"Nonlinear Parameter Estimation of a Vehicle Collision Model", Proc 13th Annual Conf on Modeling and Simulation, Univ of Pittsburgh, April 1982.

# NONLINEAR PARAMETER ESTIMATION OF A VEHICLE COLLISION MODEL

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## ABSTRACT

The velocity changes from a two-vehicle planar collision can be modeled by a set of six algebraic equations. The principal variables are the six initial velocity components, six final velocity components and three coefficients. The method of least squares is used to fit the model to several combinations of known and unknown variables such as those obtained from staged collisions. A combination of iteration and direct search is used to solve the estimation equations numerically. Results of calculations are shown using data from experimental collisions.

## INTRODUCTION

Many methods have been developed for reconstructing automobile accidents (1,2,3,4). Purposes vary, but most are principly studies for automotive safety and liability analysis. A series of staged collisions, or experimental crashes, has been conducted (5) to provide data for evaluating accident reconstruction models. All of the reconstruction models as well as the staged collisions naturally break down an accident into two sequential events, the collision and post collision motion. These two events, although linked in a full reconstruction, have two very distinct physical models. During the collision or contact phase, the major forces governing the motion of the vehicles are those between the vehicles. After separation and during post impact motion, the major forces are between each vehicle and the surface over which each is moving. In this paper, a specific model (6) of the collision phase is investigated.

The collision model studied consists of a set of six algebraic equations which follows from the laws of mechanics involving impulse and momentum. These equations contain numerous physical quantities including

1. Vehicle inertial properties
2. Vehicle geometry
3. Collision geometry
4. Velocity components
5. Energy and friction coefficients

In the classical impact problem of mechanics, the initial velocities of two rigid bodies and the coefficients are known; the final velocities are treated as unknowns. For an accident reconstruction, information from the post-impact motion can often provide "experimental values"* of the final collision-phase velocities; typically, the initial velocities are partially known. Furthermore, the coefficients may or may not be known. The approach followed here is to assume that all physical quantities involved in the collision are known except certain combinations of velocity components and three coefficients. The method of least squares is used to derive a set of equations. Solution of these equations is used to determine the unknowns based upon experimental values of the final velocity components of impact. The least squares equations are nonlinear; this is a nonlinear parameter estimation problem. Results of solutions for four of the staged collisions are compared with measurements.

## DEFINITION OF MODEL AND PROBLEM

Fig 1 shows free body diagrams of two vehicles involved in a planar collision. Each vehicle has three initial velocity components (at the instant contact begins) and three final velocity components (at the instant of separation). The quantities $v_{ax}$, $v_{ay}$, $V_{ax}$, and $V_{ay}$ are the

*Throughout this paper, the term "experimental value" refers to any value of a variable obtained independently from and used as input to the estimation problem.

velocity components* of the mass center of vehicle A measured relative to a fixed coordinate system as shown in Fig. 1. The components $\omega_a$ and $\Omega_a$ are the angular velocities of vehicle A. Similar quantities exist for vehicle B. A set of six algebraic equations has been derived (6) which relates these velocities and forms the model. In matrix form, these equations can be written as

$$f = AV - Cv = 0 \tag{1}$$

The matrices A and C are square (6 x 6) and contain all of the physical and geometrical parameters of the problem. These matrices are listed in detail in the Appendix. The symbol V is used to represent the column vector of the final velocity components where

$$V^{-1} = \{V_1, V_2, V_3, V_4, V_5, V_6\} = \{V_{ax}, V_{ay}, V_{bx}, V_{by}, \Omega_a, \Omega_b\}$$

Similarly, the symbol v is a vector of initial velocity components

$$v^{-1} = \{v_1, v_2, v_3, v_4, v_5, v_6\} = \{v_{ax}, v_{ay}, v_{bx}, v_{by}, \omega_a, \omega_b\}$$

The matrices A and C contain three coefficients, e, $e_m$ and $\mu$. The first, e, is the classical coefficient of restitution. The second, $e_m$ , is a newly defined (6) moment coefficient of restitution. The last, $\mu$ , is an equivalent coefficient of inter-vehicular friction. The coefficients in vector form are

$$c^{-1} = \{c_1, c_2, c_3\} = \{e, e_m, \mu\}$$

The parameter estimation problem associated with this model involves finding all of the velocity components and coefficients which best fit the model when experimental values exist for some of the velocities and where experimental values of the coefficients may or may not be available. Two particular cases typically arise; these are the following:

PROBLEM I: When data is available from staged collisions, certain assumptions appear reasonable. The initial velocities of the vehicles are fairly accurately controlled and can be assumed to be known exactly. Replicates may exist, or at times data from experiments with common collision geometry can be combined. The final velocity components and coefficients which best fit the model can be computed and compared with the experimentally measured final velocities. No experimental values exist for the coefficients. This is defined as Problem I.

PROBLEM II: A second problem arises when the above model is applied to an accident reconstruction. In this case, some of the initial velocity components may be known (For example, if both vehicles are travelling along straight paths prior to impact, the initial angular velocities $\omega_a$ and $\omega_b$ are zero.) Experimental values** of some or all of the final collision velocities are available. Furthermore, values of the coefficients may or may not exist. This is called Problem II.

The method of least squares is used to derive a set of equations which can be used to obtain the solution to both Problem I and Problem II. It can be noted that Problem I is a special case of Problem II.

ESTIMATION BY LEAST SQUARES

A typical approach to the parameter estimation problem of the model given by Eq. 1 is to define a sum of squares S , where

$$S = \sum_{i=1}^{6} \sum_{j=1}^{n} f_{ij}^2 \tag{2}$$

In S, $f_{ij}$ represents the ith model equation and the sum over j indicates that $f_i$ is evaluated for each of n sets of experimental values (velocity components). This approach is satisfactory for Problem I but not for Problem II. Since the number of experimental values for each velicity component can differ from one to the other, an alternative sum of squares, Q, is used. A sum Q which is general enough for Problem II is

*Throughout, upper case velocity symbols refer to final velocities and lower case symbols refer to initial velocities.

**These may be eyewitness estimates, values from post-impact trajectory calculations, estimates based upon damage, etc.

$$Q = \sum_{i=1}^{6} \sum_{p=1}^{n_i} w_i(V_i - V_{ip})^2 + \sum_{j=1}^{6} \sum_{q=1}^{n_j} w_j(v_j - v_{jq})^2 + \sum_{k=1}^{3} \sum_{r=1}^{n_k} w_k(c_k - c_{kr})^2 + \sum_{t=1}^{6} \lambda_t f_t \quad (3)$$

$$Q = Q_1 + Q_2 + Q_3 + Q_4 \quad (3\text{-a})$$

In Eq. 3-a each term of Eq. 3 is represented, respectively, by a separate symbol. With Q, each variable independently can have a different number, $n_i$, $n_j$, or $n_k$ of experimental values. The $w_j$'s are arbitrary weighting factors which can be used to force uniform dimensionality of Q. The last sum in Eq. 3 includes Lagrange multipliers, $\lambda_t$, to introduce the model equations as constraints. The sum Q is minimized with respect to each of the 6 final velocity components, each of the 6 initial velocity components and the 3 coefficients. This provides 15 equations which along with the original 6 model equations (constraints) constitutes a set of 21 nonlinear algebraic equations. The 15 equations obtained from minimization of Q will be placed into 3 groups.

Setting to zero the partial derivatives of Q with respect to the final velocity components, $V_i$, gives

$$A^T\lambda + V^* = 0 \quad (4)$$

In Eq. 4, $\lambda$ is the column vector of 6 Lagrange multipliers and $V^*$ is a vector whose ith component is

$$V_i^* = 2w_i n_i (V_i - \bar{V}_i) \quad (5)$$

Where

$$\bar{V}_i = \Big(\sum_{p=1}^{n_i} V_{ip}\Big)/n_i .$$

Similarly a second set of 6 equations comes from $\partial Q/\partial v_j = 0$. These are

$$C^T\lambda - v^* = 0 \quad (6)$$

where the jth component of the vector $v^*$ is

$$v_j^* = 2w_j n_j (v_j - \bar{v}_j) \quad (7)$$

and

$$\bar{v}_j = \Big(\sum_{q=1}^{n_j} v_{jq}\Big)/n_j \quad .$$

The last set of equations, from $\partial Q/\partial c_k = 0$, is

$$\lambda_s \frac{\partial f_s}{\partial c_k} + 2w_k n_k (c_k - \bar{c}_k) = 0 \quad (8)$$

$k = 1, 2, 3$ and $s = 3, 6, 4$ resp

and

$$\bar{c}_k = \Big(\sum_{r=1}^{n_r} c_{kr}\Big)/n_r$$

The subscript s takes on the peculiar values because of the manner in which the coefficients appear in the matrix A (see the Appendix).

At this point, Eq. 4, 6, 8 and the original model, Eq. 1 form a set of 21 equations and 21 unknowns. The unknowns are the 6 final velocity components $V_i$, 6 initial velocity components, $v_j$, the three coefficients $c_k$ and the Lagrange multipliers, $\lambda_\ell$.

SOLUTION OF ESTIMATION PROBLEM

Three different methods of solution were tried for the estimation problem. One was to treat the 21 equations as a set of 21 nonlinear algebraic equations and to obtain a numerical solution from an existing computer program (7). No meanigful solutions were obtained (probably not through the fault of the existing program, however). A second unsuccessful approach with several variations was based upon iteration of linear subgroups of the 21 equations. For example, if a set of coefficients and initial velocities are assumed known, Eq. 4 can be solved for the Lagrange multipliers. When these multipliers are placed into Eq. 6, a new set of initial velocities are obtained. Similarly, Eq. 8 can provide a new set of coefficients. The entire process is repeated. For this scheme and other variations, convergence was poor at best. A technique which ultimately worked exceptionally well is best explained by discussing Problem I and Problem II individually.

Fig. 2 summarizes the definition of Problem I and the solution strategy. By definition of the Problem, $Q_2$ and $Q_3$ of Eq. 3-a are identically zero. The last term $Q_4$ , is zero because the solution is sought by direct search using Eq. 1; thus Lagrange multipliers are not required. The search method used consisted of finding a gradient line based upon numerical, incremental derivatives of Q with respect to each coefficient. The search for the minimum along a gradient line followed the method of golden section (8).

Fig. 3 summarizes the definition of and solution scheme for Problem II. The difference between the solution schemes for the two Problems is that an iteration for the Lagrange multiplier solution values is added to that of Problem I and the direct search is for the minimum of the entire sum Q , as shown.

## RESULTS AND CONCLUSIONS

The least square equations have been solved numerically for both Problem I and II on an Apple II digital computer. Results are presented here for examples of Problem I only. Ref. 5 reports the results of 12 staged collisions; data from 4 collisions is used with the corresponding test numbers. (The acronym of the title of Ref. 5 is RICSAC and is used with the test numbers.) Fig. 4 shows the collision geometry. Except for initial speeds, tests 6 and 7 were identical as were tests 9 and 10. Table 1 lists pertinent data used in the least square solutions.

The solution of Problem I (see Fig. 2) involves a direct search of the surface defined by the sum of squares. The mean of the estimates for each unknown was used as an initial guess. A typical search covered 12 to 16 gradient points on the response surface to find the minimum.

Table 2 contains a detailed summary of the data and results from one staged collision, RICSAC 7. Part b of the table can be used to compare the least square solution values for the final velocity components with the measured values. Two cases are listed. For one, the coefficient $e_m$ was held to the value 1. For the other, $e_m$ was "unconstrained". A value of $e_m$=1 corresponds physically to zero moment impulse, M = 0, at the impact point (see Fig. 1). The significance of the range of permissible values of $e_m$ is discussed in more detail in Ref. 6. From the results in Table 2, it appears that both the constrained and unconstrained values of $e_m$ give results not remarkably different in so far as the final velocities are concerned. Part c of Table 2 contains the solution values of all of the coefficients. These coefficients are the model parameter values which the least square solution provides. (They are analagous to the slope and intercept values of the classical least square fit of a straight line). Because of the complexity of the model and because each staged collision provides only one estimate for each final velocity component, no statistical confidence bounds are sought.

One of the overall measures of the severity of a collision (9) is the magnitude of the velocity change of a vehicle, ΔV. Table 3 shows the initial speeds for RICSAC Tests 6, 7, 9 and 10. Table 4 shows the velocity changes, both measured and computed. The table also contains the corresponding coefficients of restitution, e , and equivalent friction coefficient, μ (for these examples, the impulse moment coefficient $e_m$ was arbitrarily constrained to be 1). Prior to the formulation of this estimation problem, no systematic method has ever been developed to estimate these coefficients. The values in Table 4 represent the first experimentally determined values from actual vehicle collisions.

In general, the accuracy of the least square estimates of the ΔV's are fairly good. The differences between the measured and computed values range from 0.2 mph to 4.1 mph. The estimates of the coefficients are reasonably consistent within each pair of the collisions but do differ significantly with the collision geometry (see Fig. 4).

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## APPENDIX. MATRICES A AND C FOR COLLISION MODEL, EQ 1

MATRIX A:

$$
\begin{matrix}
m_a & 0 & m_b & 0 & 0 & 0 \\
0 & m_a & 0 & m_b & 0 & 0 \\
\cos\Gamma & \sin\Gamma & -\cos\Gamma & -\sin\Gamma & \eta & \zeta \\
-\alpha m_a & \beta m_a & \alpha m_b & -\beta m_b & 0 & 0 \\
-d_{13}m_a & d_{24}m_a & d_{13}m_b & -d_{24}m_b & I_a & I_b \\
\gamma e_m m_a & -\delta e_m m_a & -\gamma e_m m_b & \delta e_m m_b & 1-2e_m & 2e_m-1
\end{matrix}
$$

MATRIX C:

$$
\begin{matrix}
m_a & 0 & m_b & 0 & 0 & 0 \\
0 & m_a & 0 & m_b & 0 & 0 \\
-e\cos\Gamma & -e\sin\Gamma & e\cos\Gamma & e\sin\Gamma & -e\eta & -e\zeta \\
-\alpha m_a & \beta m_a & \alpha m_b & -\beta m_b & 0 & 0 \\
-d_{13}m_a & d_{24}m_a & d_{13}m_b & -d_{24}m_b & I_a & I_b \\
\gamma e_m m_a & -\delta e_m m_a & -\gamma e_m m_b & \delta e_m m_b & -e_m & e_m
\end{matrix}
$$

Where

$$2d_{13} = d_b\sin(\theta_b + \phi_b) + d_a\sin(\theta_a + \phi_a) \qquad \alpha = \sin\Gamma + \mu\cos\Gamma$$

$$2d_{24} = d_b\cos(\theta_b + \phi_b) + d_a\cos(\theta_a + \phi_a) \qquad \beta = \cos\Gamma - \mu\sin\Gamma$$

$$2\gamma = d_a\sin(\theta_a + \phi_a)/I_a - d_b\sin(\theta_b + \phi_b)/I_b \qquad \eta = d_a\sin(\theta_a + \phi_a - \Gamma)$$

$$2\delta = d_a\cos(\theta_a + \phi_a)/I_a - d_b\cos(\theta_b + \phi_b)/I_b \qquad \zeta = d_b\sin(\theta_b + \phi_b - \Gamma)$$

TABLE 1. VEHICLE AND COLLISION DATA

| | RICSAC | 6&7 | RICSAC | 9&10 |
|---|---|---|---|---|
| | Veh A | Veh B | Veh A | Veh B |
| MAKE | Chevrolet | VW Rabbit | Honda | Ford |
| YEAR | 1974 | 1975 | 1975 | 1974 |
| TEST MASS, m, kg | 1955. | 1197. | 1036. | 2189. |
| YAW INERTIA, I, kg-m$^2$ | 4723. | 2279. | 1348. | 5287. |
| DISTANCE, d, m | 1.8 | 0.9 | 1.4 | 1.6 |
| ANGLE, $\phi$, deg | 0 | -140 | 0 | -30 |
| ORIENTATION, $\theta$, deg | 0 | 60 | 0 | 90 |
| IMPACT SURFACE ANGLE, $\Gamma$, deg | -30 | | 0 | |

TABLE 2. DETAILED RESULTS OF COLLISION 7

a. Initial Velocity Components, m/sec (mph)

| | $v_{ax}$ | $v_{ay}$ | $\omega_a$ | $v_{bx}$ | $v_{by}$ | $\omega_b$ |
|---|---|---|---|---|---|---|
| Measured Values | -13.01(-29.1) | 0 | 0 | 6.50(14.6) | 11.27(25.2) | 0 |

b. Final Velocity Components, m/sec (mph)

| | $V_{ax}$ | $V_{ay}$ | $\Omega_a$ | $V_{bx}$ | $V_{by}$ | $\Omega_b$ |
|---|---|---|---|---|---|---|
| Measured Values | -7.60(-17.0) | 1.38(3.1) | -1.13 | -2.55(-5.7) | 8.73(19.5) | -3.67 |
| Least Squares, $e_m$ Constrained | -7.71(-17.3) | 1.42(3.2) | -1.08 | -2.15(-4.8) | 8.94(20.0) | -4.28 |
| Least Squares, $e_m$ Unconstrained | -7.33(16.4) | 1.42(3.2) | -1.87 | -2.78(-6.2) | 8.94(20.0) | -2.94 |

TABLE 2. RESULTS OF COLLISION 7 (cont'd)

c. Coefficients

| | $e$ | $e_m$ | $\mu$ |
|---|---|---|---|
| $e_m$ Constrained | .085 | 1.00 | 1.001 |
| $e_m$ Unconstrained | .015 | -.774 | .968 |

TABLE 3. INITIAL SPEEDS, m/sec(mph)

| Collision | Veh A | | Veh B | |
|---|---|---|---|---|
| RICSAC 6 | 9.61 | (21.5) | 9.61 | (21.5) |
| RICSAC 7 | 13.01 | (29.1) | 13.01 | (29.1) |
| RICSAC 9 | 9.48 | (21.2) | 9.48 | (21.2) |
| RICSAC 10 | 14.89 | (33.3) | 14.89 | (33.3) |

TABLE 4. SUMMARY OF FOUR COLLISIONS

($e_m$ = 1 by Constraint)

| | Collision 6 | | Collision 7 | | Collision 9 | | Collision 10 | |
|---|---|---|---|---|---|---|---|---|
| $\Delta V_a$, Measured | 4.15 | (9.3) | 5.58 | (12.5) | 9.78 | (21.9) | 15.86 | (35.5) |
| $\Delta V_a$ Least Squared | 4.33 | (9.7) | 5.48 | (12.3) | 8.87 | (19.8) | 14.05 | (31.4) |
| $\Delta V_b$, Measured | 7.00 | (15.7) | 9.41 | (21.0) | 3.68 | (8.2) | 5.69 | (12.7) |
| $\Delta V_b$, Least Squares | 7.09 | (15.9) | 8.96 | (20.0) | 4.20 | (9.4) | 6.65 | (14.9) |
| e (restitution) | .073 | | .085 | | .467 | | .475 | |
| μ (friction) | 1.138 | | 1.001 | | .486 | | .492 | |

| | | | | |
|---|---|---|---|---|
| $\Delta V_a$ SMAC | (10.8) | (11.4) | (20.0) | (36.5) |
| $\Delta V_a$ CRASH | (12.4) | (11.6) | (24.2) | (32.6) |
| $\Delta V_b$ SMAC | (16.7) | (17.9) | (8.7) | (15.7) |
| $\Delta V_b$ CRASH | (20.4) | (25.3) | (11.2) | (15.9) |

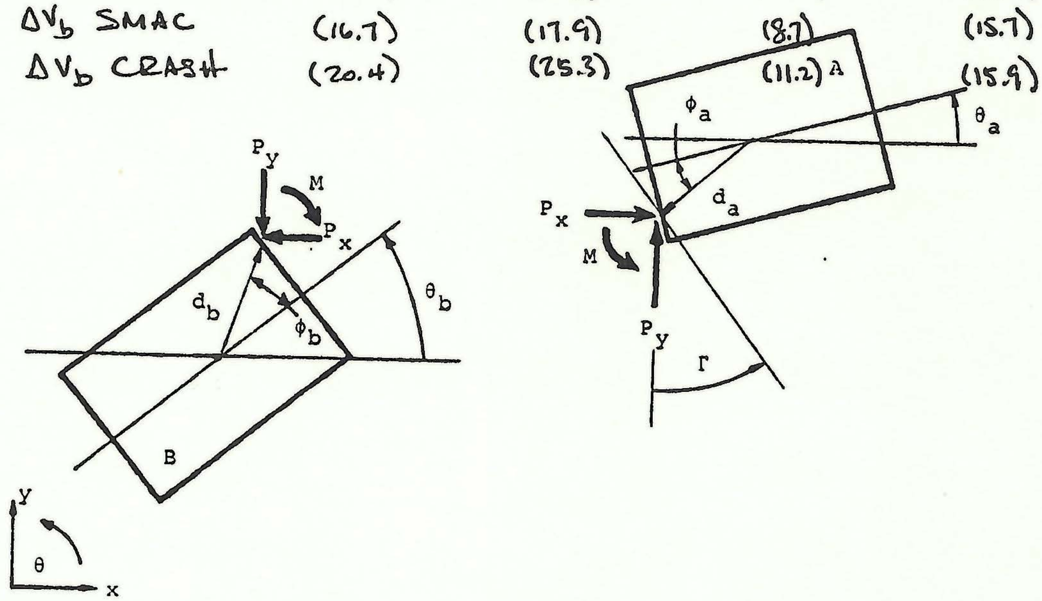



FIG 1. VEHICLE FREE BODY DIAGRAMS

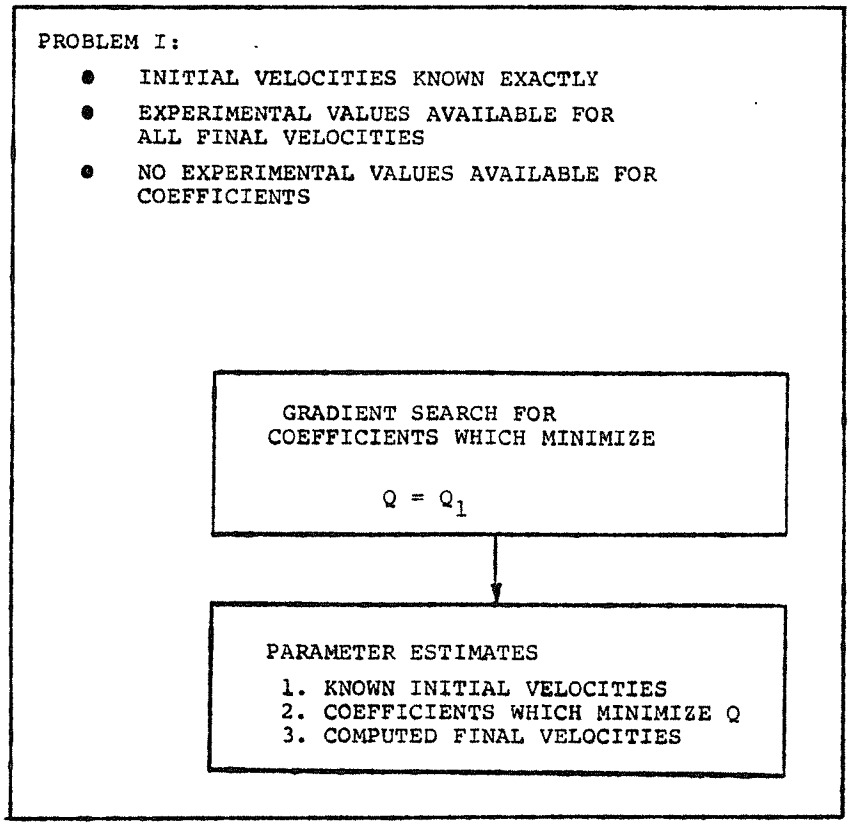



FIG 2. DEFINITION AND SOLUTION SCHEME FOR PROBLEM I

PROBLEM II:

- EXPERIMENTAL VALUE(S) AVAILABLE FOR SOME INITIAL VELOCITY COMPONENTS
- EXPERIMENTAL VALUE(S) AVAILABLE FOR ALL FINAL VELOCITY COMPONENTS
- EXPERIMENTAL VALUE(S) AVAILABLE FOR SOME COEFFICIENTS

GRADIENT SEARCH FOR COEFFICIENTS WHICH MINIMIZE

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

PARAMETER ESTIMATES

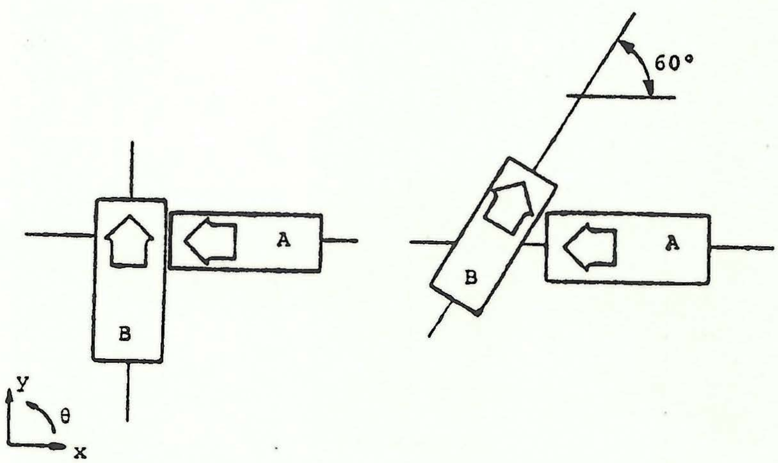
1. UNKNOWN INITIAL VELOCITIES
2. COEFFICIENTS WHICH MINIMIZE Q
3. COMPUTED FINAL VELOCITIES

CALCULATE LAGRANGE MULTIPLIERS
COMPARE WITH PREVIOUS VALUES

SIGNIFICANT CHANGE

INSIGNIFICANT CHANGE
CONVERGENCE

FIG 3. DEFINITION AND SOLUTION SCHEME FOR PROBLEM II




a. RICSAC 9 & 10 b. RICSAC 6 & 7

FIG 4. COLLISION GEOMETRY