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Sensitivity Analysis of Simulated Postimpact Vehicle Motion Using Design of Experiments (DOE)

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Abstract

An important component of the process of the reconstruction of a vehicle crash involves the modeling of the motion of the vehicle(s) before and after a collision. Depending on the conditions, this motion might be modeled using a vehicle dynamics simulation program. In the simulated dynamics of vehicle motion, the tire forces are the predominant means by which the path of the vehicle is determined, with aerodynamic loads being the other force acting on the vehicle. Recent literature on this topic investigated the effect of the steer angle of the front wheels on the postimpact trajectory of a light vehicle for a large initial angular velocity. This paper looks more broadly at the modeling of light vehicle postimpact motion using vehicle dynamics simulation but for a wider range of factors.

Design of experiments (DOE) is used to rank the effect of various physical factors of vehicle postimpact motion. The response variable used in the DOE analysis uses the rest position of the vehicle (characterized by the x and y coordinates of the CG and the vehicle heading, θ) for a given combination of factor changes. The results of the study show that in the four different designs that were conducted, a trend in the response was consistent. The single factor that consistently appeared in the various DOE analyses (with various factor combinations) was the tire-to-roadway frictional drag coefficient. Various other factors and 2-factor combinations were also found to be significant. Some of the significant factors are not intuitively obvious, such as aerodynamic drag.

Introduction

A recent article [1] examined the motion of yawing and translating vehicles following a crash. The authors looked at the use of a variety of models applicable to this type of vehicle motion. These models ranged from relatively simplistic formulas that can be applied to the full distance of vehicle motion in aggregate or applied sequentially to the vehicle motion divided into segments, to more complex formulas that explicitly involve the rotational motion of the vehicle. Ultimately, the use of vehicle dynamics simulation is studied. That paper also used vehicle dynamics simulation to examine the trends in the effective frictional drag of a vehicle that is both yawing and braking. Of particular interest to the author of this paper, is that paper included analysis of the sensitivity of vehicle dynamics simulation to the steering angle at the front wheels on the (rotational and translational) postimpact motion of a vehicle.

The research presented here extends the sensitivity analysis of vehicle dynamics simulation for rotating and translating vehicles to include multiple factors rather than concentrating on a single factor (the steering angle). The sensitivity analysis is carried out using design of experiments (DOE). The use of this method for sensitivity analysis was introduced to the field of crash reconstruction previously [2]. The methodology employed there analyzed the sensitivity of planar impact mechanics to changes in various parameters, or factors. The method is used similarly here to simultaneously evaluate the sensitivity of the vehicle dynamics simulation process to changes in a group of eight input parameters rather than just one. The vehicle dynamics simulation program used in the analysis is part of the *VCRware*® software package [3]. The details of that simulation program and the tire force model, including the accuracy using test data, have been previously presented [4], [5]. This paper addresses the sensitivity of the program rather than the accuracy.

The intent of this paper is twofold. First, the research presented in this paper further explores the application of the DOE method for sensitivity analysis in the field of crash reconstruction. Second, more information about the method for examining the behavior and (relative) sensitivity of the various factors in a vehicle dynamics simulation program is presented.

Numerous papers have been published that investigate the effect of various parameters/factors on vehicle dynamics simulation programs. Some examine various parameters used in simulation that are selected as they relate to vehicle rollover [6], [7]. Most notable as an examination of the various (single) factors involved in vehicle dynamics simulation was done by Heinrichs et al. [8]. These various analyses generally vary one or several factors over some applicable range and examine the effect on the vehicle motion. An issue with this type of analysis is that, while providing insight into the behavior of the vehicle (and the simulation model), the relative uncertainty/sensitivity of the various parameters is not quantified. In contrast, the use of DOE to examine the sensitivity provides a rank of the influence of the various factors, and numerous factors can be included in the analysis. Additionally, the DOE method provides information about the influence of factor interactions. As will be shown by the results of these analyses, interactions have significant effects on the behavior of the simulation program.

Design of Experiments (DOE)

The formulation of the concepts of the design of experiments (DOE) originate in the beginning of the 20th century. Numerous treatments of the topic exist containing the details of the method [9], [10]. Generally, the DOE method is used with experimental measurements of a physical system/process. DOE is an analytical method in which changes in the process response due to changes in the factor values is determined and analyzed.

The method can be applied to computer "experiments", i.e. computer simulations. The application of the use of DOE applied to computer simulation has been a more recent development. Early work was done in the late 1970's and early 1980's [11], [12] with the advent of widespread availability of digital computers and development of simulation models. The topic now is given its own treatment in various books [9], [13] and has been included in books dealing with forensic engineering applications [14].

The application of DOE used here belongs to the category of $2k$ Factorial Design. In this DOE construct, each of the k factors selected in the analysis has two levels, or values, generally designated as high/low (+1/−1 or +/−). If the number of factors, k, becomes large, then 2^k becomes large, requiring numerous trials. An approach can be used whereby the number of trials can be reduced by p half fractions with little loss in effectiveness by taking into account that with many factors, the effects of higher-order interactions are likely to be negligible. The number of runs is reduced from 2^k to 2^{k-p}. With this approach, the specific DOE method used is referred to as a *Fractional Factorial Design*.

The result of a trial, in this instance a simulation run, is referred to as the system response. Through the application of DOE, a linear model may be developed to model or predict the response as a function of one or more predictor variables. As an example, a linear model for consideration of two predictor variables (τ, β) on a response (y) may be written as:

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \tag{1}$$

In this equation, μ is the mean response, τ_i is the influence of the i^{th} level of Factor A, β_j is the influence of the j^{th} level of Factor B, $(\tau\beta)_{ij}$ represents interaction of the i^{th} level of Factor A and j^{th} level of Factor B. The last term represents the error of the single observation to the predicted response.

Readers interested in the details of DOE methods will find copious resources available starting with [9], [10], [13], [14].

The Simulation Model

The work done in this study uses the vehicle dynamics simulation model that is part of the *VCRware*® computer software suite [3]. The details of the model and the validation are presented elsewhere [15]. This simulation model is for a two-axle, four-wheeled vehicle pulling a semitrailer. The tire forces are modeled using a nonlinear tire force model. The details of the tire force model are presented in detail elsewhere [5], [16]. The application here did not involve a semitrailer thus the model for the vehicle motion is just using the two-axle vehicle resulting in a three degree-of-freedom system. The differential equations of motion are put in first order form and are integrated using a Runge-Kutta-Gill numerical integration scheme. The result is the motion of the vehicle for a given set of initial conditions and set of vehicle parameters. The model does not include a vehicle suspension but does include dynamic weight shift requiring that the height of the vehicle CG be defined.

The input parameters of the program include geometric properties, (e.g. wheelbase, track widths, etc.), vehicle inertial properties (e.g. vehicle mass, yaw inertia, CG height, etc.), parameters associated with the tire force model (C_α, C_s, etc.), environmental parameters (e.g. tire roadway friction, f) and initial conditions (v_x, v_y, and ω). With this number of inputs, DOE provides an excellent means to evaluate the sensitivity of the model on the numerous parameters while also evaluating the interactions of the parameters.

The output of the model is the motion of the vehicle (generally the successive positions of the vehicle CG) and the angular position (heading) as a function of time. The "sampling rate" of the motion is the time-step of the Runge-Kutta-Gill integration procedure. The analysis conducted here did not include the time-step as a factor in the DOE analysis and it was fixed throughout the analyses.

More information will be presented regarding the selection of the system response using numerical examples. Initially, the crash to be studied will be presented and then analyzed using a fractional factorial DOE.

Simulated Crash

This analysis follows work done in two papers. In [1], a sensitivity analysis of vehicle dynamics simulation to impact induced steering is presented for vehicles with high initial rotational speed. In [3], various different vehicle dynamics simulation programs were analyzed and compared using experimental data that was previously presented [17]. The vehicle used in the driving tests presented in [17] was a 1991 Honda Accord. This same vehicle was studied in [5] including generalizing the tire model amongst the various simulation packages. This generalization of the tire models (and the simulation parameters as presented in [5]) is used here again for consistency in the comparison of the results of the DOE analysis.

Figure 1 shows the impact orientation of the two vehicles that are studied here using DOE. This impact is modeled using planar impact mechanics (PIM) [18] to generate realistic initial conditions for the simulation. The crash geometry and

FIGURE 1 Impact geometry for the crash used to generate the initial conditions for the DOE analysis. The rest position from the simulation is also shown. Intermediate positions shown lighter. Large arrows on the vehicles indicate direction of preimpact velocity.

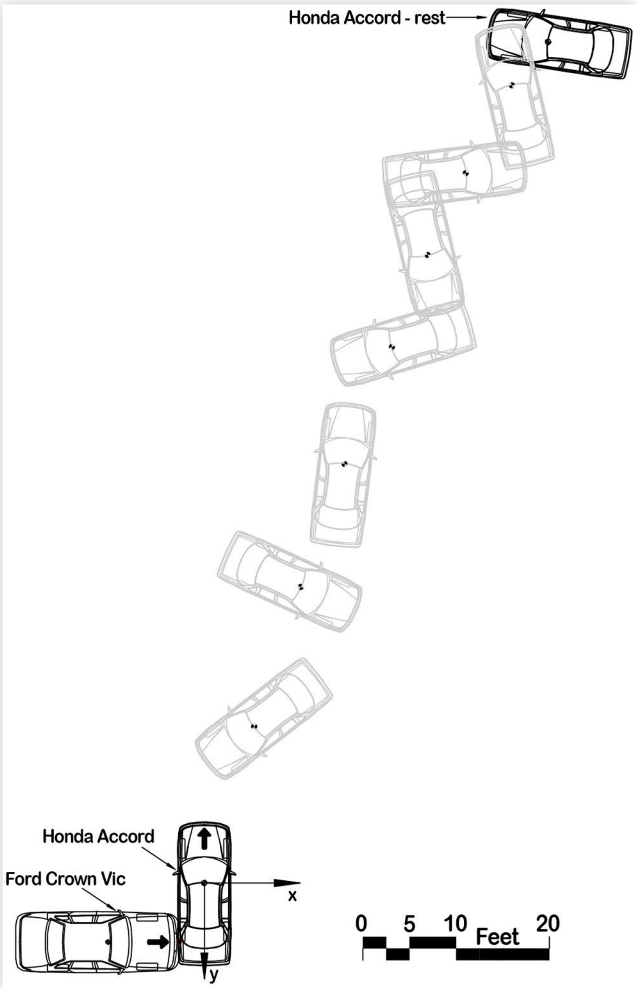


TABLE 1 Results of the impact analysis for use as the initial condition for the Honda Accord in the DOE study.

Preimpact speed of both vehicles	50 mph (80.6 kph)
Postimpact velocity in x-direction (Accord)	18.17 ft/s (5.54 m/s)
Postimpact velocity in the y-direction (Accord)	59.04 ft/s (18.00 m/s)
Postimpact angular velocity (Accord)	436.43 deg/sec

conditions are meant to mimic those of a perpendicular intersection with one vehicle failing to stop at a stop sign. This collision scenario produces a large postimpact angular speed of the Honda. Prior to impact, neither vehicle has an initial angular velocity. The impact analysis produced the initial conditions for the Honda Accord shown in Table 1 to be used in the DOE analysis. The output sheet of the PIM analysis is included in Appendix A.

DOEs and Simulation Results

Nominal Results

For simplicity in the analysis, the position of the CG of the Honda at the time of separation is placed at the origin of the coordinate system and the initial angular orientation is at $\theta = -90°$. Table 2 gives the rest position information of the vehicle for the case with all the factors at their nominal value (neither +1 or −1).

The factors selected, the nominal values and the +1/−1 values will be presented below for each of the cases considered for this impact configuration.

Initial DOE (DOE 1)

The purpose of this DOE was to (1) establish a baseline on the different response variables related to the postimpact vehicle position, and (2) identify/confirm significant factors that carry the largest influence on the postimpact position and orientation. To begin, a summary is made with regards to the structure of the DOE, starting with a summary of the factors and factor levels considered. The simulations were designed to study the following $k = 8$ factors:

A. Wheel lateral stiffness, C_α: front 13,000 lb/rad, rear 11,000 lb/rad (varied ±5%)
B. Wheel longitudinal stiffness, C_s: 10,000 lb/unit slip (varied ±5%)
C. Mass/Inertia, m/I: 3186 lb/2000 ft-lb-s^2 (varied ±5%, both in same direction)
D. CG height, h_{CG}: 1.76 ft (varied ±3% based on data in [19])
E. Tire-road frictional drag, f: $f = 0.7$ (varied ±7%, $0.65 \leq f \leq 0.75$)
F. Front wheel steer angle, δ: $\delta = 0°$ (varied ±5°)
G. Aerodynamic drag, *Aero*: no drag is the nominal, $C_{DCX}A_{XC} = 10$ ft^2, $C_{DCY}A_{YC} = 48$ ft^2
H. Initial conditions, *IC*: $V_x = 18.17$ ft/s, $V_y = -59.04$ ft/s, and $\Omega = -436.43$°/s (each of the three velocity components was varied by the same percentage such that the kinetic energy increased or decreased by approximately ±5%)

With the above list of factors there are two notes to be made:

TABLE 2 Postimpact velocity components of the Accord for the nominal conditions

Preimpact speed of both vehicles	50 mph (80.5 kph)
Rest Position - x (Accord)	36.86 ft (11.24 m)
Rest Position - y (Accord)	−90.5 ft (27.59 m)
Angular Position - θ (Accord)	−891.34°

1. The front and rear wheel lateral stiffness, $C_{\alpha f}$ and $C_{\alpha r}$ were assigned different values, 13,000 lb/rad for the front and 11,000 lb/rad for the rear. These values were varied in unison, thereby creating a condition that the balance, front to rear, of the wheel lateral stiffness was constant.
2. The mass and yaw inertia parameters of the Accord were coupled such that both would be set to their high value or both were set to their low value for a given simulation. These values are related to each other in that the yaw inertia is equal to the mass times the square of the radius of gyration, i.e. $I = mk^2$.

A study incorporating all the combinations of these eight factors at their respective +1/−1 levels would require:

$$n = 2^k = 2^8 = 256\ simulations$$

Rather than run all of these simulations, the number of runs in the analysis was reduced utilizing a standard method in the application of DOE. In this application, the DOE is referred to as having been *fractionalized*. In this first DOE, a $\frac{1}{16}$ fractional factorial DOE was used. The fractionalized DOE allowed the 8 factors to be studied in

$$\frac{1}{16}2^8 = \frac{1}{2^4}2^8 = 2^{-4}2^8 = 2^{8-4} = 16\ simulations$$

The structure of these 16 simulations were defined using Yate's algorithm to ensure the design is balanced, i.e. orthogonal. This approach defined the 16 trials in the following standard order:

The variables A through H correspond to the listed factors and the "-1" and "+1" correspond to the low and high levels of the factor, respectively. In addition to running simulations at the factor limits, a single run was performed at the nominal conditions. This is the Center Point (CP) of the DOE, with the coded values of the factor levels all at 0.

A byproduct of fractionalizing the full factorial DOE is the risk of confounding, where the effects of the individual factors and their different interactions are combined and making it difficult to determine which factor or factor interactions are significant. In the design presented in Table 3, columns E, F, G, and H were defined as the products of the "-1 s" and "+1 s" associated with factors A through D. For this DOE, factor levels for columns E, F, G and H were defined as follows:

1. E = ABC
2. F = ABD
3. G = ACD
4. H = BCD

These *definitions* results in the following aliasing structure of the DOE:

From the above alias structure, 2-factor interactions are confounded with other 2-factor interactions; however, it can also be shown that single factor effects will be confounded with 3-factor interactions, e.g. for aliasing structure 1:

AB=CE = DF = GH

A(BB) = BCE = BDF=BGH (multiply through by Factor B)

→ A = BCE = BDF=BGH (noting that BB=B^2 = I).

TABLE 3 DOE structure for 8 factors, each at 2 levels, and 16 trials.

Run	A	B	C	D	E	F	G	H
1	−1	−1	−1	−1	−1	−1	−1	−1
2	+1	−1	−1	−1	+1	+1	+1	−1
3	−1	+1	−1	−1	+1	+1	−1	+1
4	+1	+1	−1	−1	−1	−1	+1	+1
5	−1	−1	+1	−1	+1	−1	+1	+1
6	+1	−1	+1	−1	−1	+1	−1	+1
7	−1	+1	+1	−1	−1	+1	+1	−1
8	+1	+1	+1	−1	+1	−1	−1	−1
9	−1	−1	−1	+1	−1	+1	+1	+1
10	+1	−1	−1	+1	+1	−1	−1	+1
11	−1	+1	−1	+1	+1	−1	+1	−1
12	+1	+1	−1	+1	−1	+1	−1	−1
13	−1	−1	+1	+1	+1	+1	−1	−1
14	+1	−1	+1	+1	−1	−1	+1	−1
15	−1	+1	+1	+1	−1	−1	−1	+1
16	+1	+1	+1	+1	+1	+1	+1	+1
CP	0	0	0	0	0	0	0	0

As a result, the DOE is classified as a Resolution IV design, with the final notation being:

$$2^{8-4}_{IV}$$

The design of the DOE permits the development of a linear model of the form:

$$y = \beta_0 + \sum_{i=1}^{8} \beta_i \cdot X_i + \sum_{j=1}^{7} \beta_j \cdot X_A \cdot X_j + \epsilon$$

where,

- β_0 is the y-intercept
- β_i is the coefficient associated with the Main Effect of the i^{th} factor
- β_j is the coefficient associated with the 2-Factor Interaction between Factor A and the other remaining factors
- Note that the 2-Factor Interactions are aliased per the definitions shown in Table 4
- ϵ is the Error Term

Given that the trials of the DOE are based on a simulation that does not exhibit inherent variability, the Error Term (ϵ) is zero. All 16 trials of the DOE contribute to establishing the 1 + 8 + 7 = 16 coefficients.

The system response used in the analysis presented herein is selected based on the reconstruction task. Generally, a

TABLE 4 Aliasing Definitions

1. AB=CE = DF = GH	2. AC=BE = DG = FH
3. AD = BF=CG = EH	4. AE = BC = DH=FG
5. AF=BD = CH = EG	6. AG = BH=CD = EF
7. AH=BG = CF = DE	

reconstructionist will endeavor to match the available physical evidence. The physical evidence associated with the postimpact motion of a vehicle is typically used as the acceptance criteria of the simulation of vehicle postimpact motion is the rest position and orientation. Thus, the coordinates of the CG of the vehicle at rest are used here as the response vehicle.

Initial DOE Simulations Results In all trials of the DOE, the simulations were performed with the wheels locked, i.e. the slip values at each of the four wheels set equal to one, $s = 1$. This was done primarily to prevent rollout of the vehicle at the end of the angular motion. While vehicle roll-out does occur in real crashes, it was avoided here to effectively eliminate roll-out as a component in the analysis in that roll-out would have no effect on the rest position of the vehicle. The time step for the Runge-Kutta-Gill integration (in all cases) was 5 milliseconds. The results of the post impact X, Y, and θ values from the simulations for the 16 trials of the DOE are shown in Table 5.

To introduce the different analyses that support the confirmation of significant factors, a more in-depth analysis will be presented for this first DOE that considers the correlation of the response variables.

Correlation in Response Variables X, Y, and θ Given that there are multiple response variables associated with the simulations, the first step in the analysis evaluates the correlation between the responses of X, Y, and θ. A Correlation Matrix provides a measure of dependency between all the combinations of the response variables. Table 6 shows the Correlation Matrix for X, Y, and θ based on Pearson's Product Moment Correlation Coefficient ($\rho_{x,y}$), where:

$$\rho_{x,y} = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{xy}}{\sqrt{\sum_{i=1}^{n} x_i^2 - n\overline{x}^2}\sqrt{\sum_{i=1}^{n} y_i^2 - n\overline{y}^2}} \tag{2}$$

TABLE 5 Summary of postimpact position components for all 16 DOE trials.

Run	X	Y	θ
1	36.26	−92.43	−902.56
2	31.32	−78.25	−779.09
3	35.86	−88.26	−871.69
4	38.33	−99.21	−967.89
5	35.07	−86.26	−858.51
6	39.34	−102.08	−982.09
7	35.56	−90.18	−887.92
8	31.99	−80.02	−789.67
9	39.93	−98.94	−976.02
10	37.57	−88.07	−886.23
11	32.86	−77.99	−789.52
12	37.49	−92.35	−912.79
13	33.56	−79.80	−801.05
14	36.94	−90.02	−899.29
15	40.97	−101.77	−990.74
16	36.83	−86.03	−872.48
CP	36.86	−90.05	−891.34

with n being the number of x_i, y_i pairs of observations, with $n = 16$ here. Note that the correlation of $\rho_{x,x} = 1$ and that $\rho_{x,y} = \rho_{y,x}$. Also, a coefficient of 1 means perfect association between the two variables. The correlation matrix of X, Y, and θ as observed from the simulations is the 3 × 3 symmetric matrix shown in Table 6.

Table 6 shows a very strong dependency between the Y position and the angular orientation (θ), i.e. $\rho_{Y,\theta} = 0.955$. The Coefficient of Determination can be obtained by squaring the correlation coefficient to get 0.912. This value can be interpreted by stating that 91.2% of the variation in the Y position can be explained through the variation of the angular orientation (θ).

The correlation between X and θ also appears to be significant at $\rho_{X,\theta} = -0.6018$. The dependency between X and Y was less significant with $\rho_{X,Y} = -0.4179$. To provide a graphical interpretation to the correlation a scatter plot of the (Y_i, θ_i) pairs of observations is shown in Figure 2.

Due to the correlation between response variables, the application of a response function to combine them presents issues. An example of a response function is given by Equation 3.

$$R_1 = \sqrt{\left(x_i - x_n\right)^2 + \left(y_i - y_n\right)^2 + \left(\theta_i - \theta_n\right)^2} \tag{3}$$

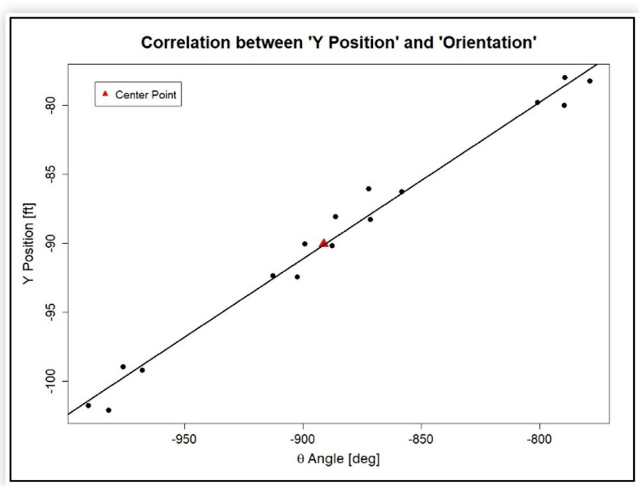
As a result, this analysis continues with the analyzing the responses of X, Y, and θ individually.

The use of a response function as given by R_1, for example, rather than a response variable has been previously presented

TABLE 6 Pearson Product-Moment Correlation Matrix for the response variables of the simulations

	X	Y	θ
X	1.0000	−0.935	−0..6018
Y	−0.935	1.0000	0.955
θ	−0.6018	0.955	1.0000

FIGURE 2 Scatter Plot of paired measures between the Y post impact position and angular orientation (θ), where the fitted line shows the dependency between Y and θ.



(22). This approach was attempted here but was deemed unacceptable due to the correlation of the variables defining the rest position. Appendix C provides some additional background on this topic.

DOE 1 Analysis (X Position Only) The following is a detailed summary on the analysis of the postimpact X position of the vehicle. The first step in this effort is to produce a Main Effects Plot to show the difference in the mean response between respective *coded* levels (−1, +1) for each factor, e.g. the effect of Factor A is $A = \bar{y}_{A^+} - \bar{y}_{A^-}$. Figure 3 shows the Main Effects Plot specific to the postimpact X position of the vehicle. The plots suggest that factors D, E, and H (h_{CG}, f, and IC) to have the largest effect on the X position, with Factor G (aerodynamic drag) as a possible 4th factor of influence.

An interpretation of a result from Figure 3, would be the effect of moving Factor D (h_{CG}) from its low level (1.71 ft) to its high level (1.81 ft) changed the average of X position by +0.09 ft. More specifically, assuming a linear function, the change in the X position per unit change in the CG is:

$$\beta_D\big|_{CODED} = \frac{37.01875\,ft - 35.46625\,ft}{1-(-1)} = 0.77625\,ft$$

In its *natural* units,

$$\beta_D\big|_{NATURAL} = \frac{37.01875\,ft - 35.46625\,ftt}{1.81\,ft - 1.71\,ft} = 15.525\,{}^{ft}\!/_{ft}$$

This implies that the x position will increase 15.5 feet for a 1-foot increase in h_{CG}. The 0.77625 ft and the 15.525 ft/ft values represents the slope for h_{CG} term in the coded and un-coded linear models.

Further analysis and confirmation of the significant terms can be made with a Normal Probability Plot of the effects, which is sometimes referred to as a Daniel Plot [20]. Figure 4 shows the Normal Probability plot for the effects as related to the X position. This plot confirms factors D, E, G and H to be

FIGURE 3 Main Effects Plot on postimpact X position of vehicle suggesting Factors C, D, G, and possible Factor E as being significant to the X position.

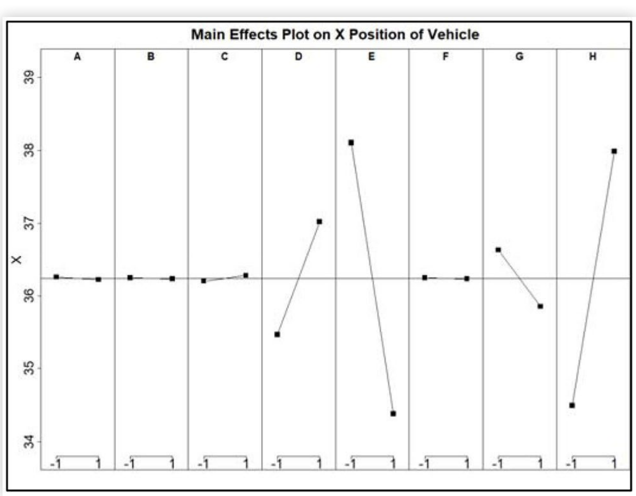
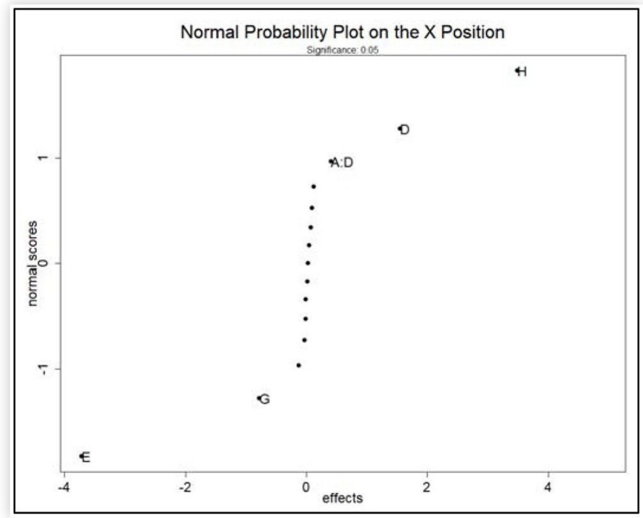


FIGURE 4 Normal Probability Plot of DOE 1 Factor Effects showing Factors D, E, G and H as being statistically significant at α=0.05. Plot also suggests the AD 2-factor interaction (which is actually the EH interaction due to confounding) as significant at α=0.05.



statistically significant at α=0.05. The same plot also shows that the A-D 2-factor interaction term to be significant. However, caution should be take when interpreting the 2-factor interaction terms. The plot shows the AD interaction as significant, but does not show that Factor A by itself to be significant. It is unlikely to have a 2-factor interaction to be significant if the factors that comprise it are not also significant. As a result, the aliasing within the DOE must be considered. Going back to Table 4, it can be noted that the AD interaction is confounded with the BF, CG and EH interactions. From the Main Effects plot it was confirmed that the factors E and H are significant; therefore, it is more likely that the 2-factor interaction seen in Figure 4 is due to the EH interaction and not the AD interaction.

The following table summarizes the effects of the model factors in *coded* units. It also shows the rank of the absolute value of the coefficients.

DOE 1 Analysis (Y Position and Angular Orientation (θ) The statistical analyses for the two remaining response variables of the postimpact Y Position and Angular Orientation θ was performed in the exact same manner as that used for the postimpact X Position. Main Effects Plots, Daniel Plots and estimates in the model coefficients were made. For brevity, this section will only show the results of the Daniel Plots that confirm the statistical significance of those factors that have the greatest influence on Y and θ. Figure 5 compares the Daniel Plots for the two additional response variables.

Just as with the postimpact X position, the significant factors for the postimpact Y position and angular orientation θ are Factors D, E G and H. A summary and comparison of their *coded* model coefficients are provided in Tables 8 and 9.

For the Y Position, Table 8 shows several significant factors - 4 main effects and 3 interactions. The significant main effects are the CG Height, Tire-Roadway Frictional Drag, Aerodynamic Drag, and Initial Conditions.

Just as with the X Position, attention must be made with regards to confounding of the interaction terms. Again, the same rationale is applied such that only those interaction terms that also have significant main effects will be considered. In case for the Y Position, the significant 2-way interactions are the Aerodynamic Drag and Initial Conditions,

TABLE 7 Summary of model coefficients, in Coded Units, for the postimpact X position of vehicle. Significant Factors are identified with asterisk.

	Factor	Coefficients (X Response)	Rank of |Coefficient|
1.	A: Wheel Lateral Stiffness	−0.01625	11
2.	B: Wheel Longitudinal Stiffness	−0.00625	14
3.	C: Mass - Inertia	0.04000	9
4.	D: Center of Gravity*	0.77625	3
5.	E: Tire Road Friction Drag*	−1.86000	1
6.	F: Front Wheel Steer Angle	−0.00625	15
7.	G: Aerodynamic Drag*	−0.38750	4
8.	H: Initial Conditions*	1.74500	2
9.	AB = CE = DF = GH Interaction	−0.06000	7
10.	AC = BE = DG = FH Interaction	0.00875	13
11.	AD = BF=CG = EH Interaction*	0.20500	5
12.	AE = BC = DH=FG Interaction	0.06125	6
13.	AF = BD = CH = EG Interaction	0.02500	10
14.	AG = BH=CD = EF Interaction	0.01625	12
15.	AH = BG = CF = DE Interaction	0.04625	8

FIGURE 5 Normal Probability Plot of DOE 1 for Response Variables X and θ. Factor Effects showing Factors C, D, E, and G as being statistically significant at α=0.05.

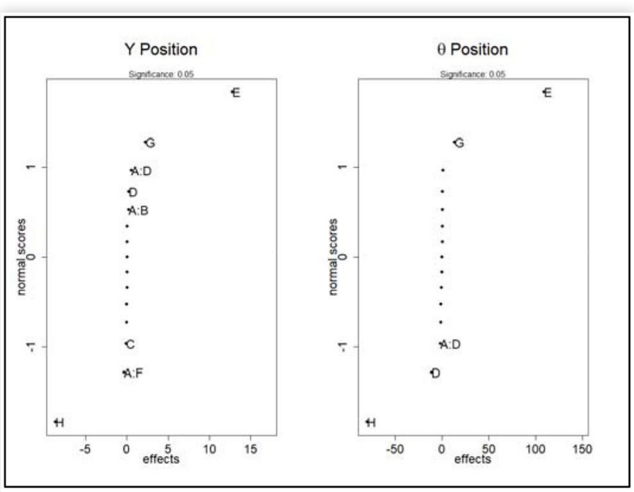


TABLE 8 Summary of model coefficients, in Coded Units, for the postimpact Y position of vehicle. Significant Factors are identified with an asterisk.

	Factor	Coefficients (Y Response)	Rank of |Coefficient|
1.	A: Wheel Lateral Stiffness	−0.02500	9
2.	B: Wheel Longitudinal Stiffness	0.00250	15
3.	C: Mass - Inertia	−0.04125	8
4.	D: CG height*	0.10750	6
5.	E: Tire Road Frictional Drag*	6.39375	1
6.	F: Front Wheel Steer Angle	−0.00750	11
7.	G: Aerodynamic Drag*	1.11875	3
8.	H: Initial conditions*	−4.34875	2
9.	AB = CE = DF = GH Interaction*	0.09875	7
10.	AC = BE = DG = FH Interaction	0.00750	13
11.	AD = BF=CG = EH Interaction*	0.27875	4
12.	AE = BC = DH=FG Interaction	0.01750	10
13.	AF = BD = CH = EG Interaction*	−0.16625	5
14.	AG = BH=CD = EF Interaction	0.00750	12
15.	AH = BG = CF = DE Interaction	0.00500	14

TABLE 9 Summary of model coefficients, in Coded Units, for the postimpact θ position of vehicle. Significant Factors are identified with an asterisk.

	Factor	Coefficients (θ Response)	Rank of |Coefficient|
1.	A: Wheel Lateral Stiffness	−0.72000	7
2.	B: Wheel Longitudinal Stiffness	0.13375	11
3.	C: Mass - Inertia	0.25250	10
4.	D: Center of Gravity*	−5.54375	4
5.	E: Tire Road Frictional Drag*	54.44125	1
6.	F: Front Wheel Steer Angle	0.08000	14
7.	G: Aerodynamic Drag*	6.63125	3
8.	H: Initial conditions*	−40.23500	2
9.	AB = CE = DF = GH Interaction	0.35000	9
10.	AC = BE = DG = FH Interaction	0.05625	15
11.	AD = BF = CG = EH Interaction*	−0.96250	5
12.	AE = BC = DH=FG Interaction	−0.11750	13
13.	AF = BD = CH = EG Interaction	−0.50125	8
14.	AG = BH = CD = EF Interaction	−0.12750	12
15.	AH = BG = CF = DE Interaction	−0.74625	6

Tire Friction and Initial Conditions, and Tire Friction and Aerodynamic Drag. Because these interactions are implied from the significant main effects, they should be confirmed with further simulations.

For the Angular Orientation (θ) the significant main effects are CG Height, Tire-roadway Frictional Drag, Aerodynamic Drag and the Initial Conditions. The analysis also suggests an interaction between the Tire Friction and Initial Conditions. Note that these are the exact same significant factors as for the X Position, as well as a subset of the significant factors for the Y Position.

Summary of Results for DOE 1 In this DOE, eight different factors were investigated using a Fractional Factorial design. These 8 factors were studied using 16 trials, by varying their factor levels between low and high levels. Statistical analysis of the simulation data has shown that the response variables of interest, i.e. the postimpact positions of X, Y, and θ, are correlated among one another. This was confirmed first with the calculation of the correlation matrix, and then with the DOE results, where all three response variables were identified as being significantly dependent on the same factors of Mass-Inertia, Center of Gravity, Tire Road Friction Drag and Aerodynamic Drag.

For the Y and θ response variables, the ranks of the significant factors are identical. In addition, the signs on the coefficients are also identical, meaning if the variable factor level is increased (or decreased) the change in the Y and θ values will increase (or decrease) in the same direction, i.e. both will increase or both will decrease. This may present an issue when trying to optimize or "tune" the simulation to match field observations, as it appears that factors that influence both Y and θ will compete amongst one another.

With regards to the significant factors for the X position, even though the significant factors are the same as those for Y and θ, the ranks of these factors are slightly different. This allows for some flexibility when trying to "tune" the model to any field results for the X position. However, caution must still be made since the signs associated with the significant factors are the exact opposite as those for Y and θ. This means that if the simulation is tuned to increase the X position, the Y and θ positions will move in the opposite direction. Reconstructionists who have used vehicle dynamics simulation in have invariably encountered this trade-off in attempting to achieve the desired rest position of a vehicle only to have the success for one position coordinate foiled in attempts to achieve success in the other coordinates.

The linear models (in coded units) for the X, Y, and θ positions based on the significant factors are:

$$X = 36.2425 + 0.77625 \cdot x_{CG} - 1.8600 \cdot x_f - 0.38750 \cdot x_{Aero} + 1.745 \cdot x_{IC} + 0.20500 \cdot x_f \cdot x_{IC}$$

$$Y = -89.51235 + 0.10750 \cdot x_{CG} + 6.39375 \cdot x_f + 1.11875 \cdot x_{Aero} - 4.34875 \cdot x_{IC} - 0.16625 \cdot x_f \cdot x_{Aero} + 0.27875 \cdot x_f \cdot x_{IC} + 0.09875 \cdot x_{Aero} \cdot x_{IC}$$

$$\theta = -885.8165 - 5.5438 \cdot x_{CG} + 54.4412 \cdot x_f + 6.6312 \cdot x_{Aero} - 40.2350 \cdot x_{IC} - 0.9625 \cdot x_f \cdot x_{IC}$$

DOE with Consideration for Yaw Inertia and Lateral Position of Center of Gravity (DOE 2)

This DOE is an extension of DOE 1 as a means to introduce three new factors:

1. Mass
2. Yaw Inertia
3. Longitudinal Position of the Center of Gravity

As noted in DOE 1, the levels of mass and inertia were coupled together. In DOE 2, a specific consideration was made for mass and inertia to be treated as separate factors.

With regards to the longitudinal position of the CG, this value is frequently taken from databases [21] and is a variable that generally is used with no uncertainty; however, within the context of this paper it was an opportunity to investigate it as part of the sensitivity analysis.

In order to accommodate the addition of the new variables just listed, the factors of Aerodynamic Drag and Initial Conditions were removed from the original factor list. As a result, the new list of factors (with their nominal and limit values) is as follows:

A. Wheel lateral stiffness, C_α: front 13,000 lb/rad, rear 11,000 lb/rad (varied ±5%)
B. Wheel longitudinal stiffness, C_s: 10,000 lb/unit slip (varied ±5%)
C. Mass, m: 3186 lb (varied ±5%)
D. CG height, h_{CG}: 1.76 ft (varied ±3% based on data in [19])
E. Tire-road frictional drag, f: $f = 0.7$ (varied ±7%, $0.65 \leq f \leq 0.75$)
F. Front wheel steer angle, δ: $\delta = 0°$ (varied ±5°)
G. Yaw Inertia, I: $I = 2000$ ft-lb-s^2 (varied ±5%, both in same direction)
H. Longitudinal position of the center of gravity: CG: Initial position is 59%/41% per [21] (varied by ±5% for 64%/36%, and 54%/46%)

The initial conditions for all of the runs were fixed at: $V_x = 18.17$ ft/s, $V_y = -59.04$ ft/s, $\Omega = -436.43°/s$.

The sixteen simulations were run with the appropriate high/low values as presented in Table 3. The results of the simulations for the X, Y, and θ are given in Table 10.

DOE Analysis The analysis of the DOE was the same as presented for DOE 1. First a correlation analysis was made using the 3 different response variables.

These correlations are confirmed in Figure 6, which makes a pairwise comparison between the results of the three response variables. Note the "clusters" that exist within each plot are likely due to dependencies among the different factors.

Table 11 and Figure 6 show the dependencies among the three response variables to be considerably stronger than with the initial DOE. Again, preventing the use of response

TABLE 10 Summary of postimpact position components for all 16 trials of DOE 2.

Run	X	Y	θ
1	37.61	−97.33	−936.39
2	34.91	−85.12	−862.20
3	33.97	−84.22	−840.45
4	38.18	−98.61	−994.05
5	33.97	−84.24	−840.55
6	37.09	−96.14	−921.43
7	37.61	−97.35	−936.32
8	32.87	−83.05	−788.59
9	39.85	−98.37	−1001.73
10	35.65	−83.92	−854.02
11	36.66	−84.96	−875.54
12	39.08	−97.10	−944.43
13	34.40	−82.84	−799.58
14	39.06	−97.08	−944.29
15	38.33	−95.95	−931.34
16	35.64	−83.97	−854.09
CP	36.86	−90.05	−891.34

FIGURE 6 Pairwise Comparison in results of the 3 response variables - DOE 2.

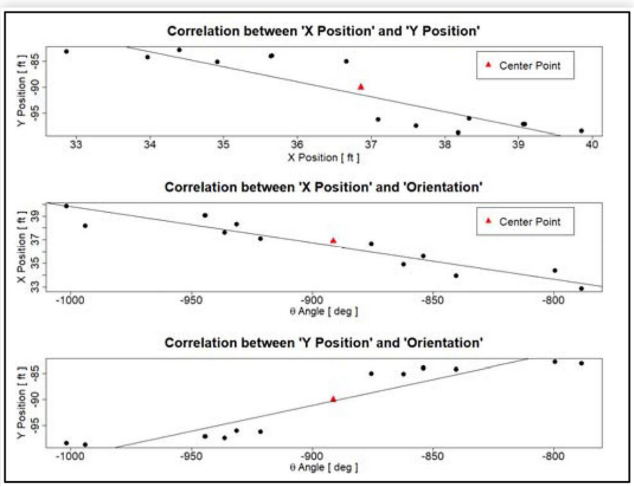


TABLE 11 Pearson Product-Moment Correlation Matrix for the response variables of the simulations.

	X	Y	θ
X	1.0000	−0.8932	−0.9364
Y	−0.8932	1.0000	0.9360
θ	−0.9364	0.9360	1.0000

functions that would combine the three response variables algebraically into a single variable, such as given in Equation 3.

The second part of the analysis was specific to the analysis of the factors. Main Effects and Normal Probability Plots of the Effects were produced to gain insight as to the significant factors. Figure 7 shows the Normal Probability Plots for the effects are related to each of the three response variables.

FIGURE 7 Normal Probability Plot of factor effects for the three response variables - DOE 2.

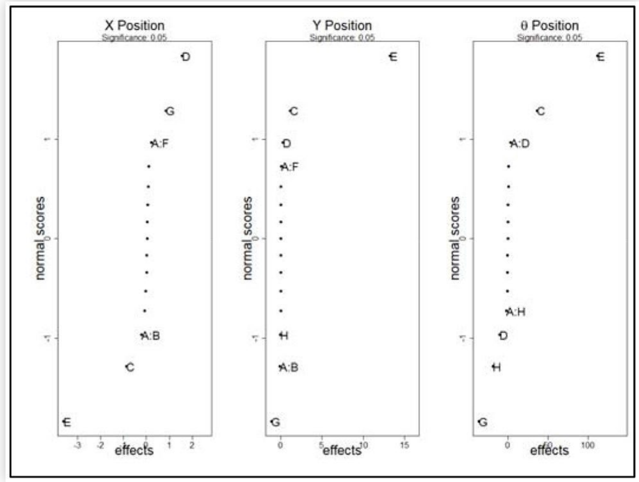


TABLE 12 Summary of model coefficients, in Coded Units, for the postimpact position of vehicle for DOE 2 - only the coefficients for the significant factors are given (α = 0.05).

	Factor	Coefficients (X)	Coefficients (Y)	Coefficients (θ)
	Main Effects			
1.	A: Wheel Lateral Stiffness	-	-	-
2.	B: Wheel Longitudinal Stiffness	-	-	-
3.	C: Mass	−0.43375	0.56313	18.2887
4.	D: CG Height	0.77875	0.11688	−5.3150
5.	E: Tire Road Friction Drag	−1.79625	6.60063	55.9350
6.	F: Front Wheel Steer Angle	-	-	-
7.	G: Yaw Inertia	0.43000	−0.57187	−18.2838
8.	H: Longitudinal Position of CG	-	-	−9.3950
	2-Factor Interactions			
9.	AB=CE = DF = GH	−0.10500	-	-
10.	AC=BE = DG = FH	-	-	-
11.	AD = BF=CG = EH	-	-	−1.4950
12.	AE = BC = DH=FG	-	-	-
13.	AF=BD = CH = EG	0.10625	-	-
14.	AG = BH=CD = EF	-	-	-
15.	AH=BG = CF = DE	-	-	−1.1150

The above plots show Main Effects as well as some 2-Factor Interaction terms to be statistically significant (α=0.05). The following table summarizes the relevant and significant factors for the models for the X, Y, and θ positions in coded units for the reduced models.

The results of Table 12 summarize the significant main effects and 2-factor interaction terms that would be considered significant in the linear models. Just as with the first DOE analysis, the 2-factor interaction terms should be reviewed to consider which combination would most likely be most significant based on which main effects are supporting the model. For the X position, the model shows that there would be a pair of 2-factor interactions. Based on the significant main effects it would be proposed that the interaction terms are the Mass-Inertia and the Friction Drag-Yaw Inertia.

For θ, the suggested 2-factor interactions will not be easy to determine. For example, the first 2-factor interaction may be interpreted as a Mass-Yaw Inertia interaction OR as a Tire Friction Drag-CG Longitudinal Position interaction. The second interaction term for the θ position is between the CG Height and the CG Longitudinal Position.

The Y position did not identify a significant 2-factor interaction in the reduced model.

Summary of Results for DOE 2 for Yaw Inertia and Longitudinal Position of CG Even though several factors are identified as being significant for each of the response variables, it can be seen by the magnitude of the absolute values of the coefficients that some factors will '*dominate*' over the others. For example, all three responses are most sensitive to the Tire Road Friction Drag.

The Mass, CG Height, Tire-roadway Frictional Drag, and Yaw Inertia proved to be statistically significant to all three responses. While the Longitudinal CG Position only had an impact on the angular orientation.

DOE with Lateral Stiffness Imbalance (DOE 3)

Recall in DOE 1, the front and rear wheel lateral stiffness, $C_{\alpha f}$ and $C_{\alpha r}$ were assigned different values, 13,000 lb/rad for the front and 11,000 lb/rad for the rear. However, together they were treated as a single factor. That is, the front and rear wheel lateral stiffnesses varied together and in the same proportion to ensure the same balance across low and high levels of Factor A in Table 3. In this DOE, the value of the lateral stiffness of the front wheels was varied from the nominal value by ±5%. As a result, another factor was introduced, C_{bal}, that specified the balance of front to rear wheel lateral stiffness.

Therefore, another eight factors, their nominal values and the variations were selected. Some of the factors and the ranges were the same as in DOE 1 and DOE 2. These were:

1. Wheel lateral stiffness front, $C_{\alpha f}$: 13,000 lb/rad (varied ±5%)
2. Wheel longitudinal stiffness, C_s: 10,000 lb/unit slip (varied ±5%)
3. Mass, m: 3186 lb (varied ±5%)
4. CG height, h_{CG}: 1.76 ft (varied ±3% based on data in [19])
5. Tire-road frictional drag, f: $f = 0.7$ (varied ±7%, $0.65 \leq f \leq 0.75$)
6. Front wheel steer angle, δ: $\delta = 0°$ (varied ±5°)
7. Yaw Inertia, I: $I = 2000$ ft-lb-s^2 (varied ±5%, both in same direction)
8. Balance of front to rear wheel lateral stiffness coefficients, C_{bal}: The nominal value is 13,000/11,000 = 1.18 (varied by ±5%, $1.12 \leq C_{bal} \leq 1.24$) the value of $C_{\alpha r}$ was selected to achieve the required balance.

TABLE 13 Summary of postimpact position components for all 16 trials of DOE 3.

Run	X	Y	θ
1	0.82	−19.89	−310.41
2	0.90	−17.82	−294.98
3	0.76	−17.36	−284.54
4	0.99	−20.52	−322.90
5	0.75	−17.36	−284.50
6	0.65	−19.38	−298.60
7	0.82	−19.89	−310.41
8	0.69	−16.94	−275.20
9	1.38	−20.66	−325.25
10	1.13	−17.45	−286.18
11	1.31	−17.96	−297.53
12	1.19	−19.99	−312.19
13	1.03	−17.05	−276.80
14	1.18	−19.99	−312.14
15	0.99	−19.47	−300.21
16	1.15	−17.49	−286.88
CP	0.97	−18.56	−297.17

The initial conditions for all of the runs were again fixed at: $V_x = 18.17$ ft/s, $V_y = -59.04$ ft/s, $\Omega = -436.43°$/s.

The structure for this DOE is the same as the previous ones. The sixteen simulations were run with the appropriate high/low values as presented in Table 3. The results of the simulations for the X, Y, and θ are given in Table 13.

DOE Analysis As with the previous DOE analyses, the first check is with regards to the correlations among the response variables of X, Y, and θ. Table 14 shows the Pearson Product-Moment Correlations. This time the strength of the correlations between X-Y and X-θ have decreased considerably, while the correlation between Y-θ remains high at 0.9622.

Figure 8 also shows large amounts of scatter in the plots for X versus Y and X versus θ, while the plot of Y versus θ show points that exhibit behavior closer to linear than the other two combinations.

Identification of the significant parameters for this model are confirmed in Figure 9 - the Daniel Plots for the three response variables.

Figure 9 shows that the X position may only be dependent on a few main effects and no 2-factor interactions, while the Y position and the angular orientation (θ) show both significant main effects and possible 2-factor interactions. As with

TABLE 14 Pearson Product-Moment Correlation Matrix for the response variables of the simulations - DOE 3.

	X	Y	θ
X	1.0000	−0.2595	−0.3943
Y	−0.2595	1.0000	0.9622
θ	−0.3943	0.9622	1.0000

FIGURE 8 Pairwise comparison in results of the 3 response variables - DOE 3.

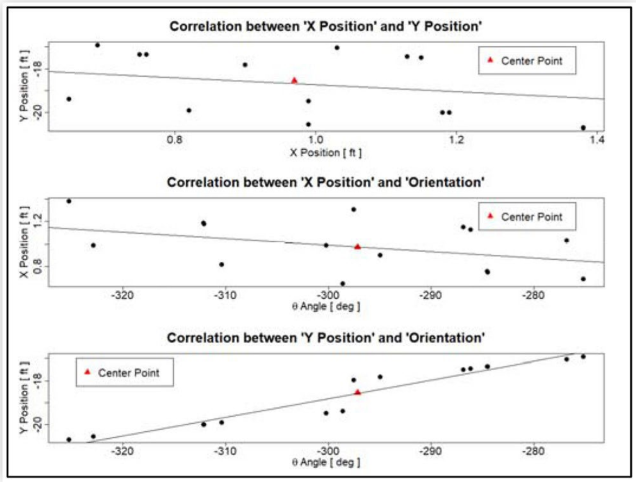
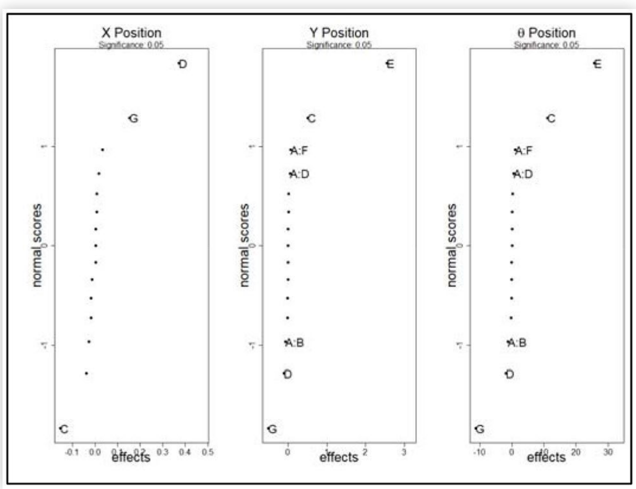


FIGURE 9 Normal Probability Plot of factor effects for the three response variables - DOE 3.



the previous DOEs, it would appear that the three response variables are all dependent on the same subsets of factors. All three are dependent on the Mass, CG Height and the Yaw Inertia. Response variables of Y and θ also share the Tire Road Friction and the same 2-factor interactions.

For each of these responses, a separate, reduced model was develop using only the suggested significant parameters. The results of the three response models are summarized in the Table 15.

Summary of Results for DOE 3 This DOE demonstrated a lower strength in the correlation between the X and Y and X and θ response variables, but maintained a very strong correlation between Y and θ.

The response variables shared many of the same significant factors, while the X position appears dependent only on single factor main effects and Y and θ include common 2-factor interactions.

TABLE 15 Summary of model coefficients, in Coded Units, for the postimpact position of vehicle for DOE 3 - only the coefficients for the significant factors are given (α = 0.05).

	Factor	Coefficients (X)	Coefficients (Y)	Coefficients (θ)
	Main Effects			
1.	A: Wheel Lateral Stiffness	-	-	-
2.	B: Wheel Longitudinal Stiffness	-	-	-
3.	C: Mass	−0.07625	0.25500	5.5775
4.	D: CG Height	0.18625	−0.05625	−0.9775
5.	E: Tire Road Friction Drag	-	1.27250	12.8438
6.	F: Front Wheel Steer Angle	-	-	-
7.	G: Yaw Inertia	0.07625	−0.26000	−5.6537
8.	H: Longitudinal Position of CG	-	-	-
	2-Factor Interactions			
9.	AB=CE = DF = GH	-	−0.03625	−0.5963
10.	AC=BE = DG = FH	-	-	-
11.	AD = BF=CG = EH	-	0.02375	−0.2637
12.	AE = BC = DH=FG	-	-	-
13.	AF=BD = CH = EG	-	0.03125	0.5075
14.	AG = BH=CD = EF	-	-	-
15.	AH=BG = CF = DE	-	-	-

DOE with Lateral Stiffness Imbalance and Change in in Vehicle Orientation (DOE 4)

The DOE analyses presented thus far have all dealt with the crash depicted in Figure 1. This is a crash with a closing speed of 50 mph (73.3 ft/s, 80.6 kph). In an effort to understand the generality of the results of the DOE analyses presented thus far, the analysis method is applied to a crash with a lower closing speed and a different impact orientation. Figure 9 shows the impact orientation of the vehicles for DOE 4.

The impact analysis produced the initial conditions shown in Table 16 for the Honda Accord to be used in the DOE analysis. The output sheet of the PIM analysis is included in Appendix B.

TABLE 16 Summary of the results of the impact analysis for use as the initial condition for the Honda Accord in the DOE 4 study.

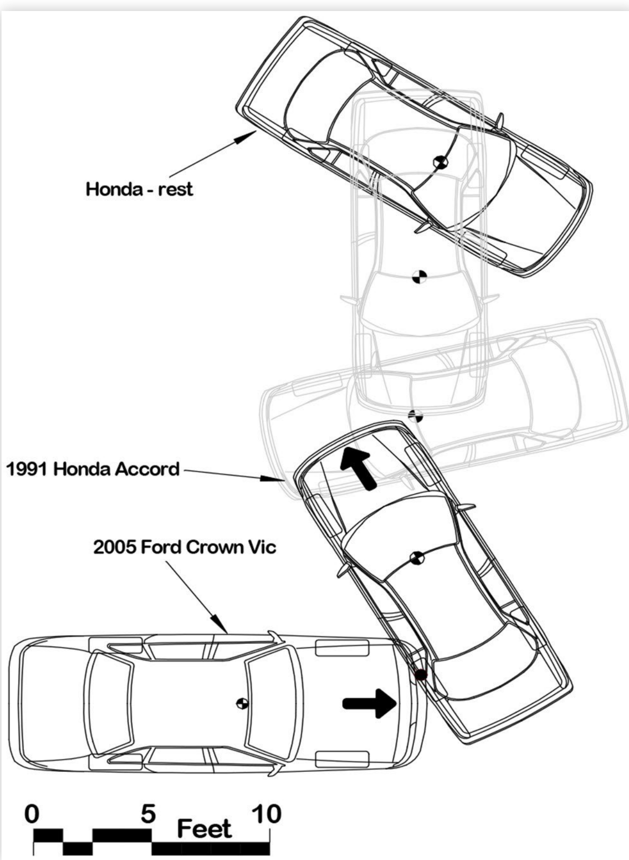
Preimpact speed of both vehicles	25 mph (40.2 kph)
Postimpact velocity in x-direction (Accord)	−1.20 ft/s (0.37 m/s)
Postimpact velocity in the y-direction (Accord)	−24.88 ft/s (7.58 m/s)
Postimpact angular velocity (Accord)	−264.08 deg/sec

The initial kinetic energy of the Honda Accord for this crash is 20% of the initial (postimpact) kinetic energy of the previous crash analyzed in DOE 1 through DOE 3.

The same eight factors were used in the DOE analysis for this crash that were used in DOE 3. These eight factors, their nominal values and the variations were:

1. Wheel lateral stiffness front, $C_{\alpha f}$: 13,000 lb/rad (varied ±5%)
2. Wheel longitudinal stiffness, C_s: 10,000 lb/unit slip (varied ±5%)
3. Mass, m: 3186 lb (varied ±5%)
4. CG height, h_{CG}: 1.76 ft (varied ±3% based on data in [19])
5. Tire-road frictional drag, f: $f = 0.7$ (varied ±7%, $0.65 \leq f \leq 0.75$)
6. Front wheel steer angle, δ: $\delta = 0°$ (varied ±5°)
7. Yaw Inertia, I: $I = 2000$ ft-lb-s^2 (varied ±5%, both in same direction)
8. Balance of front to rear wheel lateral stiffness coefficients, C_{bal}: The nominal value is 13,000/11,000 = 1.18 (varied by ±5%, $1.12 \leq C_{bal} \leq 1.24$) the value of $C_{\alpha r}$ was selected to achieve the required balance.

FIGURE 10 Impact geometry for the crash used to generate the initial conditions for the DOE 4 analysis involving the Honda Accord. (The location of the impact center is indicated by the dot. Arrows indicate direction of preimpact velocity.)



The initial conditions for all of the runs were again fixed at: $V_x = -1.20$ ft/s, $V_y = -24.88$ ft/s, $\Omega = -264.08°$/s. The sixteen simulations were run with the appropriate high/low values for the eight factors.

The structure for this DOE is the same as the previous ones. The sixteen simulations were run with the appropriate high/low values as presented in Table 3. The results of the simulations for the X, Y, and θ are given in Table 17.

DOE Analysis Unlike in DOE 3, the strength of correlation among all pairs of the response variables of X, Y, and θ appear to be significant. Table 18 shows the Pearson Product-Moment Correlations and shows the minimum magnitude for a correlation to be as low as 0.8970.

Identification of the significant parameters for this model are confirmed in Figure 9 - the Daniel Plots for the different response variables.

The identification of significant model parameters appears to be similar to DOE 3. Figure 11 shows that all three response variables depend on factors of Mass, CG Height, Tire Road Friction and the Yaw Inertia. The Y position includes two 2-factor interactions - Mass-Tire Roadway Frictional Drag and Tire Road Friction-Yaw Inertia. The θ orientation includes just one interaction term between CG Height and Tire Road Frictional Drag.

For each of these response, a separate, reduced model was develop using only the significant parameters. The results of the three response models are summarized in the Table 19.

TABLE 17 Summary of postimpact position components for all 16 trials of DOE 4.

Run	X	Y	θ
1	0.82	−19.89	−310.41
2	0.90	−17.82	−294.98
3	0.76	−17.36	−284.54
4	0.99	−20.52	−322.90
5	0.75	−17.36	−284.50
6	0.65	−19.38	−298.60
7	0.82	−19.89	−310.41
8	0.69	−16.94	−275.20
9	1.38	−20.66	−325.25
10	1.13	−17.45	−286.18
11	1.31	−17.96	−297.53
12	1.19	−19.99	−312.19
13	1.03	−17.05	−296.80
14	1.18	−19.99	−312.14
15	0.99	−19.47	−300.21
16	1.15	−17.49	−286.88
CP	0.97	−18.56	−297.17

TABLE 18 Pearson Product-Moment Correlation Matrix for the response variables of the simulations - DOE 4

	X	Y	θ
X	1.0000	−0.8970	−0.9435
Y	−0.8970	1.0000	0.9438
θ	−0.9435	0.9438	1.0000

FIGURE 11 Normal Probability Plot of factor effects for the three response variables - DOE 4.

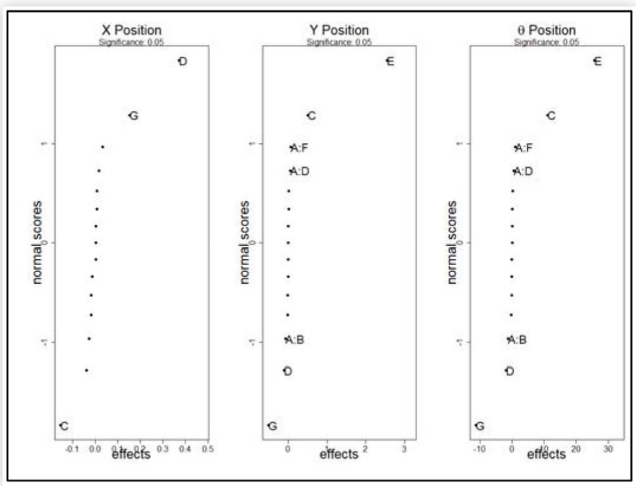


TABLE 19 Summary of model coefficients, in Coded Units, for the postimpact position of vehicle for DOE 3 - only the coefficients for the significant factors are given (α = 0.05).

	Factor	Coefficients (X)	Coefficients (Y)	Coefficients (θ)
	Main Effects			
1.	A: Wheel Lateral Stiffness	-	-	-
2.	B: Wheel Longitudinal Stiffness	-	-	-
3.	C: Mass	−0.42125	0.58875	18.9119
4.	D: CG Height	0.77625	0.10750	−5.3994
5.	E: Tire Road Frictional Drag	−1.81375	6.6125	56.0094
6.	F: Front Wheel Steer Angle	-	-	-
7.	G: Yaw Inertia	0.41750	−0.59375	−18.9019
8.	H: Longitudinal Position of CG	-	-	-
	2-Factor Interactions			
9.	AB=CE = DF = GH	-	−0.05000	-
10.	AC=BE = DG = FH	-	-	-
11.	AD = BF=CG = EH	-	-	-
12.	AE = BC = DH=FG	-	-	-
13.	AF=BD = CH = EG	-	0.04500	-
14.	AG = BH=CD = EF	-	-	-
15.	AH=BG = CF = DE	-	-	−1.0481

Summary of Results for DOE 4 - Same as DOE 3 with Exception in Initial Conditions and Orientation When compared to DOE 3, the revised conditions used in DOE 4 allowed the significant correlations between all pairwise comparisons to return. Also, the different impact orientation and closing speed lead to more significant main effects, but fewer 2-factor interactions. The main effects were common among all three response variables.

Summary/Conclusions

Several different DOEs were used to evaluate the effect of a variety of input parameters on the different response variables that are commonly used in programs that simulate the postimpact motion of vehicles. Among the different factors considered, the common significant factors to appear in all simulation experiments are tire-roadway frictional drag, *f*, the CG height, h_{CG}, and vehicle mass, m. Other important factors include the aerodynamic drag, initial velocities, longitudinal position of the CG and the yaw inertia (when included as factors). In addition, all models identified multiple two-factor interactions that were significant for the different response variables.

The analysis shows that the three coordinates that describe the position and orientation of the vehicle at rest, (X, Y, θ), which are used as the response variables for the experimental designs presented in this paper, show correlation. This correlation explains the trade-offs experienced by reconstructionists when analyzing the postimpact motion of a vehicle and trying to match the (known) rest position components. In these circumstances, changes made to one factor by the analyst when iterating to achieve a match in the *x*-coordinate of the rest position, might create the desired result, but simultaneously cause the *y*-coordinate, and/or the θ-coordinate, to move further from the desired result. Knowledge of the dependencies between the model predictors is helpful in understanding these trade-offs.

The identification of the various significant factors, significant 2-factor interactions, and an analytical basis for recognition of the dependencies of the factors are the most important results of the work presented here. Identification of the tire-roadway frictional drag is consistent with the expectations of the authors. The influence of this factor has been previously identified [8]. But the nature of the DOE analyses further identifies other factors, and factor interactions, as significant that are perhaps not as intuitive. Included in this category are aerodynamic drag, yaw inertia and CG height. Therefore, reconstructionists may need to consider including factors possibly not previously included in postimpact analyses (e.g. aero drag) or incorporating uncertainties in parameters previously assigned a fixed value (e.g. yaw inertia and CG height).

An explanation of the correlation of the various factors, such as the CG height, with the travel distance of the vehicle is not provided by the results of the DOE analysis. Correlation was not anticipated by the authors. An explanation would be a useful result. However, the differential equations that govern the motion are extremely nonlinear. Significant effort would be needed to explore the behavior of these differential equations with regard to even a single parameter such as the CG height. This topic is left for future research.

Reconstructionists generally use vehicle dynamics simulation programs iteratively in a "reverse" manner whereby the rest position of the vehicle is known and the initial translational and rotational speeds of the vehicle are desired.

This paper analyzed the sensitivity of a simulation program using DOE with the initial conditions known and the rest position being calculated. Either approach could have been used. However, with the goal of assessing the sensitivity rather than the accuracy, the authors selected the latter approach due to expediency as no iteration or subjectivity was needed in assessing the accuracy of the reconstruction for each set of conditions. Whether the two approaches yield the same sensitivities to the parameters is left for future research.

Another important observation is the recognition of the correlation that exists between the three coordinates that describe the rest position and orientation of the vehicle that were used as the response variables. These correlations prevented the use of a response function rather than a response variable in the analysis of the postimpact motion.

The analysis presented here needs to be expanded to include other vehicle dynamics simulation programs to determine whether the same factors and factor interactions are significant. Other simulation programs that include additional parameters than those analyzed here provide opportunities for these parameters, and interactions with these parameters to be analyzed. For example, the simulation program used in the analyses here does not include a vehicle suspension. Thus, this analysis cannot determine whether factors associated with the suspension may be significant.

Part of the motivation for the analysis performed in this paper was prior analysis that looked at the influence of the steer angle in the postimpact motion of a light vehicle [1]. The analysis here shows that this factor has little effect relative to the other factors considered here. It should be notes that in all the simulations conducted here the wheels were locked. For conditions in which some or all of the wheels can rotate and generate lateral forces, this result may be different. Additionally, the analysis here considered angles at the front wheels of ±5° whereas angles in [1] used angles at the front wheels as large as 18°.

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Appendix A

Results of the planar impact mechanics analysis for the impact between the 1991 Honda Accord and the 2005 Ford Crown Victoria used for DOE 1 through DOE 3.

Analysis of a Planar Vehicle Collision

impact.xls
ver 3.3
9/18/2016

			HONDA			FORD
			Vehicle 1			Vehicle 2
5280/3600	1.467		99.01	mass, m	lb-s²/ft	126.10
g	32.174	ft/s²	2000.00	inertia, I	ft-lb-s²	2970.59
			6.89	distance, d	ft	7.55
e	0.100		156.89	angle, ϕ	deg	0.00
μ (% μ₀)	100.0		-90.00	angle, θ	deg	0.00
μ	-0.789					
μ_0	-0.789			INITIAL Velocity		
Γ	0.000	deg		v	kph	
			0.00	v_x	ft/s	73.363
mbar	55.464		73.363	v_y	ft/s	0.00
k_1^2	20.199	ft^2	0.00	ω	deg/sec	0.00
k_2^2	23.557	ft^2	73.36	v	ft/s	73.36
q	0.402		0.00	v_n	ft/s	73.36
v_{rn}	73.363	ft/s	73.36	v_t	ft/s	0.00
v_{tn}	-73.363	ft/s	0.00	v_{cn}	ft/s	73.36
r	-1.000		73.36	v_{ct}	ft/s	0.00
η_1	-90.000	deg				
η_2	0.000	deg		FINAL Velocity		
d_a	0.000	ft	Vehicle 1			Vehicle 2
d_b	7.546	ft	18.17	V_x	ft/s	59.10
d_c	6.337	ft	59.04	V_y	ft/s	11.25
d_d	2.704	ft	436.43	Ω	deg/sec	206.45
d_e	8.469	ft	61.77	V	ft/s	60.16
d_f	5.951	ft	18.17	V_n	ft/s	59.10
A	2.114		59.04	V_t	ft/s	11.25
B	0.475		66.44	V_{cn}	ft/s	59.10
C	1.266		38.44	V_{ct}	ft/s	38.44
D	2.266					

	Initial speeds	
Vehicle 1		Vehicle 2
50.0	mph	50.0
	Final speeds	
Vehicle 1		Vehicle 2
42.1	mph	41.0
	|ΔV|	
Vehicle 1		Vehicle 2
15.8	mph	12.4
23.1	ft/s	18.2

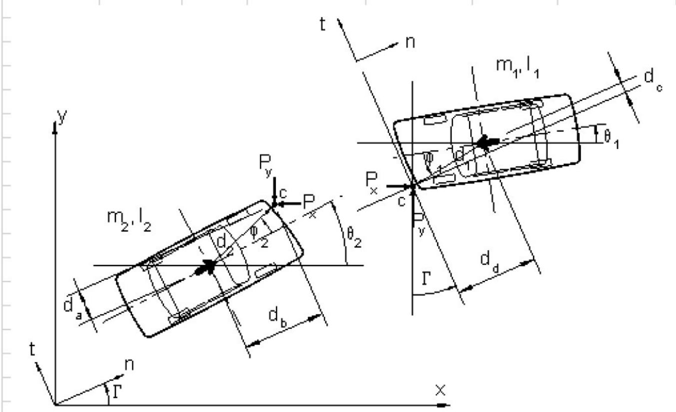
System Kinetic Energy, ft-lb		
Initial	605,799.9	
Final	494,385.3	
Loss	111,414.6	18.4%

Normal (Crush) Energy Loss:
59,382.3 9.8%
Tangential Energy Loss:
52,032.3 8.6%
Total System Energy Loss
111,414.6 18.4%

	Impulses, lb-s	
P_x	P_y	P
1798.7	-1418.5	2290.7
P_n	P_t	P
1798.7	-1418.5	2290.7

	PDOF, deg	
Vehicle 1		Vehicle 2
-51.7		38.3

	|ΔV| Components, mph	
Vehicle 1		Vehicle 2
12.39	$\|\Delta V_{lat}\|$	7.67
9.77	$\|\Delta V_{long}\|$	9.73



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Appendix B

Results of the planar impact mechanics analysis for the impact between the 1991 Honda Accord and the 2005 Ford Crown Victoria used for DOE 4.

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Analysis of a Planar Vehicle Collision

Un
Conve

			HONDA Vehicle 1			FORD Vehicle 2
5280/3600	1.467		99.01	mass, m	lb-s²/ft	126.10
g	32.174	ft/s²	2000.00	inertia, I	ft-lb-s²	2970.59
			5.50	distance, d	ft	7.70
e	0.100		151.64	angle, ϕ	deg	10.17
μ (% μ_0)	100.0		-60.00	angle, θ	deg	0.00
μ	-0.750					
μ_0	-0.750			INITIAL Velocity		
Γ	15.000	deg	25.00	v	kph	25.00
			-18.33	v_x	ft/s	36.67
mbar	55.464		31.754	v_y	ft/s	0.000
k_1^2	20.199	ft^2	0.00	ω	deg/sec	0.00
k_2^2	23.557	ft^2	36.67	v	ft/s	36.67
q	0.534		-9.49	v_n	ft/s	35.42
v_{rn}	44.907	ft/s	35.42	v_t	ft/s	-9.49
v_{tn}	-44.907	ft/s	-9.49	v_{cn}	ft/s	35.42
r	-1.000		35.42	v_{ct}	ft/s	-9.49
η_1	-75.000	deg				
η_2	-15.000	deg		FINAL Velocity		
d_a	-0.648	ft	Vehicle 1			Vehicle 2
d_b	7.673	ft	-1.20	V_x	ft/s	23.22
d_c	5.351	ft	24.88	V_y	ft/s	5.40
d_d	1.271	ft	264.08	Ω	deg/sec	143.98
d_e	6.304	ft	24.91	V	ft/s	23.83
d_f	5.105	ft	5.28	V_n	ft/s	23.82
A	1.802		24.34	V_t	ft/s	-0.79
B	0.096		29.94	V_{cn}	ft/s	25.45
C	1.144		18.49	V_{ct}	ft/s	18.49
D	2.144					

Vehicle 1	Initial speeds	Vehicle 2
25.0	mph	25.0
	Final speeds	
Vehicle 1		Vehicle 2
17.0	mph	16.3
	|ΔV|	
Vehicle 1		Vehicle 2
12.6	mph	9.9
18.5	ft/s	14.5

System Kinetic Energy, ft-lb

Initial	151,326.6	
Final	97,157.8	
Loss	54,168.8	35.8%

Normal (Crush) Energy Loss:	
29,549.0	19.5%
Tangential Energy Loss:	
24,619.7	16.3%
Total System Energy Loss	
54,168.8	35.8%

Impulses, lb-s

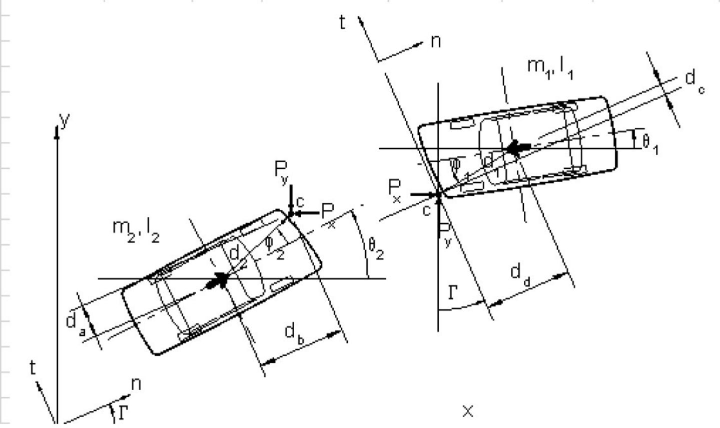
P_x	P_y	P
1696.2	-680.7	1827.7
P_n	P_t	P
1462.2	-1096.5	1827.7

PDOF, deg

Vehicle 1		Vehicle 2
-38.1		21.9

|ΔV| Components, mph

Vehicle 1		Vehicle 2
7.77	$\|\Delta V_{lat}\|$	3.68
9.90	$\|\Delta V_{long}\|$	9.17



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Appendix C

Considerations of System Model Response

In the general application of vehicle dynamics simulation, the response is the physical motion of the vehicle. For the purposes conducting a sensitivity analysis of a simulation program using DOE, the main effects (*ME*) need to be calculated using Equation C1. The DOE construct requires that a singular number be assigned as the response (the y_i's in Equation C1) of the system for a given run. In experimental applications, the response of the system satisfies this requirement in that it is typically an experimentally measured quantity, e.g. temperature, time to failure, process yield, current, voltage, etc.

$$ME_{x_j} = \frac{1}{2^{k-p-1}}\left[\sum_{i=1}^{2^{k-p}} \pm y_i\right] \quad \text{(C1)}$$

For vehicle dynamics simulation, the selection of a response variable is complicated by two issues: 1) the time-based output is not easily distilled into a singular number amenable to the calculation of main effects using Equation 1, although as will be shown shortly, options do exist to satisfy this need, and 2) the ultimate selection of a singular number characterizing the simulated vehicle motion does not provide the significance needed for the sensitivity analysis; a comparison and/or normalization of some sort is needed to facilitate the determination of the sensitivity of the process. The need for this characteristic of the response makes sense in that the assessment of sensitivity is a comparative operation. In accommodating these two issues it may appear that instead of designating a singular response **variable** that is a direct output of the simulation process, a response **function** will be needed. Some discussion regarding this notion of a response function and the application (or misapplication) to the problem studies here follows.

In addressing the first issue, various options were identified for ascribing a singular number to characterize the postimpact motion of the vehicle. Two examples identified by the authors are 1) the distance of travel of the CG of the vehicle from separation to rest and 2) the time duration of the vehicle from separation to rest. (Other options certainly exist.) Evaluating these two options, the postimpact distance of motion was selected as it relates directly to the crash reconstruction task. Generally, the reconstructionist would use vehicle dynamics simulation to model the (postimpact) motion of the vehicle until a match is achieved with the physical evidence. That physical evidence is typically the rest position and orientation of the vehicle(s) but might also include positions of the vehicle between separation and rest. The duration of time of the postimpact motion would generally not be a quantity available to the reconstructionist (unless perhaps the entirety of the motion was captured on video). Thus, selection of the time of motion as the response variable, while satisfying the criteria 1) above, has practical limitations.

A second issue is that the rest position of the vehicle is comprised of three components, the x and y coordinates of the CG and the vehicle heading, θ. This issue might be mitigated by treating the three coordinates as belonging to three space. Thus the "distance" traveled by the vehicle can be defined as given in Equation C1:

$$D = \sqrt{(x_i)^2 + (y_i)^2 + (\theta_i)^2} \quad \text{(C1)}$$

In this equation, x, y, and θ are associated with the rest position of the vehicle and the i subscript is associated with the i^{th} run of the simulation. Here the distance computed is relative to the origin of the coordinate system. Note that the three coordinates (x, y, θ) do not form an orthogonal system. Additionally, the units of the three variables are not consistent.

Addressing the second issue, that a comparison is needed for the sensitivity analysis, can be solved by using the nominal vehicle motion as a value for comparison. The nominal value is the motion of the vehicle for a given set of initial conditions with all the factors at their nominal values (as opposed to either the high or the low values associated with the DOE). Under this construct, Equation C1 can be changed to the following form:

$$R_1 = \sqrt{(x_i - x_n)^2 + (y_i - y_n)^2 + (\theta_i - \theta_n)^2} \quad \text{(C2)}$$

In this form of the equation, the n subscript is associated with the nominal run of the simulation. While Equation C2 resembles the distance formula in three space, here the three variables do not form an orthogonal system, so the equation can be more generally viewed as providing a magnitude rather than a physical distance.

Equation C2 is only one form that a response function using the magnitude approach might take. Consider that the response function might also take the following forms:

$$R_2 = \sqrt{\left(1 - \frac{x_i}{x_n}\right)^2 + \left(1 - \frac{y_i}{y_n}\right)^2 + \left(1 - \frac{\theta_i}{\theta_n}\right)^2} \quad \text{(C3)}$$

$$R_3 = \frac{\sqrt{(x_i)^2 + (y_i)^2 + (\theta_i)^2}}{\sqrt{(x_n)^2 + (y_n)^2 + (\theta_n)^2}} \quad \text{(C4)}$$

$$R_4 = \sqrt{(x_i)^2 + (y_i)^2 + (\theta_i)^2} - \sqrt{(x_n)^2 + (y_n)^2 + (\theta_n)^2} \quad \text{(C5)}$$

These four different response functions were used with one of the experimental designs. The analyses with R_1 and R_2 as the response functions produced similar results and the results with R_3 and R_4 produced similar results. The results between these two pairs of response functions are very different. Using R_1 and R_2 as the response function, the significant main effects are all interactions. Using R_3 and R_4 as the response functions yielded the same significant main effects and no significant main effects that were interactions.

Moreover, the significant main effects for these two response functions were the same. These results demonstrate that the selection of the response function has a noticeable effect on the results of the analysis.

Considerable effort was expended in examining the nature of the distinctly different results generated with these four response functions and an experimental design. This exercise led to the observation that the correlations between the variables caused these response functions to give the different results. The use of, and investigation into, response functions with experimental design was abandoned by the authors in favor of using the coordinate components of the rest positions.

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