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Modeling of Low-Speed, Front-to-Rear Vehicle Impacts

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ABSTRACT

Front-to-rear crashes between vehicles at speeds well below 20 mph account for a surprisingly large number of significant injuries, usually classified as Whiplash Associated Disorders (WAD). Although an efficient model or process that relates the vehicle-to-vehicle collision conditions and parameters to the level and characteristics of injury is desirable, the complexity of the problem makes such an overall crash-to-injury model impractical. Instead, this paper develops and explores a reasonably effective model of the vehicle-to-vehicle impact that determines the forward/rearward accelerations, velocities and the contact force as functions of time for both the striking and struck vehicles. Tire drag due to braking is included to allow the assessment of its effects.

Each vehicle is given a single degree of freedom consisting of translation of the center of gravity in the direction of vehicle heading. It is assumed that vehicles are centrally aligned and that suspension effects are negligible. The contact force is modeled using nonlinear spring and damping elements each with a coefficient and exponent. The coefficients and exponents are parameters of the model and are varied to obtain realistic contact pulse shapes, durations and velocity changes. Tire drag is modeled using Coulomb friction with equal static and dynamic coefficients.

Data from experimental collisions typically is found to include high-frequency variations due to conditions such as initial bumper misalignments as well as short-duration, local structural conditions (such as spot weld failures, buckling and failure of brackets, etc.). At least some of the high-frequency content is due to structural conditions near the accelerometers. The model is developed to match the overall acceleration pulse shape and magnitude and not reproduce the high-frequency variations. By selecting stiffness and damping coefficients to match contact duration and coefficient of restitution, the model can be used to determine peak accelerations and ΔV's for the purpose of accident reconstruction and for occupant motion studies.

INTRODUCTION

Low-speed vehicle collisions have specific characteristics that differ from high speed collisions and must be treated differently. The choice between the descriptors *low* and *high* is not only a matter of the magnitude of vehicles' closing velocity but includes a decision on how to analyze or reconstruct a crash. Orner (1992) defined a low speed impact as having a closing speed of 5 to 10 miles per hour or less with vehicular damage often being minor. McConnell et al. (1993) and Szabo et al. (1994) defined a low speed collision as one in which the struck vehicle speed change is less than or equal to 8 miles per hour. Schott, et al., (1993) defined a low speed collision as having a struck vehicle speed change of less than or equal to 5 miles per hour. Thomason et al. (1989) defined a low speed collision as one in which the barrier equivalent velocity associated with the impact is less than 12.5 mph. Finally, Bailey et al. (1995) defined a minor impact as one in which restitution effects or tire forces cannot be neglected. The coefficient of restitution typically is near zero for high speed crashes but experimental data indicate values as high as 0.6 for some front-to-rear, low-speed collisions (Cipriani, et al., 2002) and even as high as 0.8 (Siegmund, et al., 1996). Typically values of the coefficient of restitution are found to have an inverse relationship to closing speed and range roughly between 0.2 and 0.6 for unbraked vehicle-to-vehicle collisions (Siegmund, 1994).

Front-to-rear crashes between vehicles at speeds well below 20 mph account for a surprisingly large number of significant injuries, usually referred to as Whiplash Associated Disorders (WAD). The relationship between the vehicle collision conditions and human cervical injuries remains unclear (Siegmund, et al, 1997) primarily because it involves a large number of mechanical, physiological and anatomical variables. Through mathematical modeling and from experiments it has been found that head and neck kinematics are largely a function of the ΔV of the target vehicle (Anderson, et al., 1998) under certain conditions. More recent epidemiological and biomechanical studies suggest that long-term WAD may be related more directly to accelerations than ΔV (Siegmund, et al., 2001).

Although a comprehensive model of the process that relates vehicle-to-vehicle collision conditions and parameters to the level and characteristics of injury would be very useful, the complexity of the problem makes such an overall model impractical. A more fundamental model is developed and presented here of the vehicle-to-vehicle impact and is intended to simulate the forward and rearward vehicle accelerations, velocities and intervehicular contact force as functions of time. Tire drag forces are included so that the effect of braking, when it occurs, can be taken into account. The equations derived and presented here are Newton's laws in the form of differential equations, solved numerically and which provide forces and motion as functions of time. A model of this form has been developed by Ojalvo and Cohen (1997). Their model uses a linear elastic spring and linear viscous damper system to represent the contact force. Ojalvo, et al. (1998) later show how the model represents experimental data for different bumper systems. Such a linear model has an advantage of simplicity, however, it is physically unrealistic since it develops an instantaneous rise in the contact force due to the initial relative velocity and the proportional damping force. Another disadvantage of a linear model is that for higher damping (low restitution) the linear pulse shape does not realistically represent those usually observed in low-speed vehicle experiments. These deficiencies can be overcome by using the nonlinear impact model of the form proposed by Hunt and Crossley (1975). Another advantage to the use of nonlinear mechanics is greater generality. Specifically the nonlinear model permits the inclusion of the effects of braking which can be significant (Anderson, et al., 1998).

Numerous experimental low-speed collisions have been conducted, for example, Siegmund, et al., (1994), Anderson, et al., (1998), Hintzmann (1999) and Cipriani, et al., (2002). These are almost exclusively for direct barrier or zero-offset, vehicle-to-vehicle collisions. They span various vehicles and bumper systems, initial speeds and braking conditions. They add valuable information and increase the understanding of the mechanics of low-speed collisions.

Another approach can be taken to modeling low-speed collisions, that of rigid body impact mechanics using impulse and momentum, used by Anderson, et al., (1998), Brach, (1991a) and Cipriani, et al., (2002). This is an algebraic method that can include the effects of braking (when time durations are known) but does not directly model contact forces and provides only average values of accelerations, not peak values.

The objective of the work presented in this paper is to develop a simulation model that can be used to calculate the forces, accelerations, velocities and displacements as functions of time of 2 vehicles involved in a low-speed collision. The model contains contact force parameters that can be used to match experimental data and can be varied to obtain different pulse shapes. The model is intended to be used for reconstruction of low-speed collisions and for use in occupant motion studies, particularly when the effects of braking are present.

RESTITUTION, COLLISION DURATION AND ΔV

Experimental data indicate that the force and acceleration pulse transmitted from the striking vehicle to the struck vehicle has a duration that is relatively short compared to the biomechanical response of occupants. This has led to a conjecture (Siegmund, et al., 1994) that the vehicle motion can be examined separately from the occupant response and that a reasonably good representation of the acceleration pulse can be used to characterized a vehicle-to-vehicle low-speed impact. This phenomenon needs additional study and forms part of the rationale for this paper. The focus of this study is on the simulation of a vehicle-to-vehicle collision, particularly the ability to model the properties of the intervehicular acceleration pulse as a function of the system parameters. A reconstruction perspective also is taken. So, another goal is to be able to determine an acceleration pulse for a given set of vehicles and conditions based on an estimated pulse duration (duration of contact) and level of velocity restitution (coefficient of restitution). This requires relating the vehicle structural parameters such as stiffness and damping to the coefficient of restitution, e, and contact time, Δt. The acceleration pulse then provides peak acceleration, average acceleration and ΔV. Since actual pulse shapes are not necessarily symmetric, the time of the peak can sometimes be an important parameter.

In this work the *contact time*, Δt, is considered to be the duration of physical contact and is defined as the duration over which the contact force (the force between the vehicles' bumpers) initially becomes nonzero, remains positive and then returns to zero. With this definition and based on Newton's Third Law, the contact time is the same for both vehicles. *Closing velocity* is defined as the velocity of the striking vehicle minus the velocity of the struck vehicle, $\dot{x}_2(t) - \dot{x}_1(t)$. The initial closing velocity, at the beginning of contact, is v_2 - v_1 and the closing velocity at separation, the end of contact, is V_2 - V_1. The *coefficient of restitution* is defined as the negative ratio of the final closing velocity and the initial closing velocity, that is, $e = -(V_2 - V_1)/(v_2 - v_1)$. For collinear collisions, this definition of the coefficient of restitution (velocity restitution) is the same as impulse restitution and energy restitution; see Brach (1991b).

An approach is presented where the simulation model parameters are selected to match experimental acceleration pulses. Several criteria exist for determination of an acceptable fit. One way is to use the Gaussian, minimum least-squares method. Since the area under the acceleration pulse has the physical meaning of an impulse, proportional to ΔV, common acceleration-pulse areas is another criterion for fitting. Although not optimal, the latter is physically meaningful and simpler and is used here.

Nonlinear equations are used to model the structural parameters (see the following section). If the contact force model, F_C, for a collision geometry as shown in Fig 1 were linear it would be expressed as (Ojalvo and Cohen, 1997):

$$F_C = c_L(\dot{x}_2 - \dot{x}_1) + k_L(x_2 - x_1) \tag{1}$$

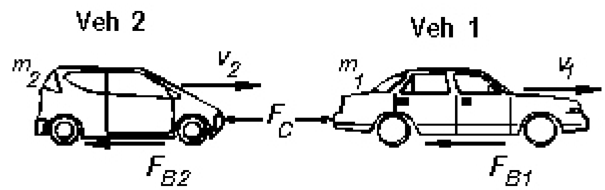

Figure 1. Diagram showing the horizontal forces on Veh 2 (striking vehicle) and Veh 1 (struck vehicle).

where k_L is the linear elastic stiffness coefficient and c_L is the linear damping coefficient. For this system and without braking, relatively simple relationships exist between the contact pulse duration and the coefficient of restitution and the stiffness coefficient and damping coefficient, These are (Brach, 1991a):

$$\Delta t = \pi / \omega_n \sqrt{1 - \varsigma^2} \tag{2}$$

and

$$e = \exp\left(-\pi\varsigma / \sqrt{1 - \varsigma^2}\right) \tag{3}$$

where $\omega_n^2 = k_L / \overline{m}$, and $\varsigma = c_L / 2\overline{m}\,\omega_n$ and $\overline{m} = m_1 m_2 / (m_1 + m_2)$. For reconstruction of low-speed collisions, it is common to have an estimate of the contact duration and coefficient of restitution. Knowing these, Eq 2 and 3 can be solved for k_L and c_L. Models such as this permit calculation of the accelerations and velocity changes, ΔV, for different vehicle combinations and different initial conditions. Drawbacks of this model are discussed elsewhere.

Experiments have been conducted that provide information about the contact duration and coefficient of restitution. Cipriani, et al., (2002) carried out 30 collisions and Antonetti (1998) collected results from several sources. Collisions were of automobiles with both foam core and piston absorber bumper systems, with target vehicles stationary and both moving and target vehicles freely-rolling with brakes not applied. Results shown in Fig 2 indicate that the coefficient of restitution decreases significantly as the closing speed increases and that the data contain considerable scatter despite being measurements from controlled experiments. Figure 2 also shows that the contact duration does not vary significantly with closing speed. Measured contact durations have a mean value of Δt_{avg} = 0.133 s and a standard deviation of $s_{\Delta t}$ = 0.031 s.

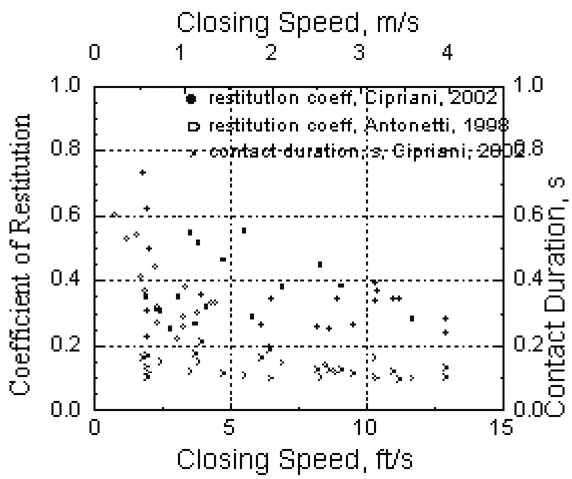

Figure 2 Results of measurements of coefficients of restitution and contact durations for tests with no braking.

Figure 3 shows that values of the coefficients of restitution from vehicle-to-vehicle collisions lie roughly in a triangular region over a range of closing speeds of 0 to 4.6 m/s (15 ft/s).In the next section a model is developed that treats the contact force as nonlinear and also allows the effects of braking to be included. Braking, particularly by the struck vehicle, can affect a low-speed impact. According to Siegmund (1994) braking tends to decrease the collision duration and increase the coefficient of restitution. This topic is covered later in this work using the model to assess the effects of braking drag.

NONLINEAR IMPACT EQUATIONS

Consider mass, m_2 (Veh 2, *striking* vehicle) colliding into the rear of m_1 (Veh 1, *struck* vehicle) as shown in Fig. 1. Initial velocities are v_2 and v_1 , respectively, where $(v_2 - v_1) > 0$. It is assumed here that the vehicles are centrally aligned (no lateral offset), vertical and forward-rearward motion due to suspension flexibility is negligible and that the bumper heights reasonably conform. An initially zero horizontal contact force, F_C, develops between the vehicles, exists uninterrupted over a contact duration, Δt, and returns to zero with the vehicles separating without additional contact (or with the vehicles coming to rest). The intention here is not to model any specific type of bumper system such as crushable foam, honeycomb or fluid dampers, but rather to develop a generic model that can provide reasonably good approximations of the pulse shape and motion for different combinations of vehicle

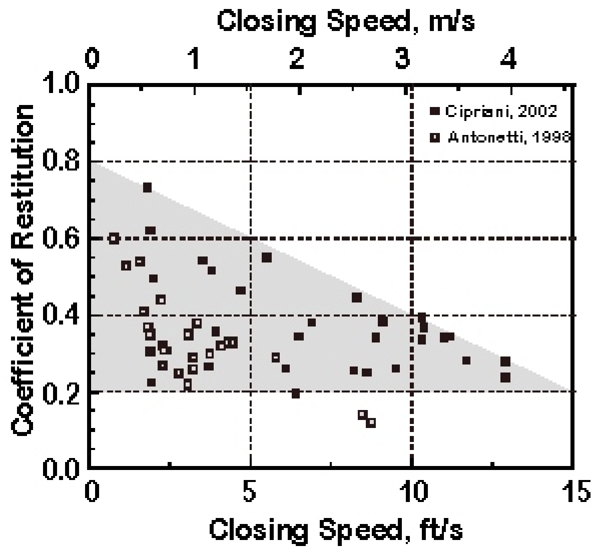

Figure 3. Trend of measured coefficients of restitution with no braking.

weights, braking conditions and initial speeds. An equation of motion can be written for each mass:

$$m\ddot{x}_1 = F_C - F_{B1} \tag{4}$$

and

$$m\ddot{x}_2 = -F_C - F_{B2} \tag{5}$$

where x_1 is the displacement of m_1, x_2 is the displacement of m_2, F_{B1} is the retardation force due to braking of m_1 and F_{B2} is the retardation force due to braking of m_2. A dot over a variable indicates a derivative with respect to time. A constant force in the form of Coulomb friction with coefficients, f_1 and f_2 and which opposes the sliding velocity, is used to represent retardation due to braking for each of the vehicles, respectively. Using lower values of this drag coefficient can represent moderate braking, that is, braking or deceleration without the wheels locked. A value of zero for either coefficient represents no braking.

The dynamic contact model of Hunt and Crossley (1975) suggests a damping force that is proportional to the product of the relative velocity and the relative displacement. In this way, the damping force does not rise instantaneously in the presence of an initial velocity. Generalization of the Hunt-Crossley model gives a contact force of the form:

$$F_C = c_d(\dot{x}_2 - \dot{x}_1)^b(x_2 - x_1)^c + k(x_2 - x_1)^a \tag{6}$$

where c_d is a damping coefficient, k is a stiffness coefficient and *a, b* and *c* are constants that are determined so that the model conforms to

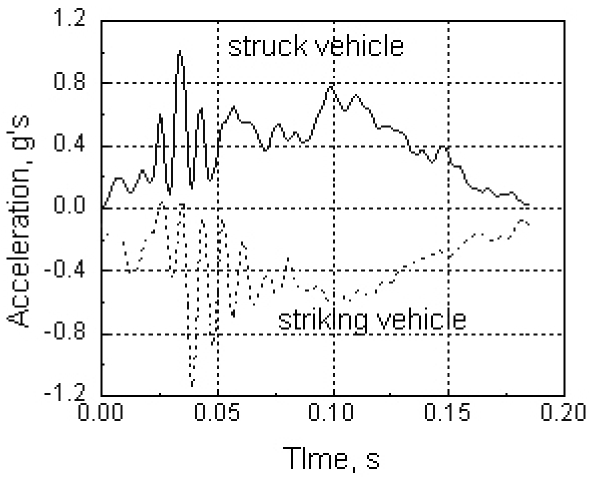

Figure 4. Experimentally measured acceleration-time curves.

experimental collision pulse shapes. Initial conditions are $\dot{x}_1 = v_1$, $\dot{x}_2 = v_2$ and $x_2 = x_1 = 0$. The assumption that (v_2 - v_1) > 0 applies to any combination of vehicle headings. These equations are solved numerically using a form of Runge-Kutta-Gill numerical integration for all results in this work.

During preliminary evaluation of the above model, comparisons with experimental contact pulses between the vehicles showed that the amount of damping during the expansion or restitution phase of the impact was considerably greater than what occurred during the initial compression phase. For that reason the damping coefficient, c_d, was changed from a constant and given the form

$$c_d = c'_d, \quad t \leq t_p$$
$$c_d = c'_d\left(t / t_p\right)^d, \quad t > t_p \tag{7}$$

where c'_d is a constant and t_p is the time of peak compression (maximum elastic force). For $t > t_p$ the term in the parenthesis is greater than 1, increases c_d and provides greater damping following the peak force. The exponent d provides additional versatility to the model.

EXPERIMENTAL CONDITIONS

Experimental results of a low-speed collision were used as a guide to establish the basic parameters of the impact model. Figure 4 shows acceleration traces from Anderson, et al. (1998). The striking car was a Pontiac Bonneville with a test weigh of 16.1 kN (1637 kg, 3610 lb) and a speed of 1.08 m/s (3.9 km/hr, 2.41 mph). The struck car was a Dodge Shadow with a test weight of 12.6 k N (1287 kg, 2837 lb) and initially at rest. Neither vehicle had its brakes applied during this test. Figure 4 shows the measured acceleration pulses. Although when multiplied by mass, these pulses are representative of the common contact force, they actually are accelerations at locations near the mass centers of the vehicles. As such the measured accelerations contain components associated with local structural vibrations. A Fourier analysis of the signals is shown in Fig 5. Examination shows that the main contact force pulse near 3 Hz is dominant. With the exception of a common peak at around 50 Hz, the higher frequency content of the accelerometer signals does not appear to have a high correlation. An implication is that the high-frequency content is due to structural responses near the accelerometers and that a model for the contact force should have the same general pulse shape but does not have to reproduce the higher frequency components of the measured accelerations.

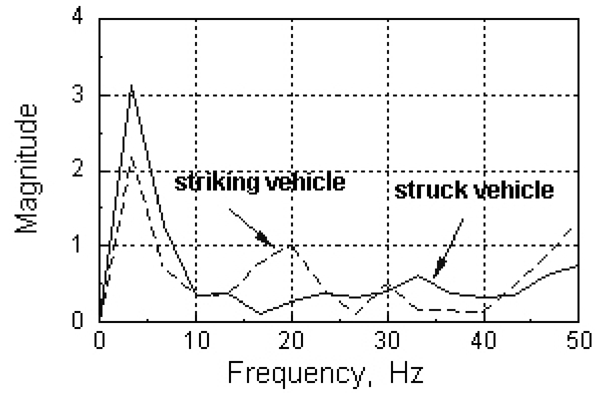

Figure 5. Frequency content of experimental acceleration signals.

DETERMINATION OF SIMULATION MODEL COEFFICIENTS AND EXPONENTS

The coefficients and exponent of the nonlinear contact force model were determined in a trial-and-error fashion by comparison of the model acceleration output and the experimental acceleration pulses. Figure 6 shows what is considered to be a good match, chosen on the basis of a similar shape and equal impulses. The experimental impulse is 960 N-s (216 lb-s) and the simulated value is 950 N-s (213 lb-s). It is interesting

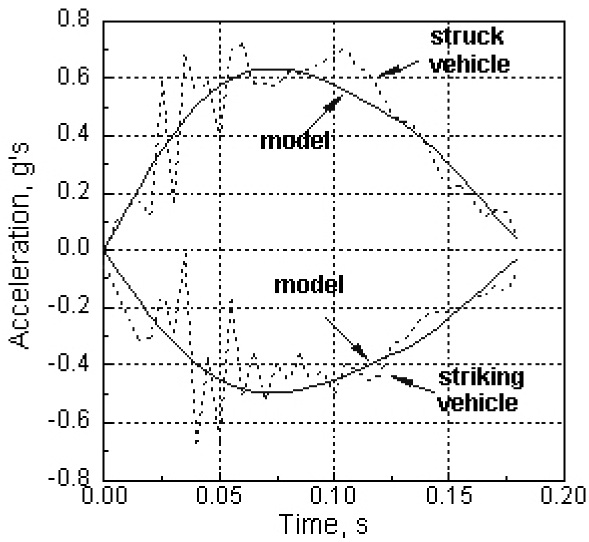

Figure 6. Nonlinear model results compared to an experimentally measured contact force (Anderson, et al., 1998); with same coefficient of restitution, *e* ~ 0.23, contact duration, Δ*t* ~ 0.186 s and impulse *P* ~ 0.95 kN-s as from the tests.

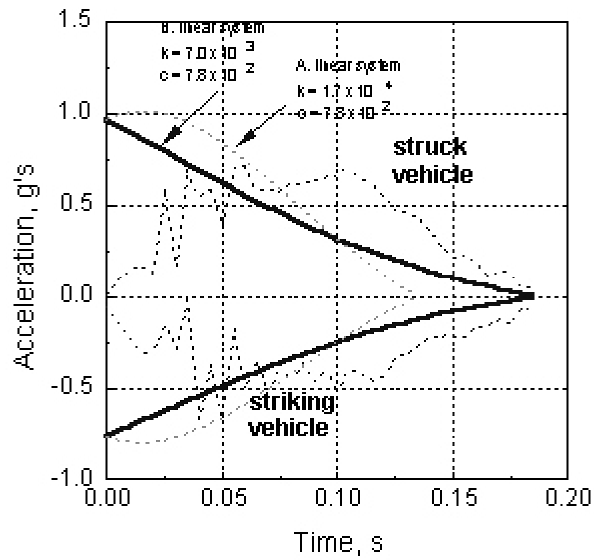

Figure 7. A, fit to linear system with same coefficient of restitution as experiment, *e* = 0.228 and contact duration (half period), Δ*t* = 0.187 s. Eq 2 and 3 give Δ*t* = 0.187 and *e* = 0.228, respectively. B, another fit to linear system with same coefficient of restitution as experiment, *e* = 0.228 and contact duration (half period), 0.187 s. Eq 2 and 3 give Δ*t* = 0.491 and *e* = 0.019, respectively.

to note that this occurs for the contact-force exponents in Eq 6 with values of $a = b = c = 1$. In addition, the exponent d, in Eq 7, has the value of $d = 3$. The value of k = 73 kN/m (5000 lb/ft) was chosen so that the contact duration of F_c is Δt, = 0.184 s compared to the experimental value of Δt, = 0.186. The damping coefficient, c_d = 96 kN-s/m^2 (2000 lb-s/ft^2), was chosen so that the coefficient of restitution of the model, e = 0.232, and from the test, e = 0.228, were nearly equal.

LINEAR MODEL

Since linear mass-spring-damper systems have been used in the past for modeling low-speed collisions some comparisons can be made. Figure 7 shows two contact-force pulses from a linear model. Curve A has spring and damper constants chosen to match the test values of coefficient of restitution and the pulse duration (half-period of the linear model) of Eqs 2 and 3. That is, Eq 2 and 3 give Δt = 0.187 and e = 0.228, respectively. Curve B is where the constants of the linear model are chosen to match the coefficient of restitution and the zero crossing of the contact force pulse. That is, e = 0.228 and contact duration (half period), 0.187 s; here, Eq 2 and 3 give Δt = 0.491 and e = 0.019, respectively. In both cases, the experimental and model impulses are identical, 0.95 kN-s (214 lb-s). Examples A and B show that the use of a linear model has significant deficiencies. The infinite, linear-system pulse rise time can never match the experimental pulse shape. The time-to-peak-acceleration can be important in relating pulse dynamics to injury and the linear model always provides a poor match.

IMPULSE AND MOMENTUM MODEL

Low-speed collisions can be modeled using the principle of rigid-body impulse and momentum such as used by Anderson, et al., (1998), Brach, (1991a) and Cipriani, et al., (2002). Two features of that approach are that collisions between 2 objects are analyzed over a common duration of contact and that the impulses of forces other than the contact force (such as a friction impulse, P_f) must be accounted for independently. Table 1 shows the results of measurements and calculations of carious physical quantities from a) experiments and b) nonlinear simulation and c) the impulse and momentum model. For no braking comparisons between the simulation and the impulse and momentum solutions are quite good as is the comparison to the experiment. For the test with the brakes of the struck car applied , the experimental results are suspect. In addition, some questions arise about the utility of the impulse and momentum method here. With a high enough frictional drag factor, it is common for both vehicles to come to rest while still in contact, or at least very shortly after contact ends. This has a couple of implications. If the time interval used is the contact duration and $v_2 - v_1 \sim 0$, then the coefficient of restitution, e, is near zero. This is true even though a collision of the same vehicles under the same conditions without braking gives a much larger value of e. Another implication is the use of the impulse and momentum method to estimate acceleration. An average value of acceleration can be obtained by using $\Delta v/\Delta t$. For the struck vehicle from Table 1 and the test without braking, this gives $\Delta v/\Delta t = -0.58/0.185 = -3.14$ m/s^2 = -0.32 g. Measurements from this test gave a momentary peak of -1.0 g (see Fig 4) and an average of -0.39 g's. The peak from the simulation is -0.5 g (see Fig 6). On the other hand, when friction impulses are included, the Δv value can be zero if the struck vehicle is initially at rest and when the effects of friction cause it to come to, or close to, rest at the end of the contact duration. A time interval, Δt, shorter than the contact duration could be used. Selecting the time interval can be arbitrary and setting up the method to determine velocity changes before contact ends requires special treatment. The advantages of the algebraic, impulse and momentum method quickly are lost, especially for purposes of reconstruction.

Table 1. Summary of Low-Speed Impacts

▪Test with no braking:

quantity	experiment	nonlinear simulation	impulse & momentum
v_{20}, m/s	1.08	1.08	1.08
Δv_2, m/s	-0.57	-0.59	-0.58
v_{10}, m/s	0	0	0
Δv_1, m/s	0.76	0.75	0.73
Δt, s	0.186	0.185	0.185
e	0.228	0.219	0.219
P_f, N-s	0	0	0

▪Test with braking of struck vehicle:

quantity	experiment	nonlinear simulation	impulse & momentum
v_{20}, m/s	1.08	1.08	1.08
Δv_2, m/s	-0.74	-1.11	-1.14
v_{10}, m/s	0	0	0
Δv_1, m/s	-0.33	0.00	0.00
Δt, s	0.432	0.432	0.411
e	-0.620	0.030	0.030
P_f, N-s	-	1815	1815

EFFECTS OF BRAKING DURING A LOW-SPEED COLLISION

Once basic values of the parameters (a, b, c, d) of the contact-force and the stiffness and damping (k and c_d) for the test vehicles were found, a comparison was made with another test of Anderson, et al. (1998). In this case the target

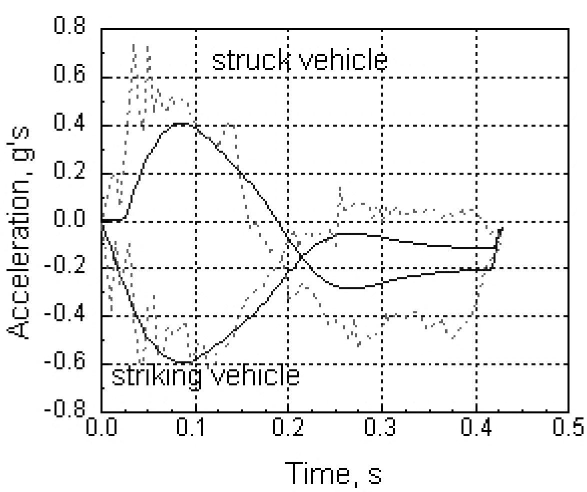

Figure 8. Target vehicle with brake application. Light curve from experiment. Solid curve from simulation; *f* = 0.35. Contact duration Δ*t* = 0.432 s and coefficient of restitution *e* = 0.030.

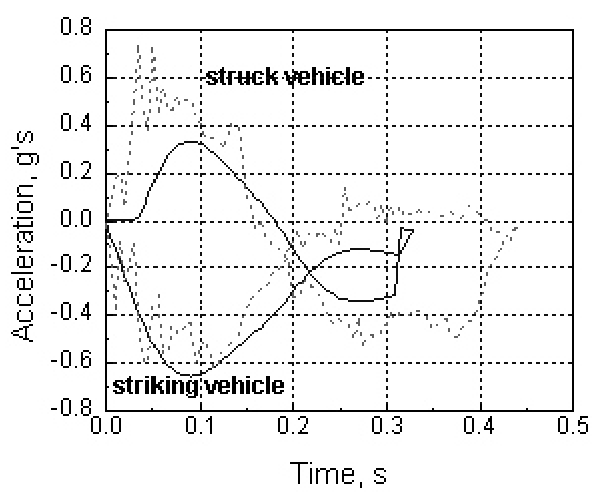

Figure 9. Target vehicle with brake application. Light curve from experiment. Solid curve from simulation; *f* = 0.50. Contact duration Δ*t* = 0.338 s and coefficient of restitution e = 0.083.

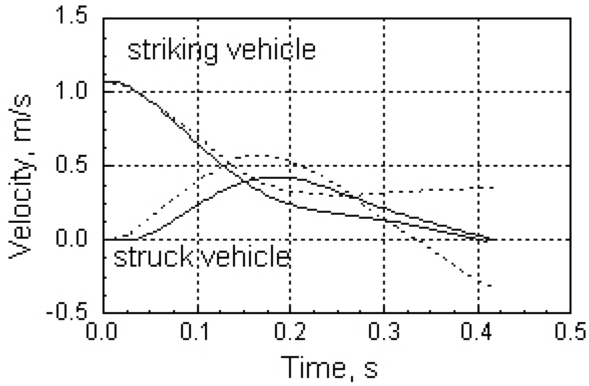

Figure 10. **Test (dashed curves) and simulated (solid curves) vehicle speeds with a fully-braked target vehicle.**

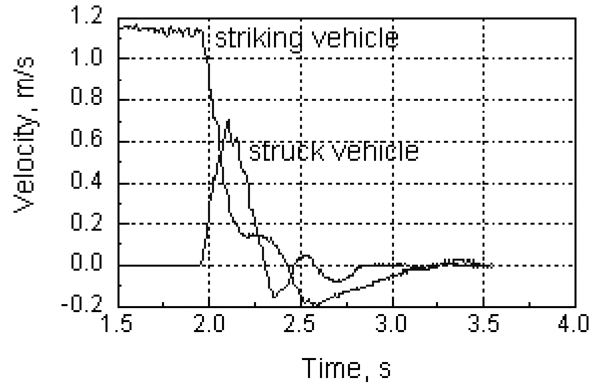

Figure 11. **Velocity of test vehicle measured using a fifth wheel.**

vehicle had its brakes fully and mechanically actuated throughout the low-speed impact. Figure 8 superimposes the experimental accelerations and the output of the model. The simulated acceleration of the struck vehicle initially (from t = 0 to about t = 0.025 s) is flat and shows that it does not begin to accelerate until the contact force reaches the level that overcomes friction. In this same range of time the experimental acceleration of the struck vehicle rises and drops with a local peak. This localized peak acceleration may be due to the flexibility between its sprung and unsprung masses. Following this, the struck vehicle begins to accelerate more rapidly. The striking vehicle should always have negative acceleration as it is retarded by the struck vehicle, including the struck vehicle's frictional drag. From the simulation, the struck vehicle accelerates positively and gains a forward velocity. Its acceleration reaches a peak and then drops and eventually decelerates as the contact force from the striking vehicle is less than the frictional drag.

The frictional drag of the experimental tire-gound interface during the test was not measured and is unknown; a value of f = 0.35 was chosen for use in the model to provide an overall match of contact duration, acceleration pulse shapes and durations shown in Fig 8. Figure 9 shows a like comparison except for f = 0.5. As expected the higher value of frictional drag causes the vehicles to come to rest sooner, but considerably sooner than in the test. So the value of f = 0.35 appears to provide a better fit by the model of the experimental trends of the accelerations. As can be seen, however, the magnitudes of the vehicle accelerations during sliding (for t > 0.2 s) is not accurately matched by the model. Another interesting result is that Δt = 0.432 s and e = 0.030 ~ 0. This indicates that braking of the struck vehicle can *increase* the contact duration significantly and significantly *reduce* the coefficient of restitution to the point that both vehicles come to rest.

Figure 10 shows the corresponding test and simulated velocities of the striking and struck vehicles. The velocity of the test vehicles was found by numerically integrating each acceleration signal. It shows that the struck vehicle in the test approaches a final velocity of -0.315 m/s and the test striking vehicle approaches a final velocity of 0.350 m/s whereas the model shows both vehicles coming to a stop. Test results imply that the struck vehicle is pushed forward and then it reverses its motion. The integrated test velocity signals also imply that the striking vehicle slows but keeps moving forward as the struck vehicle rebounds. It is not clear why this type of motion occurs or should occur. Independent measurements of the vehicle velocities were made using a fifth wheel and are shown in Fig 11 (note that Fig 11 has a different time scale). Fig 11 does seem to confirm that the striking vehicle rebounds and reverses its direction. It also shows a damped oscillatory velocity profile for the struck vehicle, further implying that it moved forward then backward, etc. Such oscillatory motion could be due to sprung-mass flexibility. If so, this would imply that sprung-mass flexibility may play a more important role than previously thought. This may require verification, however, since the striking vehicle velocity signal does not show such an oscillatory trend.

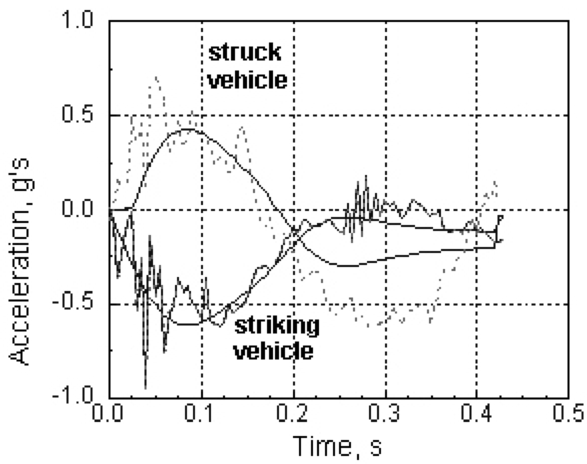

Figure 12 Target vehicle with brake application. Light curve from second experiment. Solid curve from simulation; *f* = 0.35. Contact duration Δ*t* = 0.432 s and coefficient of restitution *e* = 0.030..

Figure 12 shows a like comparison of the model with another test of Anderson, et al., under identical experimental conditions as in Fig 8. The comparison drawn above between the model and experimental results shown in Fig 8 seem to apply here as well.

In summary, the simulation seems to do a satisfactory job of matching the low-speed mechanics when there is no braking, Fig 6. With braking present, it is not clear if the match between the simulation and test results shown in Fig 8 and Fig 12 is satisfactory or not since the braking experiments some display questionable behavior.

UTILITY OF THE SIMULATION

Comparisons of the simulation model with 3 tests was carried out above by matching the model's pulse shape with the experimental ones. This was done by finding the model parameters *a*, *b*, *c*, *d*, *k* and c_d that gave a reasonable match. Other pulse shapes may occur from collisions of different vehicles, different initial speeds, etc. Figure 13 shows that varying the model parameters can produce a variety of pulse shapes. Table 2 shows the model parameters corresponding to the curves in Fig 13.

As another example of the model's utility, Fig 14 shows the accelerations of both vehicles when the striking vehicle is being braked both just before and during contact and the struck vehicle is not. Vehicle and collision conditions are identical to the

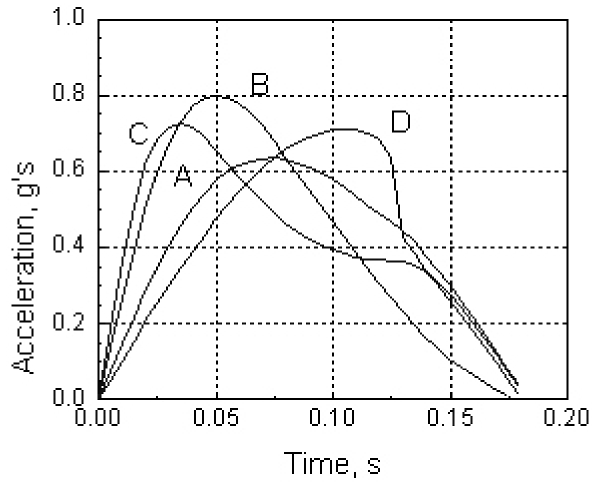

Figure 13. Pulse shapes for different model parameters (see Table 2).

experimental conditions listed earlier. The striking vehicle has a frictional drag of *f* = 0.35. The vehicles separate (When the contact force goes to zero) such that Δ*t* = 0.15 and *e* = 0.26. Compared to the case of no braking the contact duration is shorter and the coefficient of restitution is higher. The peak acceleration of the struck vehicle is lower.

DISCUSSION AND CONCLUSIONS

A simulation model using a nonlinear contact force has been developed that can be used to calculate the contact force and resulting vehicle motions for low-speed collisions. Through the inclusion of a frictional drag factor, the model simulates the effects of partial or full braking of either or both vehicles. Parameters of the model were chosen so that it matches experimental results. For no braking, the match is quite good. With full braking of the struck vehicle, the accelerations computed by the model followed the trends of the tests but differences in the levels of the contact force occurred.

An effect of braking of the struck vehicle is that the

Table 2. Model Parameters for Different Pulse Shapes (see Fig 13)

	a	*b*	*c*	*d*	*k,* kN/m	c_d, kN-s/m^2
A	1	1	1	3	73.0	95.8
B	1	1	1	1	90.5	215.5
C	1	2	1	3	73.0	100.6
D	1	0.25	1	3	81.7	95.8

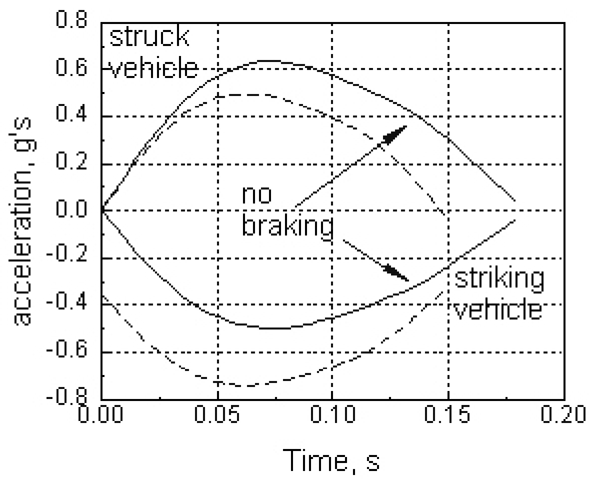

Figure 14. Dashed curves are simulation results of a collision where the striking vehicle is braked, *f* = 0.35, and the struck vehicle is not. Contact duration is Δt = 0.15 s. Solid curves are simulation results for no braking.

time the vehicles remain in contact can significantly increase compared to the same collision conditions without braking. Some analysts neglect this lengthened contact duration when examining the initial pulse. This perspective is permissible when the effect of the initial pulse is of primary concern. But the initial pulse is based on the full mechanics of the collision and when studying the impact mechanics, all of the motion must be considered. This is particularly true, both theoretically and practically, when examining the coefficient of restitution of the impact. The rebound velocities and coefficient of restitution can be significantly different with and without braking. When both vehicles come to rest during the contact duration due to the braking of the struck vehicle, the coefficient of restitution reduces to zero.

Comparison of Fig 6 and 8 shows that an overall effect of struck-vehicle braking is a significant increase in the contact duration. The vehicles remain in contact until they slide to a stop. Taking the entire contact duration into account shows that the coefficient of restitution becomes zero, that is, e = 0.0. There is a delay and definite decrease in the peak of the initial, positive, portion of the struck-vehicle acceleration. It remains to be determined what effect the change in pulse shape and contact duration has on occupant injury.

It is not clear from this study if the compliance effects of the vehicle suspensions are or are not significant. Figures 8 and 12 show early peaks in the struck vehicle forward acceleration that the model doesn't. If the occupant biomechanical response is sensitive to early peaks they could be important. Intuition indicates that the presence of struck vehicle braking should heighten the effect of suspension dynamics particularly at very low speeds. Results here are inconclusive. More analysis needs to be done. Unfortunately a simulation model including suspension effects is likely to be considerably more complicated than the model presented here. This is because in addition to 2 more degrees of freedom, bumper dynamic contact geometry must be included to take into account vertical misalignments and pitch due to and during braking.

Estimation of the ratio of the peak, or maximum, to the average acceleration of the pulse transmitted to the struck vehicle is often sought in reconstruction work. The experimental values from the test with no braking and the two with braking are 2.6, 2.4 and 2.6, respectively.

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