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Modeling Combined Braking and Steering Tire Forces

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ABSTRACT

The force distributed over the contact patch between a tire and a road surface is typically modeled in component form for dynamic simulations. The two components in the plane of the contact patch are the braking, or traction force, and the steering, or side or cornering force. A third force distributed over the contacts patch is the normal force, perpendicular to the road surface. The two tangential components in the plane of the road are usually modeled separately since they depend primarily on independent parameters, wheel slip and sideslip. Mathematical expressions found in the literature for each component include exponential functions, piecewise linear functions and the Bakker-Nyborg-Pacejka equations, among others. Because braking and steering frequently occur simultaneously and their resultant tangential force is limited by friction, the two components must be properly combined for a full range of the wheel slip and sideslip parameters. This paper examines the way in which these two components are combined for an existing approach known as the Nicolas-Comstock model.

First, performance criteria for tire modeling are proposed. Then the Nicolas-Comstock model is examined relative to the criteria. As originally proposed, this model falls short of meeting the criteria over the full range of transverse and longitudinal wheel slip values and sideslip angles. A modified version of the Nicolas-Comstock model is presented that satisfies these performance criteria. Finally, comparisons are made of the Modified Nicolas-Comstock model to other combined tire force models and to existing tire force measurements.

INTRODUCTION

Aside from gravitational forces, aerodynamic forces and intervehicular forces developed during collisions, the forces and moments generated by the interaction between the tires of a vehicle and the ground normally control the motion of the vehicle. Hence, vehicle dynamicists require a means by which these forces can be computed. Since the 1930's, numerous models have been presented for use in predicting the forces and moments at the tire-road interface.

In order to establish a standard within the industry, the SAE has recommended a reference tire axis system as shown in Figure 1. This figure shows all the moments and forces associated with a wheel. The three moments shown in the figure are not discussed in this paper because the topic of interest is the way in which F_x and F_y are modeled and related under combined braking and steering conditions. An analytical approach has been developed which includes all tire forces and moments [Gim and Nikravesh, 1990, 1991a, 1991b].

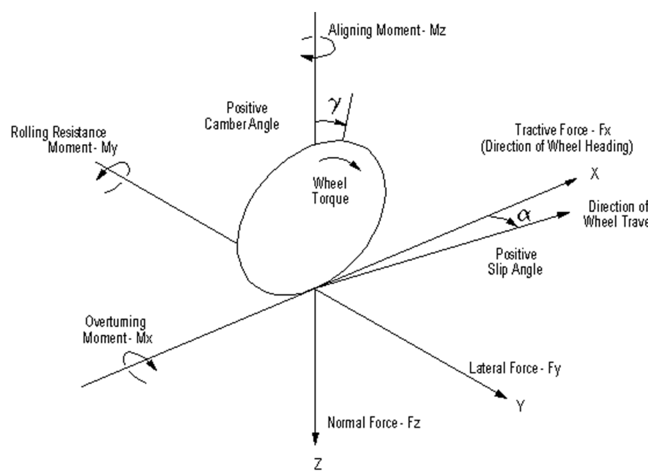

Figure 1.

The x axis in Figure 1 is the intersection of the wheel plane and the road plane and the y axis is perpendicular to x and in the plane of the road. The force F_x is referred to as the longitudinal force. It can be developed by engine torque as a driving or traction force (forward or reverse) or can be developed by brake application as a braking force. F_y is referred to as the transverse or lateral force. It is sometimes called a side force, steering force or cornering force, and is perpendicular to the heading of the wheel. The resultant of these components, F, is in the plane of the contact patch where $F^2 = F_x^2 + F_y^2$ and is generated through friction between the tire and road surface. Initially in this paper, the modeling of F_x and F_y are examined independently from each other. Later, the paper focuses on modeling these forces under the condition of combined steering and braking. This model presents an alternative to existing models [Nguyen and Case, 1975, and Schuring, et al., 1996] for use in vehicle dynamic simulations.

Over the years various mathematical functions have been use to represent F_x and F_y. Various methods have been proposed [Nguyen and Case, 1975; Schuring, et al. 1996] to combine them to model conditions of simultaneous braking/traction and steering. In order to model tire forces properly, F_x and F_y and their combination must meet certain conditions and so a set of performance criteria is presented. An existing model by Nicolas and Comstock, 1972, allows any functions to be used for F_x and F_y and is a relatively straightforward approach. However, since it does not meet all of the performance criteria, a modification is proposed to improve the model. Tire data[1] collected by the SAE [Anonymous, 1995] is used to assess the modified Nicolas-Comstock model. This modeling technique is compared to other techniques, and the results are discussed.

LONGITUDINAL AND TRANSVERSE FORCE COMPONENTS

THE LONGITUDINAL FORCE – The longitudinal force at the tire patch can be a tractive force or a braking force. Figure 2 shows a side profile of a wheel rotating about its axle with an angular velocity ω. The longitudinal force, F_x, is a traction force acting in the direction of the velocity, V_F, of the wheel at the tire-ground interface. The force, F_z, is the normal reaction force of the ground on the wheel. The longitudinal force is known to be a nonlinear function of the wheel slip s. Wheel slip can be defined in several ways [see Wong, 1993].

For a braking wheel,

$$s = \frac{V_F}{V_W} = \frac{V_W - R\omega}{V_W} = 1 - \frac{R\omega}{V_W}$$

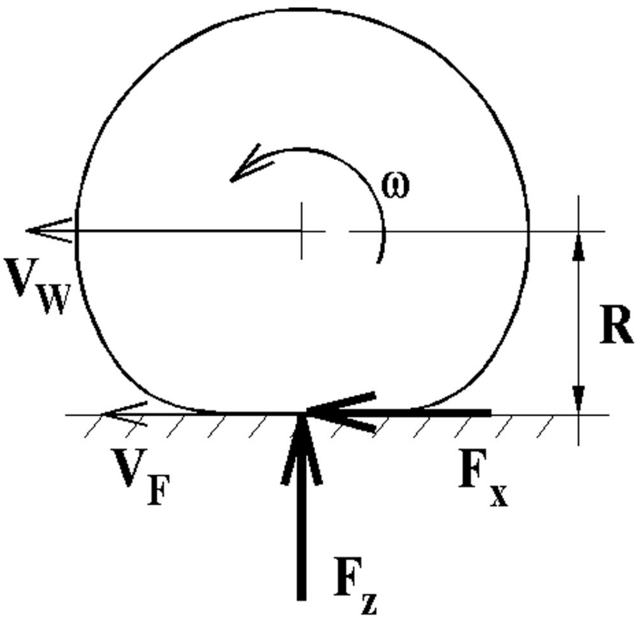

Figure 2.

where R is the effective rolling radius of the wheel and V_W is the hub velocity of the wheel and V_F is the forward velocity at the contact patch. For a freely rolling wheel, the forward velocity $V_W = R\omega$ and s = 0, i.e. no slipping occurs between the wheel and the ground. If the wheel is locked from rotation by braking, ω = 0 and s = 1, corresponding to a locked wheel skid. For a wheel with positive traction such as F_x in Figure 2, slip can be defined as

$$s = -\frac{V_F}{R\omega} = \frac{R\omega - V_W}{R\omega} = 1 - \frac{V_W}{R\omega}$$

Here, for the ideal case when engine torque is generating a forward force F_x and $V_W = R\omega$, then s = 0. If the forward velocity is restrained to be zero but the wheel is rotating, then s = 1. A reason for the two different definitions is that the latter case will produce a slip of ∞ from Equation 1. With the two different definitions of wheel slip, the range of s is $0 \le s \le 1$. In the following, Equation 1 will apply to braking and Equation 2 to forward traction, so that in all cases, $0 \le s \le 1$.

Longitudinal forces, both traction and braking, are functions of the wheel slip s, that is, $F_x = F_x(s)$. A wheel that is locked from rotation and moving in the direction of positive V_W produces a longitudinal force F_x such that $F_x = \mu_x F_z$, where μ_x is the coefficient of sliding friction between the tire and the ground in the longitudinal direction. It is convenient to look at the longitudinal force in a normalized form, where

$$Q_x(s) = \frac{F_x(s)}{\mu_x F_z}$$

[1] These data were collected by the SAE Truck Tire Characteristic Task Force pursuant to NHTSA Contract DTNH22-92-C-17189. The data are available from SAE Cooperative Research, Warrendale, PA.

Figure 3 shows a typical distribution of a longitudinal force $Q_x(s)$. Initially, a braking force is approximately linear with slope C_s, often called the slip stiffness. $F_x(s)$ grows nonlinear as s increases beyond a value of about s = 0.1, and typically reaches a maximum in the range 0.15 < s < 0.30. On most roadway surfaces the braking force decreases as the slip s is increased beyond the maximum. The normalization process in Equation 3 creates the condition that the longitudinal force is equal to $\mu_x F_z Q_x(s)$. The shape of this curve, and its associated maximum, is one of the reasons for the development of anti-lock braking systems. One function of an anti-lock brake system is to maintain the forward force, F_x, near the maximum.

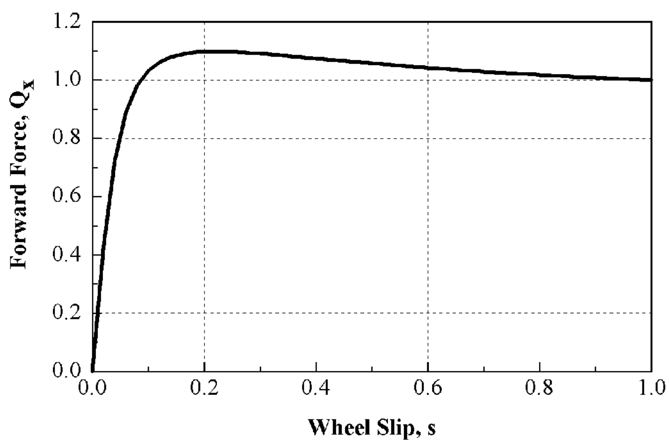

Figure 3.

THE TRANSVERSE TIRE FORCE – The transverse tire force lies in the plane of the tire patch, is perpendicular to the heading of the wheel, and opposes the transverse velocity V_T which is shown in Figure 4. It is this transverse force that provides directional control of a vehicle. During a steering maneuver, the heading angle of the wheel and the resultant velocity, V_R, differ by an angle α which is called the sideslip angle (or the slip angle) as shown in Figure 4. The sideslip angle is therefore defined as follows:

$$\alpha = \tan^{-1}\left(\frac{V_T}{V_W}\right)$$

where V_T and V_W are shown in Figure 4.

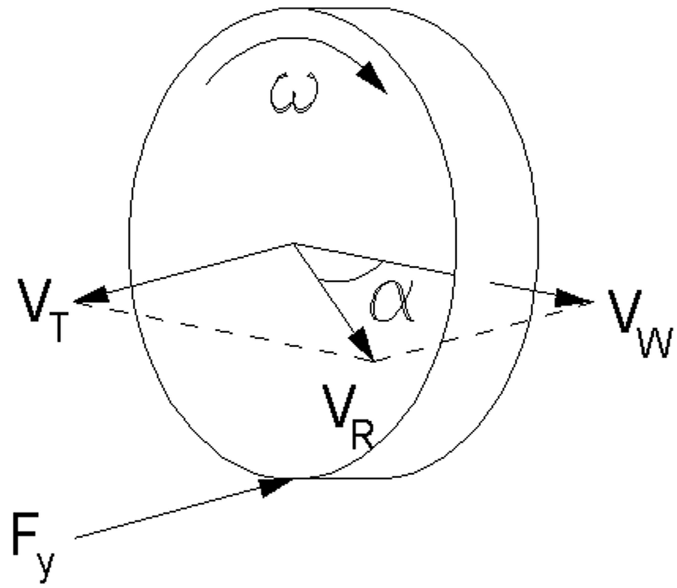

Figure 4.

For a freely rolling wheel (s = 0), the transverse force $F_y = F_y(\alpha)$ and is a nonlinear function of slip angle α. For no transverse velocity, $\alpha = 0$, the transverse force is 0 and corresponds to where the wheel velocity is aligned with its heading. At another extreme, when the slip angle $\alpha = \pi/2$, the transverse force is normal to the wheel's heading and is equal to the coefficient of sliding friction in the transverse direction times the normal force at the wheel, $F_y = \mu_y F_z$. A normalized version of the transverse force can be defined as follows:

$$Q_y(\alpha) = \frac{F_y(\alpha)}{\mu_y F_z}$$

A representative plot of a normalized transverse force $Q_y(\alpha)$, is shown in Figure 5. This shows that $Q_y(\alpha)$ is approximately linear for small values of the sideslip angle α. As α increases, $Q_y(\alpha)$ becomes nonlinear as the slope decreases and the normalized force approaches 1 as α approaches $\pi/2$. The slope of the actual force $F_y(\alpha)$ at $\alpha = 0$ is called the sideslip stiffness, C_α, also referred to as the cornering stiffness. $F_y(\alpha)$ also approaches the road friction limit $\mu_y F_z$ at $\alpha = \pi/2$.

According to Coulomb's theory of rigid body friction, the coefficient of sliding friction is independent of direction and load, i.e. $\mu_y = \mu_x = \mu$. However, experimental data has shown that for tires, differences between these values do occur, [Warner, et al. (1983)], so these coefficients are considered distinct. Following the above preliminary concepts, individual mathematical models for the longitudinal and transverse forces are discussed and presented.

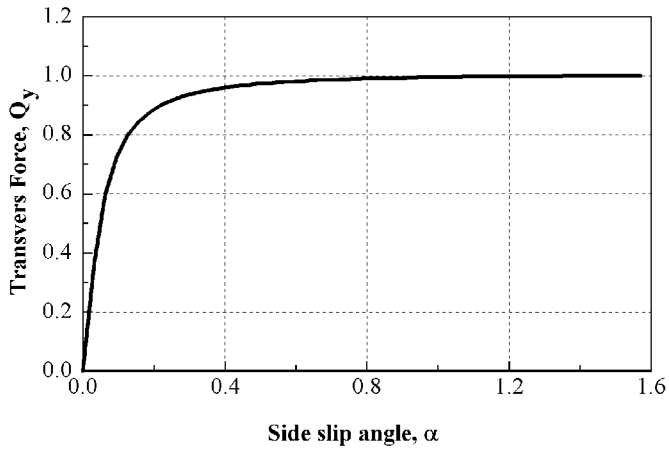

Figure 5.

TIRE FORCE EQUATIONS – In the past and depending on the application, various mathematical equations have been used to represent tire forces including piece-wise linear approximations and exponential functions, among others; for details, see Nguyen and Case, (1975). Recent work by Bakker, Nyborg and Pacejka (1987) produced a convenient tire force formula based on a combination of trigonometric and algebraic functions. This is referred to in this paper as the BNP model or the BNP equations. The BNP model applies equally well to both the longitudinal and transverse forces (as well as contact patch moments). Due to its convenient form and its ability to fit experimental data reasonably well, the BNP model has supplanted previous methods and has been adopted by various authors. These include, Schuring, et. al. (1993) and d'Entremont, (1997). It also has been called the "Magic Formula". Because of the versatility of the BNP model, it is used in this paper as well. A short description of the model follows, in which the equations are presented in a normalized form.

The BNP tire force equation expresses a tire force, P, as a function of a parameter, u, where $0 \le u \le 1$ and can represent either longitudinal wheel slip, s, or sideslip angle, α. The force Q is used here where Q(u) = P(u)/P(1), where $P(1) = P(u)|_{u=1}$. The BNP equations in normalized form can be written as:

$$Q(u) = \frac{P(u)}{P(1)} = \frac{D \sin [C \tan^{-1} (B \phi)]}{P(1)}$$

and

$$\phi = (1 - E) Ku + \left(\frac{E}{B}\right) \tan^{-1} (B Ku)$$

The parameters B, C, D and E are constants chosen to model specific wheel and tire systems and/or to give forces that correspond to specific experimental data. (The original BNP equations contain additional constants S_v and S_h that permit a vertical and horizontal offset of the origin to allow a more accurate match to experimental data; for simplicity, these are set to zero here.) For the longitudinal force model, the constant, K, is given a value of 100 so that $0 \le Ku \le 100$ corresponds to a percentage wheel slip. For the transverse force, the constant, K, is given a value of 90 so that $0 \le Ku \le 90$ corresponds to degrees of sideslip. Furthermore, since the forces are normalized to their value at u = 1, the constant D is given the value of 1. When modeling a longitudinal force, u represents longitudinal wheel slip s. When modeling a transverse force, the sideslip angle is such that $0 \le \alpha \le \pi/2$ and so $u = 2\alpha/\pi$. The initial slope of Q(u) is BCK/P(1) which is the first derivative of Q(u) evaluated at u = 0 with D = 1 (see Appendix 1). Equations 6 and 7 are used with different constants B, C and E, to model longitudinal and transverse tire force components. The force magnitudes are found simply by multiplying each Q(u) by its appropriate limiting frictional force. That is, $F_x(s) = Q(s)A\mu_x F_z$ and $F_y(\alpha) = Q(\alpha)A\mu_y F_z$. The initial slopes of $F_x(s)$ and $F_y(\alpha)$ are the stiffness coefficients, so

$$C_s = \frac{BCK}{P(1)} \mu_x F_z$$

and

$$C_\alpha = \frac{BCK}{P(1)} \mu_y F_z$$

respectively. When modeling wheel and tire systems with specific, known stiffness coefficients, C_s and C_α, the shape and curvature factors, *C* and *E*, respectively, can be chosen to establish the appropriate shape of the tire force function. Equations 8 and 9 can then be used to solve for the corresponding value of the stiffness factor, *B*.

COMBINED LONGITUDINAL AND TRANSVERSE FORCES

CRITERIA FOR COMBINING THE TRANSVERSE AND LONGITUDINAL FORCE COMPONENTS – With the appropriate coefficients B, C, D, E, and K, the BNP equations provide expressions for the longitudinal force component $F_x(s)$ when there is no sideslip angle, α, and for the transverse force component $F_y(\alpha)$ when there is no wheel slip. During combined steering and braking, however, each of these components will simultaneously be a function of both the longitudinal wheel slip s, and the sideslip angle α. Hence, the force components must be described as $F_x(s,\alpha)$ and $F_y(\alpha,s)$, respectively. The approach taken here is to combine the individual longitudinal and transverse force component equations in such a way to produce a single force, F, tangent to the roadway surface where

$$F(\alpha, s) = \sqrt{F_x^2(\alpha, s) + F_y^2(\alpha, s)}$$

with components collinear with and perpendicular to the heading of the tire, and both in the road plane.

All models of the combined tire force, regardless of the formulation, must provide realistic results. To do this, the following performance criteria are proposed for the process of combining the transverse and longitudinal components:

1. the combined longitudinal force component $F_x(s,\alpha)$ should approach the longitudinal component $F_x(s)$ as $\alpha \rightarrow 0$, i.e. $F_x(\alpha,s)|_{\alpha \rightarrow 0} \rightarrow F_x(s)$;
2. the combined transverse force component $F_y(s,\alpha)$ should approach the transverse component $F_y(\alpha)$, as $s \rightarrow 0$, i.e. $F_y(\alpha,s)|_{s \rightarrow 0} \rightarrow F_y(\alpha)$;
3. the combined force, $F(\alpha,s)$, must be friction limited,
4. the combined tire force $F(\alpha,s)$ must agree at least approximately with experimental results; and
5. the combined tire force $F(\alpha,s)$ should produce a force equal to μF_z in a direction opposite to the velocity vector for a locked wheel skid (s = 1) for any α.

It is instructional to begin the investigation of combined tire friction forces by looking at the concept referred to as the "friction circle" [see for example, Warner, et. al. (1983)]. This concept asserts that the total available friction force at the tire-road interface is given by $F_{max} = \mu F_z$, and its direction lies opposite to the direction of the resultant velocity vector, V_R. The concept further asserts that if the vector sum of the longitudinal force and transverse force is less than F_{max}, the tire will continue to track and rotate. Experiments show that tires exhibit properties that differ from the theoretical friction circle. Differences in longitudinal and transverse coefficients of friction due to such attributes as tread patterns, tire construction and tire geometry, lead to different longitudinal and transverse limiting forces. Consequently, the friction circle actually is better described as a friction ellipse. An expression for the friction ellipse can be written as:

$$\frac{F_x^2(\alpha, s)}{\mu_x^2 \ F_z^2} + \frac{F_y^2(\alpha, s)}{\mu_y^2 \ F_z^2} \leq 1$$

Equation 11 illustrates mathematically the concept that if a steering input is introduced while braking or if brakes are applied while steering, less capacity is available for each individual force than when braking or steering alone. This leads to the situation, for example, where a steering input to a wheel under severe braking (but not fully locked) can increase the force to F_{max}, causing the tire to slide out and begin to skid.

COMBINATION MODELS – While Equation 11 is useful for an intuitive understanding of the physical conditions at the tire-road interface under combined loading, it provides only a bound on the resultant force. For quantitative use of the combined force, an equation for $F(\alpha,s)$, not an equality, is needed. Various models have been proposed over the years. See for example Nicolas and Comstock (1972), Bakker, et. al. (1987), Wong (1993), and Schuring, et. al. (1996).

The model proposed by Nicolas and Comstock is based on two conditions. The first is that the resultant force, $F(\alpha,s)$ for s = 1, is collinear with and opposite to the resultant velocity, V_R. The second condition is that as α is varied from 0 to $\pi/2$, the resultant force F can be described by an ellipse of semi-axes F_x and F_y. Semi-axis F_x is the longitudinal friction force defined by a μ_x slip curve for $\alpha = 0$ and semi-axis F_y is the transverse friction force predicted by an appropriate equation for s = 0. They produce the following expressions:

$$F_x\ (\alpha, s) = F_x\ (s)\ \ F_y\ (\alpha)\ soverSQRT s^2\ F_y^2\ (\alpha) + F_x^2\ (s)\ \tan^2 \alpha$$

$$F_y\ (\alpha, s) = F_x\ (s)\ \ F_y\ (\alpha)\ \tan \alpha overSQRT s^2\ F_y^2\ (\alpha) + F_x^2 (s)\ \tan^2 \alpha$$

Note that from these two equations the ratio of $F_y(\alpha,s)$ to $F_x(\alpha,s)$ is always $(\tan\alpha)/s$.

Each equation is easily evaluated for a given s and α by first evaluating $F_x(s)$ and $F_y(\alpha)$. However, for either $\alpha = 0$ and/or s = 0, the expressions as defined above are undefined. In particular, when $\alpha = 0$, there is no transverse force on the tire, i.e. $F_y|_{\alpha=0} = 0$ by definition. With both α and $F_y(\alpha)$ equal to zero, both the numerator and the denominator of the two equations goes to zero. This is inconvenient as it is expected that $F_x(s,0)$ should reduce to $F_x(s)$. Similarly, with s and $F_x(s)$ equal to zero, both the numerator and the denominator of the two equations goes to zero. As given above the Nicolas-Comstock equations do not satisfy all of the criteria presented earlier. For small values of s and α, where it is expected that $F_x(s,\alpha) \rightarrow F_x(s)$ and $F_y(s,\alpha) \rightarrow F_y(\alpha)$, the equations produce bias factors as is now shown.

For small values of α and s, the tire is operating in the linear region of the force curves (see figures 3 and 5). Therefore, linear approximations can be used for the longitudinal and transverse forces using the wheel slip stiffness and the sideslip stiffness presented earlier. Hence, $F_x(s) \approx C_s s$ and $F_y(\alpha) \approx C_\alpha \alpha$ and $\tan\alpha \approx \alpha$. Introducing these approximations into equations 12 and 13 yields:

$$F_x\ (\alpha,\ s) = F_x\ \ (s) \frac{C_\alpha}{\sqrt{C_\alpha^2 + C_s^2}}$$

$$\text{where } 0 < \alpha << \pi/2 \text{ and } 0 < s << 1$$

$$F_y\ (\alpha,\ s) = F_y\ \ (\alpha) \frac{C_s}{\sqrt{C_\alpha^2 + C_s^2}}$$

$$\text{where } 0 < \alpha << \pi/2 \text{ and } 0 < s << 1$$

This shows that the Nicolas-Comstock equations have biased slopes in the combined linear regions of the tire force curves.

In considering the Nicolas-Comstock equations relative to the first and second performance criteria, two conditions are examined, that of straight-ahead braking (s = 1 and $\alpha = 0$) and transverse sliding (s = 0 and $\alpha = \pi/2$). Using these values in Equations 12 and 13 yields the conditions of 0/0 and ∞/∞, respectively. This demonstrates that the Nicolas-Comstock equations also do not meet the first and second performance criterion.

The bias factors shown in Equations 14 and 15 and the failure of the Nicolas-Comstock equations to meet the performance criteria can be mitigated by appropriate modification of the equations. Modified equations are presented and investigated in the next section and will be referred to as the Modified Nicolas-Comstock (MNC) equations. Typical plots of the force predicted by the MNC tire friction force model are presented. The capability of the model to match actual tire behavior is presented using data recorded by the SAE. A new way of graphically presenting the force plots is also presented.

THE MODIFIED NICOLAS-COMSTOCK MODEL – The following Modified Nicolas-Comstock equations are proposed to correct the deficiencies discussed above:

$$F_x(\alpha, s) = \frac{F_x(s)\ F_y(\alpha)}{\sqrt{s^2\ F_y^2(\alpha) + F_x^2(s)\ \tan^2\alpha}} \bullet \frac{\sqrt{s^2\ C_\alpha^2 + (1-s)^2\ \cos^2\alpha\ F_x^2(s)}}{C_\alpha}$$

$$F_y(\alpha, s) = \frac{F_x(s)\ F_y(\alpha)}{\sqrt{s^2\ F_y^2(\alpha) + F_x^2(s)\ \tan^2\alpha}} \bullet \frac{\sqrt{(1-s)^2\ \cos^2\alpha\ F_y^2(\alpha) + \sin^2\alpha\ C_s^2}}{C_s\ \cos\alpha}$$

Equations 16 and 17 provide the tire force components for any combination of the parameters α and s such that $0 \le \alpha \le \pi/2$ and $0 \le s \le 1$ for any pair of functions $F_x(s)$ and $F_y(\alpha)$. The equations have a relatively simple form for use in vehicle dynamic simulations. They easily can be extended for the range of sideslip angle such that $0 \le \alpha \le 2\pi$. It can be seen that $F_x(\alpha,s)$ and $F_y(\alpha,s)$ as given by Equations 16 and 17 approach $F_x(s)$ and $F_y(\alpha)$ respectively for $0 < \alpha << \pi/2$ and $0 < s << 1$, when $F_x(s)$. $C_S s$ and $F_y(s)$. $C_\alpha \alpha$.

A typical means of viewing models of the combined forces is to plot the combined forces for given values of α over the full range of s as shown in Figure 6 or for given values of s over the full range of α.

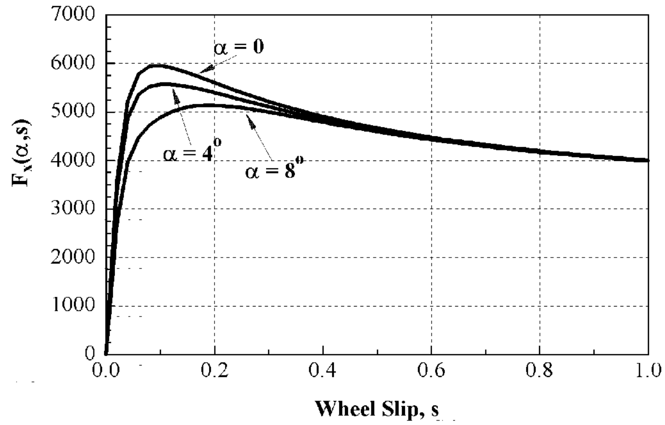

Figure 6.

An alternative means to viewing the equations is used here by displaying the data over the full range of both independent variables in one three-dimensional plot. Figures 7 and 8 are examples.

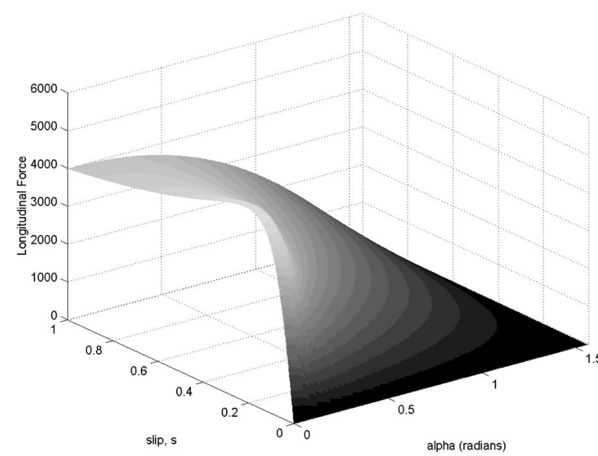

Figure 7.

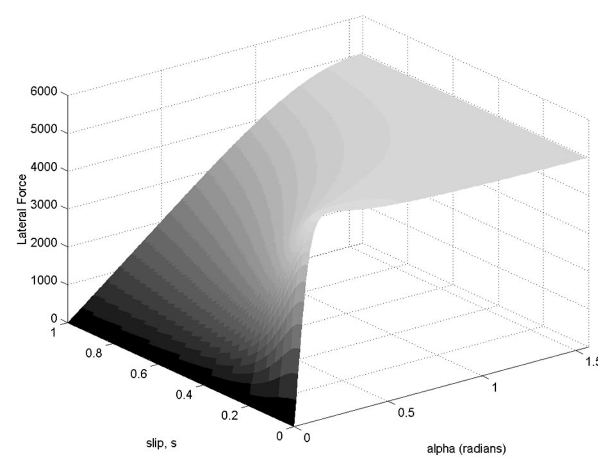

Figure 8.

COMPARISON WITH EXPERIMENTAL MEASUREMENTS OF TIRE FORCES

FIT OF BNP FORMULA TO EXPERIMENTAL DATA – The fourth performance criteria presented earlier requires that the combined tire force F(α,s) must agree at least approximately with experimental results. Heavy truck tire force data was collected through sponsorship of the SAE and has been made available for general use [Anonymous, 1995]. In addition to combined force data, free rolling cornering and straight line braking data were collected as part of the project. This individual force data has been used here with an optimization algorithm to determine the appropriate coefficients for the BNP formula, B, C, D, and E, for longitudinal forces versus s (straight line braking) and lateral forces versus α (free rolling cornering). Figures 9 and 10 show the data with the accompanying BNP curve generated by the fitting process for one normal force, F_z = 20,604 N.

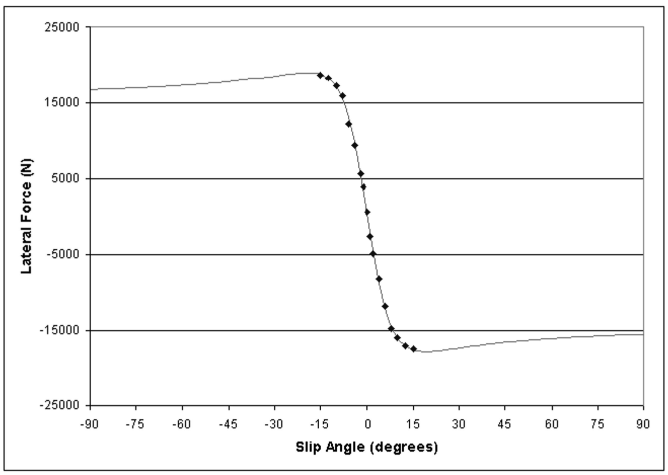

Figure 9.

The BNP coefficients and formulas then were used with the modified Nicolas-Comstock equations to compute the combined force for braking and steering F(α,s). A set of data was selected to compare the experimental to the analytical. The data chosen shows the combined force for a 295/75|275/80R22.5 truck tire for a slip angles of ±4° [Anonymous, 1995]. The resulting plot comparing the experimental and analytical data is shown in Figure 11.

The comparison shown in Figure 11 indicates that a good fit for the combined force can be obtained from MNC equations using just free-rolling cornering and straight line braking data.

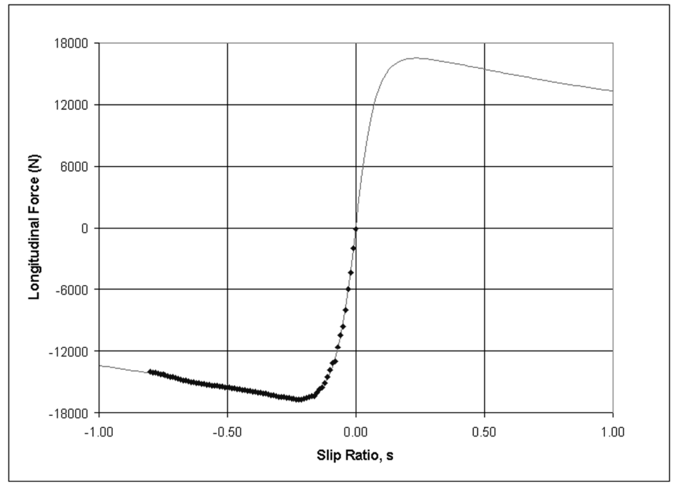

Figure 10.

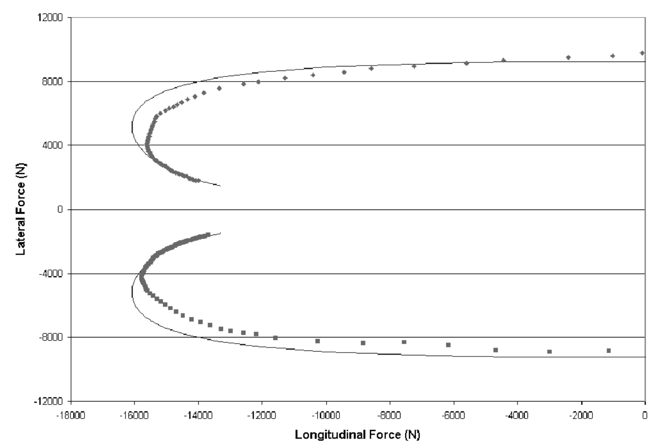

Figure 11.

DISCUSSION AND CONCLUSIONS

A model for the combined frictional force at the tire-road interface based on a modified version of the modified Nicolas-Comstock equations (MNC) has been presented. The model, in conjunction with the BNP equations or any other tire force model, provides a simple algebraic means of finding the two tire force components over a full range of combined wheel slip and side slip angles. The only requirement needed to implement the model is the availability of the straight line braking and free rolling cornering data for given conditions. By specifying the friction limits, $\mu_x F_z$ and $\mu_y F_z$, the MNC equations yield acceptable results for the prediction of the combined force at the tire-road interface. They can be used for vehicle dynamics simulations and also are useful for accident reconstruction applications.

A set of criteria has been presented that which can be applied to any model of the combined force condition at the wheel. Compliance with these criteria ensures that the model will produce useful results that represent the combined tire force system.

The Modified Nicolas-Comstock equations offers an alternative to other methods of predicting the combined force system such as the Combinator [Schuring, et al., 1996; Pottinger, et. al., 1998] which recently has been made available. That method relies on the use of the friction circle/ellipse and the free rolling cornering and straight line braking to produce a combined force model.

The expression of the BNP forces in a normalized form was used and found to be a convenient way of using that method. Once the normalized form is developed, it can be multiplied by the appropriate value of μF_z to get the actual force. A three dimensional view of the longitudinal and transverse tire force components over full ranges of wheel slip and sideslip angles was presented. This technique allows for a convenient visual inspection of the force components when under combined braking and steering.

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APPENDIX

The BNP formula is:

$$F = D \sin[C \tan^{-1}(B\phi)]$$

where

$$\phi = (1 - E) K x + (\frac{E}{B}) \tan^{-1}(B K x)$$

where the constants used to introduce horizontal and vertical shifts into the data have been omitted with no loss in generality. Consider the partial derivative of *F* with respect to x evaluated at x = 0:

$$\frac{\partial F}{\partial x}\Big|_{x=0} = \frac{\partial}{\partial x}(D \sin(C \tan^{-1}(B\phi)))$$

Performing the partial derivative:

$$\frac{\partial F}{\partial x} = D \cos(C \tan^{-1}(B\phi)) \frac{\partial}{\partial x}(C \tan^{-1}(B\phi))$$

which becomes

$$\frac{\partial F}{\partial x} = D \cos(C \tan^{-1}(B\phi)) \left(\frac{C}{1 + B\phi}\right) B \frac{\partial \phi}{\partial x}$$

But

$$\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left((1 - E) K x + \frac{E}{B} \tan^{-1}(B K x)\right)$$

$$\frac{\partial \phi}{\partial x} = (1 - E) K + \frac{E K}{1 + B K x}$$

So

$$\frac{\partial F}{\partial x} = D \cos\left(C \tan^{-1}(B\phi)\right) \left(\frac{C}{1 + B\phi}\right) B \left((1 - E) K + \frac{E K}{1 + B K x}\right)$$

Evaluating at x = 0 gives the following result:

$$\frac{\partial F}{\partial x}\Big|_{x=0} = B C D K$$