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Nonlinear Optimization in Vehicular Crash Reconstruction

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ABSTRACT

This paper presents a reconstruction technique in which nonlinear optimization is used in combination with an impact model to quickly and efficiently find a solution to a given set of parameters and conditions to reconstruct a collision. These parameters and conditions correspond to known or prescribed collision information (generally from the physical evidence) and can be incorporated into the optimized collision reconstruction technique in a variety of ways including as a prescribed value, through the use of a constraint, as part of a quality function, or possibly as a combination of these means. This reconstruction technique provides a proper, effective, and efficient means to incorporate data collected by Event Data Recorders (EDR) into a crash reconstruction. The technique is presented in this paper using the Planar Impact Mechanics (PIM) collision model in combination with the Solver utility in Microsoft Excel. Five examples, including a high-speed sideswipe collision, intersection collisions, etc., are used to demonstrate the technique.

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INTRODUCTION

The traditional speed reconstruction method developed and applied in the field of vehicular accident reconstruction over the last four decades or so, generally divides the crash reconstruction problem into three phases: the preimpact phase, the impact phase, and the postimpact phase. These phases are generally analyzed separately in a reconstruction project and are usually analyzed in reverse chronological order, using the available, quantifiable physical evidence (vehicle crush, tire mark positions and lengths, vehicle rest positions, etc.) as criteria for an acceptable solution. The three phases are linked together using common conditions at the transitions between the phases.

Early on, the models developed to reconstruct the impact phase consisted of point mass impact models comprised primarily of the conservation of linear momentum. Later, planar impact mechanics (PIM) models were introduced. Among various differences, the PIM models include the rotational motion (and rotational inertia) of the vehicles whereas the point mass impact models do not. Impact models based on energy loss calculated from the residual crush of the vehicle(s) were also developed early on and still find widespread use. These crush energy models are considered within the context of this topic later in the paper.

The application of a PIM collision model in an academic sense would consist of specifying values of the initial (preimpact) velocities of the vehicles, as well as vehicle dimensional and inertial properties, impact geometry, etc., to predict the values of final (postimpact) velocities. In accident reconstruction, information about the postimpact velocities is typically available, and it is the preimpact velocities which are generally desired. Mixed problems are also common where preimpact parameters of one of the vehicles may be known (or assumed), such as a preimpact velocity and/or preimpact orientation. Thus, in reconstruction applications, the solution procedure would typically require an iterative approach to determine a solution that satisfies the various elements of the known, quantified preimpact and postimpact physical evidence. As an alternative to an iterative method, another approach to solve this type of problem, in which a mix of input and output values is known and another mix of values is unknown, is the use of nonlinear optimization methods. A technique of solving vehicle crash reconstruction problems using nonlinear optimizations is presented in this paper.

Recent advances in the collection of data by the on-board electronic systems, on both light vehicles and heavy trucks, have changed the scope of the physical evidence available to reconstructionists. In addition to (or, sometimes in place of) traditional physical evidence such as tire marks, residual crush, rest positions, etc., the list of physical evidence in crashes involving vehicles with an EDR may include measurements of the ΔV in the longitudinal and lateral directions, preimpact speed, preimpact steering input, etc. Reconstructionists benefit from the on-vehicle acquisition of a variety of parameters when a crash occurs. Including these parameters in the traditional approach to a speed reconstruction requires further evaluation on the part of the analyst to ensure these values are achieved at each iteration of the solution algorithm until a solution is found.

Conversely, by using nonlinear optimization in solving this "mixed" problem, this information can be handled easily and conveniently through the use of constraints. By achieving a solution for a given set of conditions, the optimization algorithm specifically excludes solutions in which the constraints, i.e. specific physical evidence, are not met. This optimization approach increases the efficiency of the reconstruction process as the analyst no longer needs to use time-consuming iterative methods to search the solution space for a given set of conditions, constantly evaluating the results against the physical evidence.

After a review of the applicable literature, the fundamental aspects of the Planar Impact Mechanics (PIM) model are presented followed by some general background regarding nonlinear optimization methods. This background includes details of the optimization tool in Microsoft Excel, called Solver, which is used in the examples presented in this paper. Prior to presenting various examples which demonstrate the method, the specific scheme used in the examples in the paper is outlined.

LITERATURE REVIEW

It appears that only two previous papers [1, 2] consider the use of optimization methods in the reconstruction of vehicular crashes involving the collision. Both deal with the Optimizer utility that is part of the PC-Crash reconstruction software [3]. These two papers present a scheme for the use of optimization methods in the reconstruction of crashes. The approach presented in those papers has similarities and differences from what is presented here. The approaches are similar in that both exploit the utility of optimization in achieving a solution to a reconstruction problem and both approaches implement the process using a quality function that tallies the differences between desired values for various parameters and their corresponding calculated values.

The implementations of the two optimization processes differ in that the process described here considers only the impact phase of the crash whereas the implementation in PC-Crash optimizes both the impact and postimpact phases together. While the implementation shown here optimizes an algebraic set of equations (the PIM model), the PC-Crash implementation optimizes a similar set of algebraic equations (the PC-Crash impact model) as well as a nonlinear set of differential equations that govern the postimpact motion of one or more vehicles. The principal disadvantage of optimizing over a larger portion of the crash rather than just the impact phase is increased complexity.

The origin of PC-Crash predates the widespread adoption in the auto industry in North America of placing event data recorders (EDR) in vehicles. The crash data collected by these devices, particularly data from the impact itself, such as the ΔV of a vehicle and its preimpact speed, permits the streamlining of the optimization process such that it can be tremendously effective when applied to the impact phase alone. The examples presented in this paper demonstrate this effectiveness.

Both implementations have comparative advantages and disadvantages. Certain applications will, perhaps, be more suitable to one approach or the other. For example, a crash in which one or more of the vehicles is involved in a significant secondary collision after the initial collision might not be suitable for the PC-Crash Optimizer. Certainly applications in which the reconstructionist wishes or needs to incorporate the postimpact motion of one or more of the vehicles in an optimization scheme will need to use a more complex scheme such as the one in PC-Crash as opposed to an implementation of PIM coupled with an optimization utility.

However, scenarios such as those shown in the examples in this paper can easily be handled by the technique presented herein. The analyst will need to assess the best approach based on the circumstances of a given application. An understanding of the examples presented here and in the references will aid in that decision-making process. Additionally, nonlinear optimization has been used [18] for the fitting of a spiral to the measured tire mark from a critical speed event.

OPTIMIZATION METHODS

Least squares is defined as a method of determining the curve that best describes the relationship between expected and observed sets of data by minimizing the sums of the squares of the deviations between observed and expected values [10]. In the field of vehicular accident reconstruction, the relationship is between the physical evidence (observed) and the computed/reconstructed (expected). In the application presented here, the method is applied to achieve a match, in a least squares sense, between the physical evidence and the computed, or modeled. The least squares condition is imposed in that the deviations between the observed and expected values are squared and summed, and this summation is minimized. When the minimized value is zero (or near zero), a match between the observed and expected conditions is achieved.

This technique as presented here can be expressed as a function, Q, where Q, frequently referred to as a quality function, is defined by the equation:

$$Q\left(s_j\right) = \sum_{i=1}^{n} w_i (u_i - u_i^{data})^2 \tag{1}$$

In this equation, n is the number of parameters to be compared from the known data, w_i is a weighting factor, u_i^{data} is a parameter determined from physical evidence and u_i is the corresponding parameter calculated using the Planar Impact Mechanics (PIM) model. Each parameter, u_i, is a function of various other vehicle and crash parameters, s_j. Parameters, u_i, that are known with greater confidence, i.e. less uncertainty, are generally given a higher weighting factor than those with more uncertainty. A preferred technique is to make the u_i quantities nondimensional or to require all to have the same dimensions. Weighting factors can also be used to even out the influence on Q of competing parameters, u_i, where one parameter is naturally much larger in magnitude but not in importance than another parameter. Nonlinear optimization techniques are then used to search the solution space by varying values for certain unknown input variables, s_j, until a minimum value results for Q. This process satisfies the physical PIM model and matches the physical evidence in an optimal or "best" way. One commonly available nonlinear optimization tool useful for this method is the Solver feature in Microsoft Excel.

Microsoft documentation [7] states that Excel Solver uses the Generalized Reduced Gradient algorithm for optimizing nonlinear problems, i.e. to find a maximum, minimum or specified value of a "target cell" in a spreadsheet. It accomplishes this by varying the values of a number of "changing cells" on which the value of the "target cell" depends. Details of how Solver works can be found in [12]. The spreadsheet must contain a series of known u_i values and a corresponding series of calculated u_i values. The calculated u_i values must be contained in spreadsheet formulas that depend on the various coefficients and parameters, s_j, that will be varied by the Solver in achieving a solution [12].

Vehicle and/or impact parameters that are not known exactly in a given crash, but are known to generally fall within certain ranges, for example, frictional drag coefficients and/or the coefficient of restitution, can be bounded through the use of constraints within Solver to narrow the range of possible solutions. Additionally, these coefficients and other impact parameters, such as the angle of the vehicle-to-vehicle contact plane (Γ), which may not be known with certainty, can frequently be determined in the optimization process through the use of constraints. The utility of imposing constraints in the solution process cannot be understated. Through the imposition of constraints, the optimization algorithm limits its search for a solution to only those parts of the solution space that include the permissible values of the parameter.

The question naturally arises as to whether a solution found by the optimization algorithm is a global or local minimum. (Note that *global* here is relative in that with constraints imposed, the solution space is necessarily restricted.) The best approach to ascertain the nature of the solution is to explore the solution space near the parameter values found by the optimization algorithm and to use knowledge of the circumstances of the crash and the judgment of the reconstructionist to evaluate the solution(s). This exploration can be implemented quite effectively using the Solver utility itself through modification of the constraints, varying the input values, etc. This approach is consistent with the other sources that address this question [7, 12]. The details of the solution process used by Excel Solver are beyond the scope of this paper. Information regarding the solution process is available by contacting the developer of the code [8].

Four of the five examples presented in this paper use the Planar Impact Mechanics (PIM) model [4, 5, 6]. The solution equations of the PIM model are available [9]. Validation of this model for use in the analysis and reconstruction of vehicular crashes has been demonstrated [5]. The impact model used in PC-Crash [4] is essentially the same model presented and used here. The remaining example uses point-mass impact mechanics (applicable to in-line central) collisions) coupled with EDR data. The use of the point-mass impact mechanics model is valid in this application since it is a central collision. Care should be taken when using point-mass impact mechanics for reconstructing and analyzing non-inline, offset vehicle crashes [17].

Also worth mentioning before presenting a series of examples, is that the lack of a suitable match between the observed and the expected conditions can also provide information about the solution space. Knowing that the constrained solution space is not yielding a satisfactory match between the physics model and the physical evidence demonstrates that a review of the problem is necessary. The reason for the lack of a match may be as simple as a data entry error (e.g. use of weight instead of mass) or based on the physics (e.g. imposing the common velocity conditions when the crash being reconstructed did not satisfy these conditions). Using optimization to trouble-shoot this situation has advantages. Constraints of parameters to a certain range can be relaxed or tightened in a systematic manner and the results evaluated to assess their effect. This can be done quite efficiently and effectively to learn the nature of the solution space. This situation is discussed in Example 3 below.

EXAMPLE RECONSTRUCTIONS

Example 1 - Intersection Collision without EDR Data

The first example illustrating the use of nonlinear optimization in vehicle crash reconstruction involves two passenger cars that collided at approximately right angles on a city street (see Figure 1.1). Vehicle 1 (Veh 1) was reportedly exiting a drive along the south side of the street while Vehicle 2 (Veh 2) was traveling eastbound in the lane closest to the south curb. The front of Veh 2 collided with the left side of Veh 1. After the collision, Veh 1 continued northward across the street, mounted a curb, collided with a concrete bench and came to rest facing north. Vehicle 2 continued eastbound and came to rest in the westbound lane facing east.

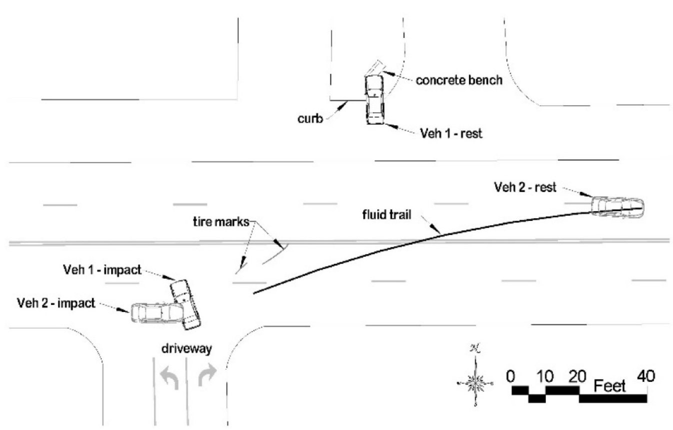

Figure 1.1. Scaled crash diagram for Example 1

Neither vehicle was equipped with an event data recorder. No measurements or photographs were taken at the crash scene. Photographs of the site were taken by the police the following day, after the vehicles had been removed. The impact and rest positions and orientations of the vehicles were estimated based on the vehicle damage and roadway markings (tire marks). The initial post-collision path of Veh 1 was indicated by curved tire marks on the street. The rest position of Veh 1 was estimated by the location of the concrete bench and the tire marks visible in the site photographs. The post-collision path of Veh 2 was indicated by a fluid trail on the street.

Crush measurements were taken of both vehicles during the vehicle inspections. Measurements were also taken at the crash site to facilitate planar photogrammetric analysis of police photographs of the site. In this way, the tire marks on the roadway and the fluid stain, which define the postimpact motion of the vehicles, were located accurately.

The reconstruction of the postimpact speed of Veh 1 is subject to considerable uncertainty due to the impacts with the curb and the concrete bench. The postimpact motion of Veh 2, which takes place entirely on the roadway, appears to be a reasonable candidate for a reconstruction. However, it is also subject to some uncertainty as the distance traveled, in this case greater than 100 feet, will produce different results depending on the drag assigned to the vehicle (wheel conditions, etc.). Thus, the decision is made to use optimization methods and, rather than reconstruct the impact based on the postimpact **speed** of one (or both) of the vehicles, the reconstruction will be based on the postimpact **directions of motion** of both vehicles. Additionally, with crush measurements, the energy loss due to crush can be calculated and used in the reconstruction. As will be shown, the energy loss can be imposed as a constraint in the optimization. Recall that the postimpact velocity has both magnitude (speed) and direction components. The reconstructed speed of Veh 2 is used at the end of the reconstruction as an additional means to check the results.

The PIM spreadsheet (Figure 1.2) was set up to calculate the initial speed of Veh 2 by optimizing the calculated departure angles, cells Y10 and Y11, for both vehicles simultaneously. From available barrier test data and crush measurements of both vehicles involved in the crash, the normal crush energy loss [20] was determined to be 55,664 ft-lb. In analyzing the vehicle velocities at impact, the crush energy calculated using PIM was constrained to be within ± 50 percent of this value. The coefficient of restitution, *e*, was also constrained to be between 0.0 and 0.2, typical for passenger car collisions [9]. Initial heading angles and the location of the impact center, *C*, were determined from scale diagrams and used as inputs to the analysis. Geometric and inertial properties of both vehicles were taken from available literature. Finally, the departure angles (angles of the final velocity vector components) of the mass center of both vehicles were determined from the scale diagram of the site and used to set up the nonlinear optimization scheme that calculated initial velocity for Veh 2 and *e* by minimizing the difference between calculated and desired values of the departure angles for both vehicles. The initial speed of Veh 1 was estimated from typical vehicle acceleration values and the distance Veh 1 traveled to impact was entered into the PIM analysis spreadsheet as an input.

Ultimately, a narrow range of initial speed for Veh 2 produced the correct rest position and orientation for Veh 1. Differences in initial speed of ± 1 mph produced substantially different post-collision movements for Veh 1. Using this initial speed for Veh 1 and the analysis techniques already described, resulted in a calculated total normal crush energy loss of 55,805 ft-lb. The normal crush energy loss calculated from Planar Impact Mechanics differed from that calculated from crush measurements by 0.25 percent. The ΔV's for Veh 1 and Veh 2 calculated using PIM were both about 2 mph greater than those calculated using crush energy. As a check of the final speed for Veh 2 calculated using PIM, the average drag coefficient required to bring Veh 2 to a stop over its rollout distance was calculated and found to be 0.14, which seemed reasonable for the circumstances. Figure 1.2 shows the solution spreadsheet for this example. The postimpact speed of Veh 1 was determined to be 17.9 mph and the preimpact speed of Veh 2 was 37.9 mph.

Example 2 - Intersection Collision with EDR Data

Figure 2.1 is a scale diagram of an intersection crash involving a westbound vehicle with an EDR, Vehicle 1 (Veh 1), and a southbound vehicle not equipped with an EDR, Vehicle 2 (Veh 2). The EDR on Veh 1 captured information for both the longitudinal and lateral speed changes of the vehicle, ΔV_{long} and ΔV_{lat}, as well as the preimpact speed of the vehicle for five seconds prior to the collision at one second intervals. As a result of the impact, Veh 1 was redirected from its westward preimpact motion to the southwest. It exited the shoulder southwest of the intersection, rolled down the berm and came to rest facing southwest. After impact, Veh 2 continued south and to the west, rotated approximately 200° clockwise and came to rest approximately 47½ feet from its position at impact.

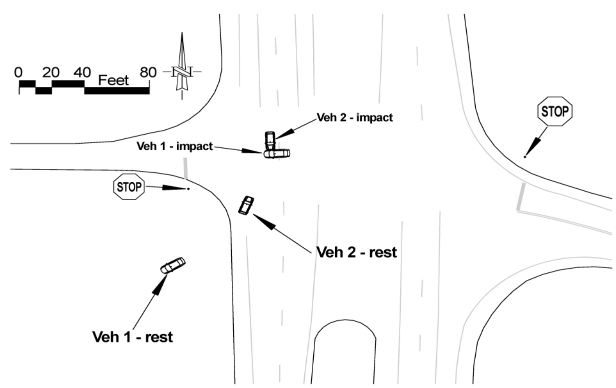

Figure 2.1. Scaled crash diagram for Example 2

The postimpact motion of Veh 1, which includes rolling down a fairly steep hill with the left front chassis furrowing and gouging the ground/soil and an impact with the front of the vehicle at the base of the hill, presents difficulties in reconstructing its postimpact speed using simulation techniques. The postimpact motion of Veh 2 takes place entirely on the roadway and therefore traditional methods of postimpact analysis, vehicle dynamics simulation in this case, could be applied to reconstruct its speed at separation. This speed could then be used to reconstruct the impact phase. However, in combination with optimization, this speed at separation becomes just one of a series of parameters used to reconstruct the impact to determine the preimpact speed of Veh 2. Due to the availability of EDR data including in this case, both lateral and longitudinal ΔV components for Veh 1, the use of crush analysis, with its considerable uncertainty [14, 16], was not used. Therefore, measurements of the residual crush of the vehicles were not made.

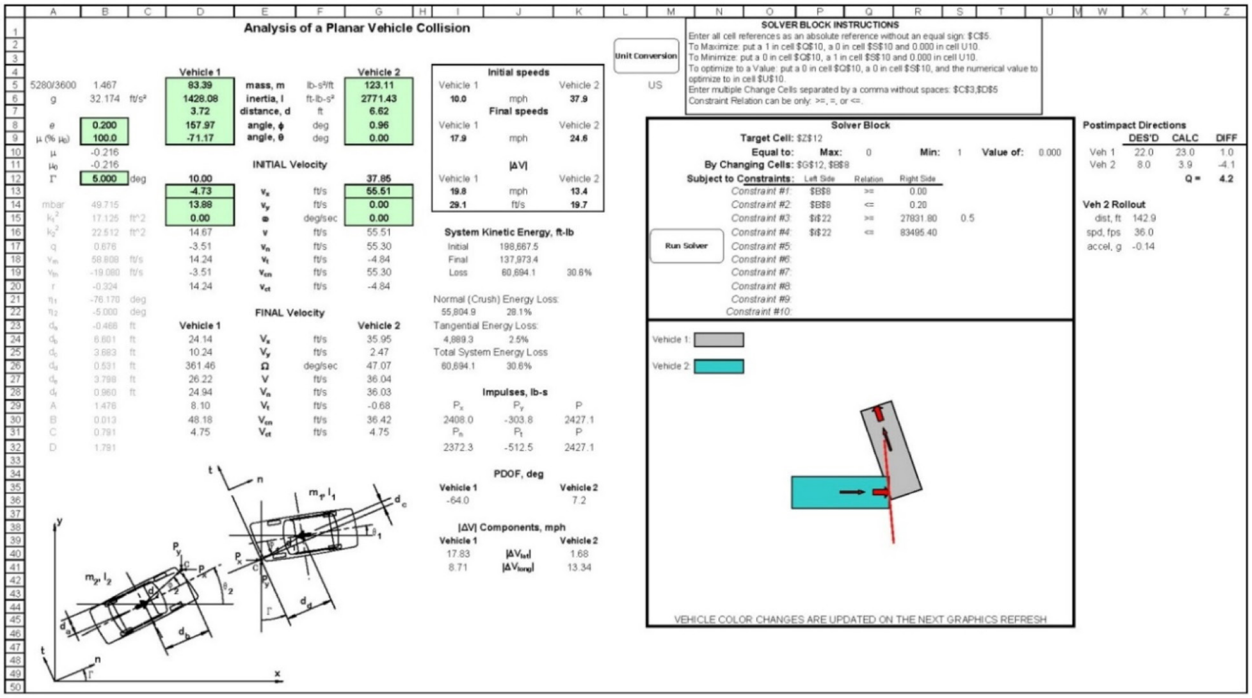

Figure 1.2. Planar Impact Mechanics solution spreadsheet for Example 1

Optimization is implemented in this reconstruction primarily by the use of the EDR data from Veh 1. Table 2.1 lists a subset of this data. In this situation, the speed of the vehicle at the time of the impact, assumed to be at algorithm enable (AE) is subject to uncertainty. The EDR data show that while the throttle level is decreasing from two seconds to one second prior to AE, likely since the driver moved his/her foot from the accelerator to the brake, the speed increases by 2 mph. Thus, the answer to the question as to the speed of the vehicle at AE depends on whether the brake application is maintained through impact and at what level. The recommended approach is to manage the uncertainty by selecting a range for the vehicle speed based on reasonable assumptions. For the purposes of this example, the reconstruction of the preimpact speed of Veh 2 is done with the preimpact speed of Veh 1 assumed to be 19 mph.

The optimization in this situation is applied to minimize the differences between the desired values of the postimpact directions of motion of the two vehicles (cells W12 and W13). These directions are determined from the physical evidence (roadway markings, rest positions, etc.) and a detailed scale diagram. The lateral speed change of Veh 1 from the EDR, ΔV_{lat} = 21.01 mph, was included in the optimization analysis as a constraint.

Figure 2.2 shows the Solver Block for this analysis and the calculation of the square root of the sums of the squares of the differences between the desired and the calculated values. Figure 2.3 shows the spreadsheet of the PIM solution. The reconstruction using optimization techniques revealed that for an initial speed of Veh 1 of 19 mph, the initial speed of Veh 2 is 48 mph (cell K6).

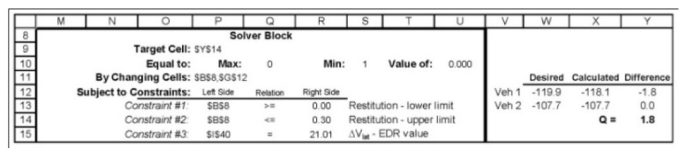

Figure 2.2. Solver block for Example 2

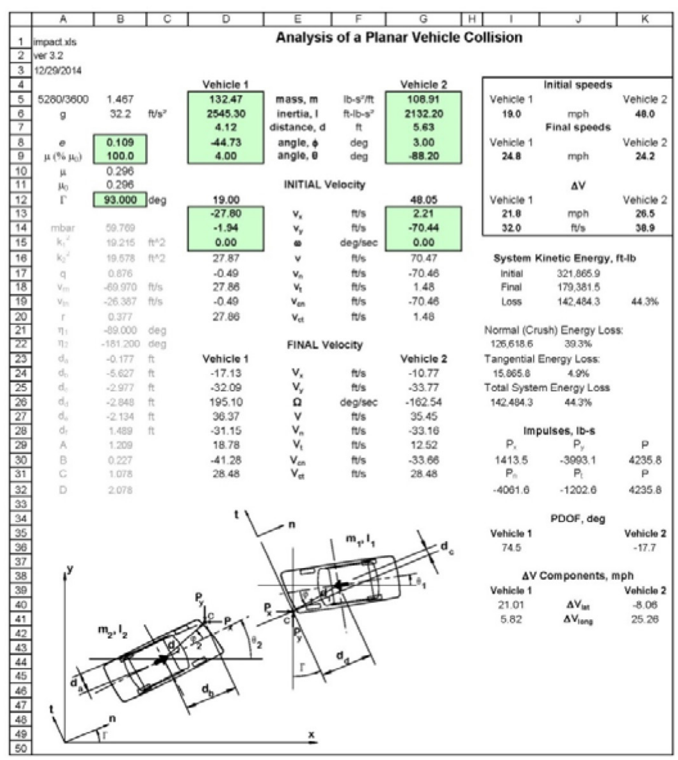

Figure 2.3. Solution spreadsheet for Example 2

Table 2.1. Partial listing of EDR data from Veh 1

Time before AE	-5	-4	-3	-2	-1
Speed (mph)	16	14	15	17	19
Brake Switch Circuit	ON	ON	OFF	OFF	ON
% Throttle	22	20	29	36	24

Example 3 - Collision example involving post-collision impacts with EDR data

Figure 3.1 is a scale diagram of an intersection crash. After first stopping, Veh 1 made a left turn into the path of Veh 2. Vehicle 1 was equipped with an EDR; Veh 2 was not. The EDR on Veh 1 captured information for both the longitudinal and lateral speed changes of the vehicle, ΔV_{long} and ΔV_{lat}, as well as the preimpact speed of the vehicle for five seconds prior to the collision at one second intervals. After the impact, Veh 1 moved laterally and interacted with a median surrounded by a raised curb. As a result of the interaction, the vehicle rolled, driver-side leading, and came to rest west of the median on its roof. In addition to interacting with the curb, the vehicle also rolled over into a steel light pole positioned in the median (not shown) and fractured it from its concrete base. After the impact, the left side wheels of Veh 2 were located on the median, indicating that it also interacted with the raised curb.

The postimpact trajectories of both vehicles in this crash, i.e. postimpact collisions with both vehicles and the environment and rollover of Veh 1, pose difficulties in reconstructing the preimpact speed of Veh 2 using traditional, reverse chronological methods. The postimpact collisions with the environment make reconstructing the speeds of the vehicles at separation via postimpact analysis subject to considerable uncertainty. Therefore, another method is used to reconstruct the preimpact and postimpact speeds of Veh 2 that avoids analysis of the postimpact motion.

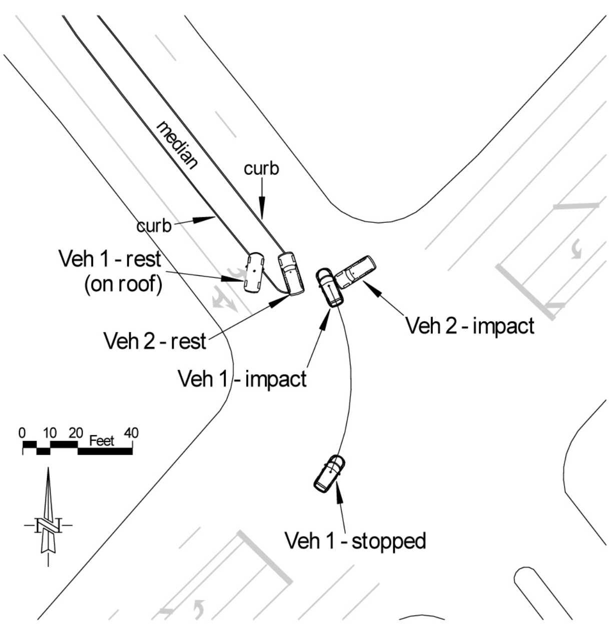

Figure 3.1. Scale crash diagram for Example 3

Optimization is implemented in this reconstruction by use of the EDR data from Veh 1. Table 3.1 lists a subset of this data. In this situation, the speed of the vehicle at the time of the impact is calculated from the data. The speed of the vehicle one second prior to impact is known, and, extrapolating the acceleration between −2 seconds and −1 second prior to impact (which is assumed to occur at 0 seconds) to between −1 second and impact, the speed at impact can be estimated. The acceleration of the vehicle between −2 and −1 is $a = 0.32g$ and assuming that this acceleration continues up until impact, the speed of Vehicle 1 at impact is 25 mph.

Table 3.1. Partial listing of EDR data of Veh 1

Time before AE	-5	-4	-3	-2	-1
Speed (mph)	0	0	5	11	18
% Throttle	14	23	34	72	100

The optimization in this situation is applied to minimize the differences between the desired values of the longitudinal and lateral speed changes of Veh 1, $\Delta V_{long} = 4.74$ mph and $\Delta V_{lat} = 6.10$ mph, obtained from the EDR and the values calculated by PIM. Figure 3.2 shows the Solver Block for this analysis and the calculation of the square root of the sums of the squares of the differences between the desired and the calculated values.

Examination of the Solver Block (Figure 3.2) shows that Q (cell X17) is to be minimized. To achieve this goal, Solver is permitted to change three values: the preimpact speed of Veh 2 (cell G12), the coefficient of restitution, e (cell B8), and the impulse ratio μ (cell B9). A solution is shown in Figure 3.3.

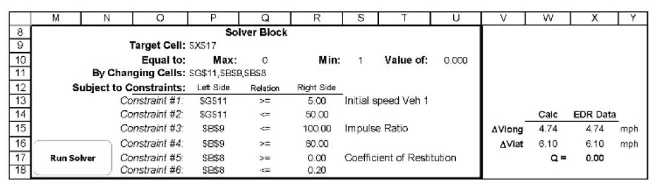

Figure 3.2. Solver block and Q calculation for Example 3

Initially, the optimization was applied with only the preimpact speed of Veh 2 and restitution being permitted to be changed in seeking a solution. After some time spent exploring the solution space, no viable solution was found for the given set of parameters. This process indicated that a review of the assumptions and conditions of the problem was needed. Review of the physical evidence, in this case the damage to the front of Veh 2, indicated that the assumption of a critical impulse ratio [9], $\mu = \mu_0$, was incorrect. This assumption was relaxed and a solution was found that satisfied all the constraints. In this case, the decision was made to allow the optimization algorithm select the value of μ (as a percent of μ_0). The result was that the speed of Veh 2 at the initiation of contact was found to be 10.6 mph. The value of $Q = 0.0$ means that the reconstructed conditions yielded an exact match to the two EDR ΔV components.

Example 4 - Collision example involving crush energy with EDR data

The analysis and reconstruction of a controlled head-on (in-line) collision has been previously presented [11]. The data from the crash presented in the article are used here to reconstruct the crash. The head-on, in-line collision involved a 2009 Chevrolet Impala (Vehicle A) and a 2008 Ford Focus (Vehicle B). Because the crash is a head-on central collision (i.e., preimpact and postimpact rotational velocities of the vehicles are negligible), point mass, impulse and momentum theory [9] can be used. Among the data available for the reconstruction of the collision are those imaged from the EDR of both vehicles. These contain the speed of the Focus in the five seconds leading up to $t = 0$ s before impact and the speed of the Impala including $t = -0.5$ s before AE. In summary:

- The Focus data show no preimpact brake lamp actuation with the speed at $t = 0$ s as 25.9 mph (38.0 ft/s, 41.7 kph).
- The Impala data show brake lamp actuation beginning at $t = -0.5$ s before AE and a speed at $t = -0.5$ s of 23 mph (33.7 ft/s, 37.0 kph).
- The EDR data include the longitudinal crash ΔV values, $\Delta V_{long} = 19.24$ mph (28.2 ft/s, 30.9 kph) for the Impala and $\Delta V_{long} = 23.15$ mph (34.0 ft/s, 37.2 kph) for the Focus.

The frontal residual crush deformation profiles of both vehicles were measured and crush energy analyses were carried out [11]. The results of these analyses are expressed as values of velocity change, computed from crush energy, ΔV^C:

- $\Delta V_A^C = 18.2$ mph (26.7 ft/s, 29.3 kph) for the Impala
- $\Delta V_B^C = 23.3$ mph (34.2 ft/s, 37.5 kph) for the Focus.

The initial speeds of each vehicle cannot be reconstructed using the results of the crush energy analysis alone; only the closing speed can be determined. If EDR data are available and are considered reliable, a reconstruction of the preimpact speeds of the vehicles can be carried out using combined data from the crush analysis and the EDR data.

This crash is reconstructed here using a combination of the EDR data and the crush energy data to illustrate the utility of optimization for reconstructing collisions. The use of the physical evidence, including electronic data, in the reconstruction of any crash will depend on the physical evidence, the data available, and the uncertainties of that data.

	A	B	C	D	E	F	G	H	I	J	K
1	impact.xls			Analysis of a Planar Vehicle Collision							
2	ver 3.0										
3	1/30/2015										
4				Vehicle 1			Vehicle 2			Initial speeds	
5	5280/3600	1.467		107.70	mass, m	lb-s^2/ft	143.59		Vehicle 1		Vehicle 2
6	g	32.2	ft/s^2	2043.65	inertia, I	ft-lb-s^2	3230.20		25.0	mph	10.6
7				2.90	distance, d	ft	7.20			Final speeds	
8	e	0.002		-78.20	angle, ϕ	deg	-8.60		Vehicle 1		Vehicle 2
9	μ (% μ_0)	69.9		-65.30	angle, θ	deg	211.80		21.2	mph	6.4
10	μ	0.769									
11	μ_0	1.100		25.00	INITIAL Velocity		10.61			ΔV	
12	Γ	25.000	deg	Vehicle 1			Vehicle 2		Vehicle 1		Vehicle 2
13				-15.32	v_x	ft/s	-13.23		7.7	mph	5.8
14	mbar	61.541		33.31	v_y	ft/s	-8.20		11.3	ft/s	8.5
15	k_1^2	18.975	ft^2	0.00	ω	deg/sec	0.00		4.7	ΔV_{long}	5.0
16	k_2^2	22.496	ft^2	36.67	v	ft/s	15.56		System Kinetic Energy, ft-lb		
17	q	1.003		0.19	v_n	ft/s	-15.45		Initial	89,783.9	
18	v_m	-15.644	ft/s	36.67	v_t	ft/s	-1.84		Final	63,503.5	
19	v_{tn}	-38.509	ft/s	0.19	v_{cn}	ft/s	-15.45		Loss	26,280.3	29.3%
20	r	2.462		36.67	v_{ct}	ft/s	-1.84				
21	η_1	-90.300	deg						Normal (Crush) Energy Loss:		
22	η_2	186.800	deg		FINAL Velocity				7,554.3	8.4%	
23	d_a	0.226	ft	Vehicle 1			Vehicle 2		Tangential Energy Loss:		
24	d_b	-7.196	ft	-20.54	V_x	ft/s	-9.31		18,726.0	20.9%	
25	d_c	-0.578	ft	23.26	V_y	ft/s	-0.66		Total System Energy Loss		
26	d_d	-2.842	ft	-43.57	Ω	deg/sec	-98.81		26,280.3	29.3%	
27	d_e	1.606	ft	31.03	V	ft/s	9.33				
28	d_f	5.758	ft	-8.79	V_n	ft/s	-8.71			Impulses, lb-s	
29	A	1.011		29.76	V_t	ft/s	3.34		P_x	P_y	P
30	B	0.018		-8.35	V_{cn}	ft/s	-8.32		-562.5	-1082.8	1220.2
31	C	1.230		27.60	V_{ct}	ft/s	15.75		P_n	P_t	P
32	D	2.230							-967.4	-743.6	1220.2
33											
34										PDOF, deg	
35									Vehicle 1		Vehicle 2
36									52.2		-30.7
37											
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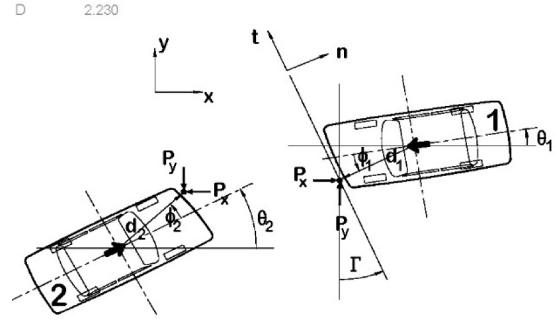

Figure 3.3. PIM solution spreadsheet for Example 3

An analysis of a combination of the crush and EDR data can be carried out using the method of least squares, described above. In this example, the values used from the data are the four values in Table 4.1 (the ΔV and initial speed for each of the two vehicles). The variables to be found (reconstructed) are the initial speeds, V_A and V_B. All values of the weighting factors, w_i, were chosen to be unity, that is, $w_i = 1$, $i = 1 \ldots 4$.

Table 4.1. Physical Data used as input to the Least-Squares Reconstruction

	Impala (Veh A)	Focus (Veh B)
ΔV from crush, mph	18.2	23.3
Initial Velocity, EDR, mph	23.0	25.9

Table 4.2. Reconstruction and Test Results

	Impala (Veh A)	Focus (Veh B)
Reconstructed v_i, mph	20.5	23.4
Reconstructed ΔV, mph	20.43	23.52
Measured Initial Speed, mph	22	25

A convenient way to carry out such a reconstruction is to place the impulse-momentum model equations into a spreadsheet [9], set up the computation of Q and let the optimization routine of the spreadsheet carry out the minimization [12]. Figure 4.1 shows a spreadsheet from the *VCRware* software package [13] that is programmed to handle central impacts (including low speed impacts) using the impulse-momentum model. Although not used in this example, the equations in this spreadsheet allow the effects of braking-tire impulses to be taken into account because for low speed collisions such effects can be significant. Figure 4.1 shows the results of the minimization process. Figure 4.2 shows the input to the spreadsheet's Solver routine including the "target" cell containing Q which is to be minimized, and the cell numbers of the quantities whose values are to be determined (reconstructed) through the minimization process.

The reconstructed values of the initial speeds are: $v_A = -20.5$ mph (30.1 ft/s, 33.0 kph) and $v_B = 23.4$ mph (34.4 ft/s, 37.7 kph). Because the spreadsheet solves the general central impact problem, many other reconstructed results are shown. Table 4.2 shows a summary of results of the speed reconstruction. Table 4.2 also contains the initial speeds measured from the crash test for comparison. Note that the reconstructed closing velocity that satisfies the impulse momentum equations is $V_C = 43.9$ mph (64.4 ft/s, 70.7 kph).

Some comments can be made concerning the reconstruction process (least-square minimization) and results:

- The Impala EDR showed a brake application in the last half-second before AE. Depending on the level of braking, this could have added an external frictional impulse to the collision. This impulse could have been taken into account in the least squares minimization process by adding the friction coefficient, f_A (see cell G5 in Figure 4.1), of Veh A to the list of unknowns to be found. This was done initially, but the closing speed of the collision was high enough to be considered a high speed collision and the change in results due to the external friction impulse, if it existed, was found to be negligible.
- The coefficient of restitution was chosen to be fixed at $e = 0$ for the least-square reconstruction. It too could have been found as part of the least-square process. Initially it was included in the least-square minimization, was found to be zero and was then simply fixed at $e = 0$.
- A question could be asked as to why the ΔV values from crush were used in the optimization process rather than the ΔV values from the EDR. This was done intentionally to illustrate the utility of the method. The reconstruction of the crash could have been done using the ΔV values from the EDR rather than from the crush analysis. This topic is explored further in the Discussion section.

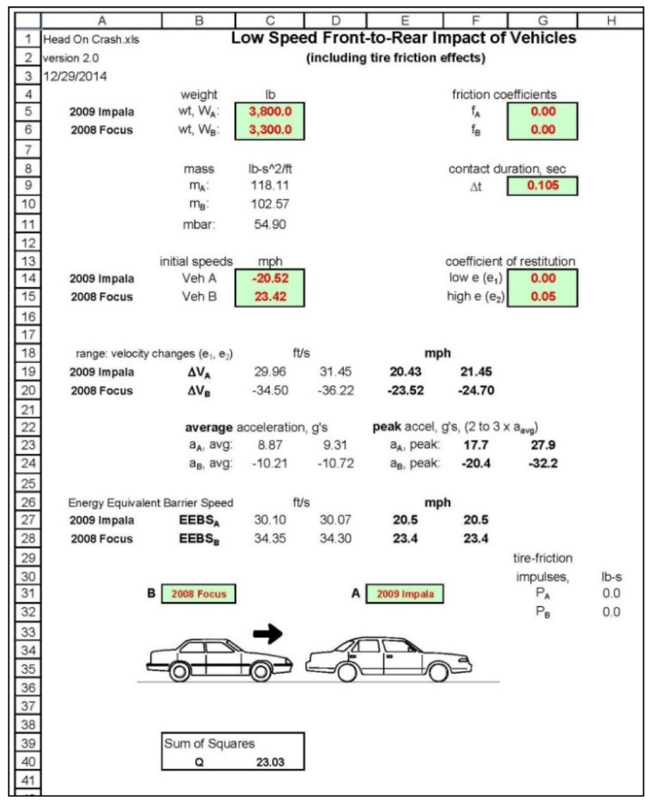

Figure 4.1. Spreadsheet with results of the least-square reconstruction of Example 4. Shaded cells are for known input values for an analysis; unknown input values can be found using the *What If* feature for a reconstruction when output information is known or specified.

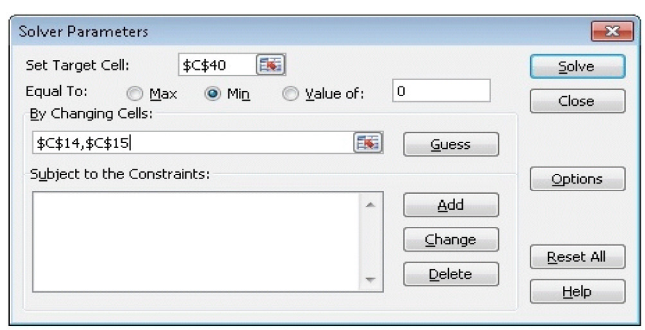

Figure 4.2. Input to Solver routine for minimization of Q (cell C40, Fig 4.1) in order to find the vehicle initial speeds (cells C14 and C15).

Example 5 - High-Speed Sideswipe with EDR Data

Collision configurations that give rise to sideswipe conditions, while not nearly as common as non-sideswipe collisions, do arise in practice. One collision configuration in particular that leads to this condition occurs when a vehicle drifts left of the centerline on a two-lane roadway and collides with another vehicle traveling in the opposite direction in which the collision overlap is perhaps a little wider than the headlight assemblies of both vehicles. The application of the method and data previously presented [15] are used here to illustrate the reconstruction of such a crash.

Figure 5.1 is a scale diagram that illustrates a crash configuration in which a crossover utility vehicle, Vehicle 2 (Veh 2), drifts left of the centerline on a two-lane roadway. Vehicle 1 (Veh 1), a full-size pickup truck, is traveling in the opposite direction. The two vehicles collide in the orientations shown in the scale diagram, which also shows the postimpact motion of each vehicle and the rest positions of the vehicles. The on-road and off-road tire marks measured at the scene are also shown.

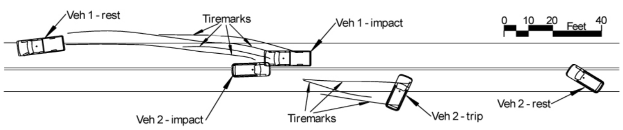

Figure 5.1. Scale diagram showing the crash scenario

In addition to this physical evidence (vehicle rest positions, tire marks, debris patterns, etc.), Veh 2 was equipped with an air bag control module (ACM) that included an event data recorder (EDR). The data from the EDR were imaged, examined and deemed reliable. The data from the EDR included the preimpact speed and the maximum recorded longitudinal ΔV, ΔV_{long}. The reconstruction of the collision was undertaken to determine the preimpact speed of Veh 1, which was not equipped with an EDR. The approach used to reconstruct the speed used PIM in combination with a least squares optimization strategy to simultaneously meet the various known crash criteria based on the physical evidence (including the EDR data). These criteria were the ΔV_{long} of Veh 2 (from the EDR) and the postimpact vector velocity directions of the center of mass of both vehicles determined from the tire marks. Additionally, the preimpact speed of Veh 2 is known from the data imaged from the EDR.

In this example, the values used from the data are given in Table 5.1. The variable to be found (reconstructed), s_1, is the initial speed of Veh 1, v_1. All values of the weighting factors, w_i, were chosen to be unity, that is, $w_i = 1$, $i = 1, \ldots, 3$.

Table 5.1. Data used as input to the Least Squares Reconstruction

	Vehicle 1	Vehicle 2
ΔV_{long}	TBD	32.5 km/h
Departure angle of CG	167.4°	-8.1°
Initial Speed	TBD	109.4 km/h

A convenient way to carry out such a reconstruction is to place the PIM impulse-momentum model equations into a spreadsheet [13], set up the computation of Q and let the optimization algorithm of the spreadsheet, in this case Excel Solver, carry out the minimization [12]. Figure 5.2 shows the input to the spreadsheet's optimization routine (called "Solver Block"), including the "Target" cell which contains the cell reference for Q which is to be minimized, the cell numbers of the quantities whose values are to be determined (reconstructed) through the minimization process and the constraints imposed on the process. Figure 5.3 shows a spreadsheet from the *VCRware* software package [13] that uses the PIM impulse-momentum model. The equations in this spreadsheet, by definition, include the coefficient of restitution, e, and impulse ratio, μ (implemented in the spreadsheet as a percentage of μ_0). Figure 5.3 also shows the results of the minimization process. The suitable ranges for the constraints on e, μ and Γ (crush surface angle) for a sideswipe collision are established using available test data [15].

The reconstructed value of the initial speed of Veh 1 is $V_1 = 40.8$ mph (59.8 ft/s, 65.6 kph). Note that the coefficient of restitution, cell B8 in Figure 5.3, and the impulse ratio, cell B9 in Figure 5.2, are found in the optimization process rather than being specified a priori. These values are $e = 0.158$ and $\mu = 70.8\%$ of μ_0. This value of the coefficient of restitution is consistent with the data from the IIHS testing [15] ($0 \leq e \leq 0.3$) as well as with other published data [1, 9]. The value of the impulse ratio of 70.8% of the critical value is on the low end of the range for the vehicle-to-barrier tests, and below the range from the six vehicle-to-vehicle tests that were sideswipes [15]. In this situation, confidence in the results (principally the speed of Veh 1) is high for two reasons:

- The values for e, μ, and Γ, all found by the optimization routine, are based on data collected by the EDR during the crash and other physical evidence (tire marks), and
- These values are consistent with the test data.

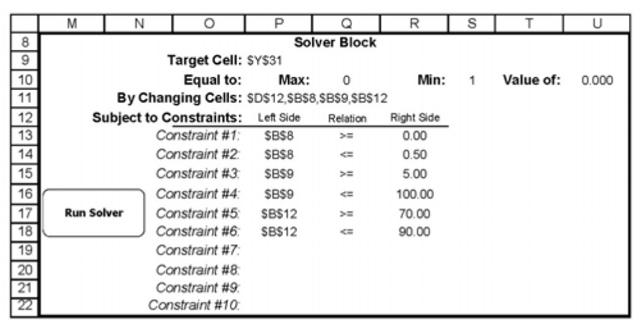

Figure 5.2. Input to Solver optimization routine for minimization of Q (calculated in cell Y31, not shown) in order to find the initial speed of Vehicle 1 (cell D12 in Figure 5.2).

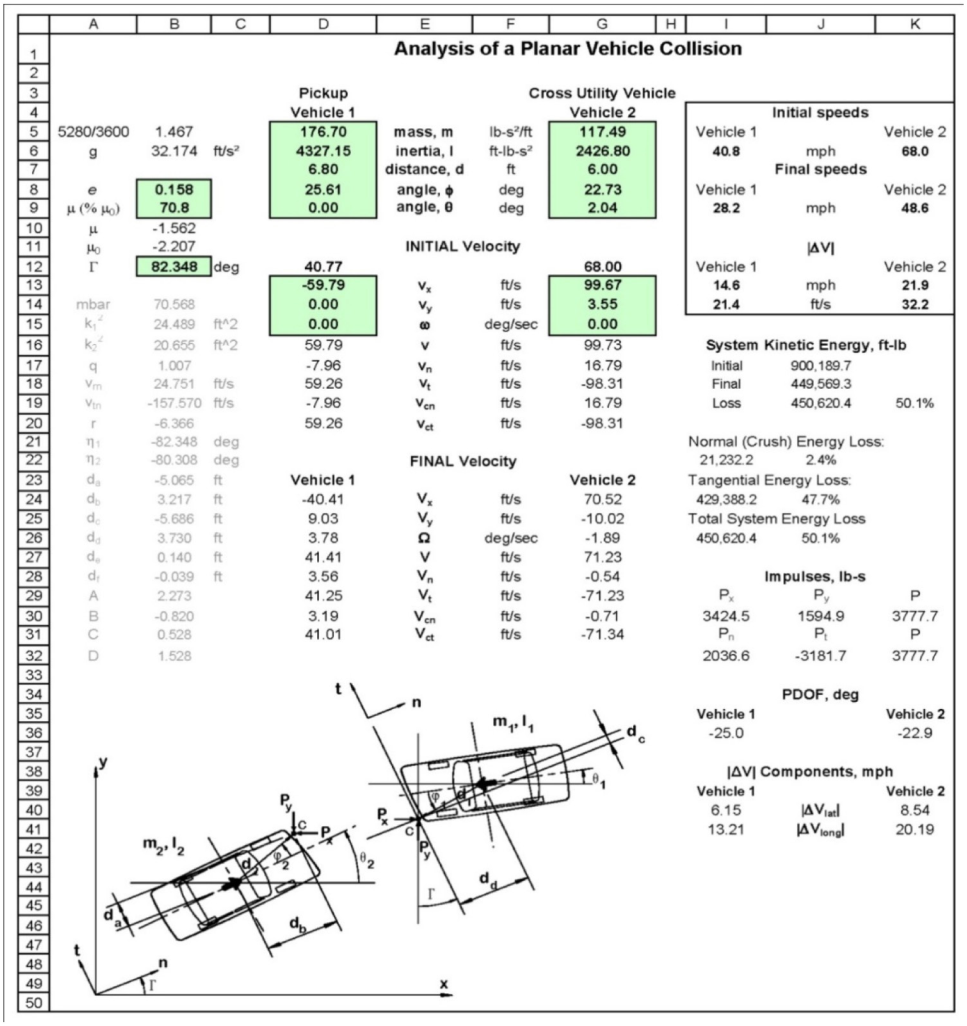

Figure 5.3. Spreadsheet showing the results of the least squares reconstruction

DISCUSSION

This primary focus of this paper is the presentation of a technique for the reconstruction of vehicular crashes based on the use of nonlinear optimization methods. The five examples provided in the paper demonstrate the utility of the method for reconstructing a wide range of vehicle crashes. The applications include a variety of intersection crashes and also include a high-speed sideswipe crash. The examples show that the technique provides an effective and efficient method for incorporating EDR data into the crash reconstruction process, chiefly through the use of constraints.

Although not explicitly stated, Examples 2, 3 and 5 approached the reconstruction problem from the perspective that the availability of EDR data from one (or both) of the vehicles precludes the need for the use of crush energy as a reconstruction method. This crush-based method, frequently referred to as the CRASH3 method, uses the residual crush of the vehicles involved in a crash, coupled with experimental barrier crash data, to estimate the (system) energy loss of the collision. This energy loss is then used to calculate the magnitude of the ΔV of the mass centers of each of the vehicles. The direction of the ΔV, commonly referred to as the PDOF, is estimated visually from the residual crush by the reconstructionist. Data from even the earliest generation of GM sensing and diagnostic modules provide the longitudinal ΔV of the vehicle computed from the acceleration measured by accelerometers during the collision. Assuming that the data from a given module is properly vetted and is deemed reliable, it provides a direct measure of the (longitudinal) ΔV. This measure does not rely on experimental crash data, measurements of the residual crush or estimates by the reconstructionist for the direction of the change in velocity. By using measured values for the magnitude and direction of ΔV, the uncertainty of the results from the reconstruction is reduced. The uncertainty of the ΔV determined using the CRASH3 method has been analyzed [14, 16]. Comparison of the uncertainty reported in those references with the uncertainty of the ΔV_{long} determined from ACM measurements [19] provides the rationale for the use of measured ΔV versus calculated the ΔV using CRASH3.

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