.3 (7.71)
Γ_1	137.2	50.2	0	-.390	.829	15.0 (4.57)	24.6 (7.50)
Γ_2	143.2	51.2	0	-.376	.517	15.9 (4.85)	26.1 (7.96)

the boundaries of the shaded areas chosen within the deformed regions. These points (impact centers) were treated as parameters, and least-square solutions were computed. The values of d, ϕ, and Γ corresponding to these points are listed in Table 4.

Since over 24 solutions were run, only summaries of the results in the form of averages are presented. For example, choice of an impact center near the undeformed impact surface would correspond to points A_1B_1, A_0B_0, or A_2B_2; an impact center near the final deformed surface would be either of points $A_1'B_1'$, $A_0'B_0'$, or $A_2'B_2'$ (for collision 4). Results averaged over the first three points and the latter three points can indicate the effect of choosing the impact point nearer to or away from the undeformed contact surface. Tables 5 and 6 give the results of the calculations. The effects of the parameter variations are assessed by comparing sums of squares, energy loss and each vehicle's velocity-change magnitude.

Effects of Parameter Variations. It is apparent from Tables 5 and 6 that varying the impact center's location and/or varying the crush-surface angle have negligible effect on energy loss and on vehicle velocity changes in the least-square solution. This is an extremely important result. It indicates that locating the impact center or the crush surface angle is not critical when fitting the model to experimental data.

A second common observation can be made concerning the coefficient of restitution, e. Negligible change in e was noted under all variations. This indicates little coupling or relationship between the crush surface angle, Γ, and the impact center location. It must be kept in mind that both of these collisions have small coefficients, e; both are less than 0.1. This conclusion may not apply to the 90 deg Front-to-Side collision category where e has larger values.

Another observation is that changing the angle, Γ, of the crush surface seems to affect principally the equivalent coefficient of friction. This makes sense because the definition of this coefficient is based directly upon the components of the force impulse along the tangent line defined by Γ.

Moving the impact center from one side of the crush surface to the other (points with subscript 1 to points with subscript 2) affects principally the moment coefficient, e_m. Again, this appears intuitively satisfying since this directly affects the moment arm of the lineal impulse P. This in turn will influence the value of the moment impulse, M, which enters directly into the definition of e_m. One other effect which appears with the side-to-side variation of the impact center is on the minimum sum of squares. This is particularly notable in collision 6. For both collisions, the smaller sum of squares corresponds to the value of e_m closer to $-1/2$. When $e_m \neq +1$, a value near -0.5 seems to be typical; see Table 2. For collision 4 this occurs for impact centers near the region of initial contact; for collision 6, this occurs near the region of final contact. More study is needed before any general conclusions can be drawn from these observations.

Conclusions

The method of least squares was used to fit experimental data to a planar, vehicle collision model based upon the principles of impulse and momentum. This procedure yielded values for the three coefficients in the model. These coefficients and the model provide final velocity components for the collisions which do not always agree with the measured values. On the other hand, the calculated velocity changes, ΔV, of each vehicle, do agree well with experimental values. In fact, the ΔV of each vehicle can be predicted with reasonable accuracy if only the fractional energy loss and initial momentum magnitude of the collision are known. This seems to be true, independent of collision geometry.

Certain general trends were observed for the coefficients.

The coefficient of restitution, e, is generally small, 0.2 or less. Exceptions were for the 90 deg Front-to-Side collisions where values near 0.4 occurred. For these same collisions (90 deg Front-to-Side) values of the moment coefficient ($e_m = +1$) corresponding to zero moment occurred. For all other collisions values of e_m near -0.5 were typical.

It was found, fortunately, that variations in the choice of the impact center and crush surface angle had little effect on the energy loss and velocity changes calculated by the least-square procedure. Some systematic changes in the values of the coefficients resulted from these variations, however. Investigation of the effect of variations in the vehicle orientations is yet to be done.

No experimental collision under identical conditions were conducted. As a consequence, it is not possible to assess how much of the observable variations are due to model inadequacies and how much is due to experimental errors.

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APPENDIX

The mathematical collision model discussed inthe body of the paper has the following form:

$$f = AV - Cv = 0$$

where

Matrix A:

$$A = \begin{bmatrix} m_a & 0 & m_b & 0 & 0 & 0 \\ 0 & m_a & 0 & m_b & 0 & 0 \\ \cos\Gamma & \sin\Gamma & -\cos\Gamma & -\sin\Gamma & \eta & \zeta \\ -\alpha m_a & \beta m_a & \alpha m_b & -\beta m_b & 0 & 0 \\ -d_{13}m_a & d_{24}m_a & d_{13}m_b & -d_{24}m_b & I_a & I_b \\ \gamma e_m m_a & -\delta e_m m_a & -\gamma e_m m_b & \delta e_m m_b & 1-2e_m & 2e_m-1 \end{bmatrix}$$

Matrix C:

$$C = \begin{bmatrix} m_a & 0 & m_b & 0 & 0 & 0 \\ 0 & m_a & 0 & m_b & 0 & 0 \\ -e\cos\Gamma & -e\sin\Gamma & e\cos\Gamma & e\sin\Gamma & -e\eta & -e\zeta \\ -\alpha m_a & \beta m_a & \alpha m_b & -\beta m_b & 0 & 0 \\ -d_{13}m_a & d_{24}m_a & d_{13}m_b & -d_{24}m_b & I_a & I_b \\ \gamma e_m m_a & -\delta e_m m_a & -\gamma e_m m_b & \delta e_m m_b & -e_m & e_m \end{bmatrix}$$

And

$$2d_{13} = d_b\sin(\theta_b+\phi_b) + d_a\sin(\theta_a+\phi_a) \qquad \alpha = \sin\Gamma + \mu\cos\Gamma$$

$$2d_{24} = d_b\cos(\theta_b+\phi_b) + d_a\cos(\theta_a+\phi_a) \qquad \beta = \cos\Gamma - \mu\sin\Gamma$$

$$2\gamma = d_a\cos(\theta_a+\phi_a)/I_a - d_b\sin(\theta_b+\phi_b)/I_b \qquad \eta = d_a\sin(\theta_a+\phi_a-\Gamma)$$

$$2\delta = d_a\cos(\theta_a+\phi_a)/I_a - d_b\cos(\theta_b+\phi_b)/I_b \qquad \zeta = d_b\sin(\theta_b+\phi_b-\Gamma)$$

Notation

- e = coefficient of restitution
- e_m = moment coefficient of restitution
- d = distance between mass center and crush center
- I = vehicle yaw inertia about its mass center
- m = mass of vehicle
- V = velocity component of a vehicle following impact
- v = velocity component of a vehicle before impact
- μ = equivalent coefficient of friction along the impact surface
- θ = heading angle of vehicles relative to the x axis
- Γ = angle of impact surface relative to the y axis
- Ω = angular velocity of a vehicle following impact
- ω = angular velocity of a vehicle before impact
- ϕ = angle between the length axis of a vehicle and a line between its center of gravity and the center of impact

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