An Impact Moment Coefficient for Vehicle Collision Analysis

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THE COLLISION OF VEHICLES is an unfortunate yet common experience. The resulting damage can range from almost none to situations where vehicles are literally torn in half. Simulation of the deformation and forces in a collision has recently been accomplished with some success, primarily with the use of finite element analysis (6).* This problem is complex; structure geometry and material properties must be known reasonably well and the resulting computer solutions be long. Other collision studies have been made, also with some success. These involve an approach using the equations of impulse, momentum and energy for the initial and final states of impact. This latter approach is useful in vehicle collision studies and is often coupled with other vehicle motion simulations (3,5,7). In studying the motion of two vehicles during an accident, the impact is treated as an isolated event following pre-impact motion and preceding the post-impact motion of the vehicles. In (3) actual deformation energy data is used.

Philosophically, the study of impact using elasticity and plasticity with finite elements attempts to model impact forces and deformation as closely as possible. Classical impulse and momentum techniques seek the overall motion changes and imbed the entire deformation characteristics into coefficients of restitution, friction and/or energy losses. Potentially, the vehicle deformation studies using finite elements can be more accurate and informative. The impulse-momentum-energy approaches are much simpler. Both have their uses and can complement each other. This paper concentrates on the latter approach. Specifically, it introduces a new coefficient, the "impact moment coefficient," associated with the moment that develops between two vehicles in contact. It is related to changes in angular velocities due to impact.

When two objects moving on a plane collide, deformation takes place and forces are generated. In a real collision the force between the bodies is distributed over the contact surface. This surface, the deformations and the orientations of the bodies change with time during the impact, however it is assumed that suitable average quantities

*Numbers in parentheses designate References at end of paper.

ABSTRACT

Many investigators have used the equations of impulse, momentum and energy to analyze the changes in velocities when two vehicles collide. The equations generally include the classical coefficient of restitution which is used as a measure of energy loss. These equations and the coefficient are based upon large forces and short-duration contact between the two bodies. In all real collisions contact is over a surface, and in many vehicle collisions, momentary or permanent interlocking of deformed parts occurs over this surface. This causes a moment to develop whose impulse can significantly affect the dynamics; most authors neglect or ignore this moment (1,2,3,4,5)*. In this paper, the equations of impact of two vehicles are derived including the moment impulse. An impact moment coefficient is defined. The value of this coefficient determines the extent to which a moment is developed between the two vehicles during impact. Two examples are presented. The first is a simple classical problem of two rigid bodies impacting over a common surface and is presented to illustrate the concept of the impact moment coefficient. The second example uses data from an actual collision of two automobiles and shows that in accident reconstruction problems, an *a priori* value of the impact moment coefficient is often not needed.

exist. In all impact analyses the surface force is replaced by its resultant; actually the impulse of this resultant is considered. Since this resultant impulse is derived from a surface force, its line of action and position relative to the surface depends upon the surface force distribution. In a real impact such as when two vehicles collide, this average distribution is seldom known with any accuracy. In other words, the point of application of the resultant impulse is never known precisely. Consequently when formulating the equations of impact, the resultant of the total surface contact forces must consist of both a force impulse (at some arbitrary point) and a moment impulse. The need for this moment impulse has been overlooked by many people working in vehicle collision analysis. Only one reference (11) has been found which considers a surface moment impulse; this was a torsional moment and in a non-vehicular context.

When another unknown (the moment impulse) is introduced into the impact problem, another equation is needed. It is shown in this paper that this additional equation can take a form analogous to the one containing the classical coefficient of restitution. This requires the introduction of another coefficient, the moment coefficient.

The above discussion points out that unless the exact point of action of the resultant impulse is known, a moment impulse must be included. This leads to an impulse moment coefficient. Another argument can be made to justify the need for this new coefficient. It is well known that a completely inelastic impact can occur without any plastic deformation; coupling of rail cars is an example. In this case interlocking of the two bodies causes zero relative velocity following the impact. In the classical approach to solving this problem, the coefficient of restitution is set equal to zero. Now, it is certainly physically possible to have a planar impact where the bodies rebound with a non-zero relative translational velocity but where both bodies have the same angular velocity. If this zero relative angular velocity is caused by a constraint, current classical theory has no simple way of treating this. If another coefficient of restitution corresponding to angular velocities existed, independent of the familiar coefficient, the hypothesized problem could be easily handled. In vehicular crashes, mechanical interlocking of parts certainly occurs and although the final relative angular velocity may seldom be zero, the moment generated by this interlocking can often be significant. For this reason, a moment coefficient is necessary.

In order to introduce the concept of the moment coefficient, a relatively simple impact problem is worked in the next section. It is solved with and without including a moment impulse for contrast. In another section, the equations of planar impact of two vehicles is derived which includes the moment coefficient. Finally, an example using these equations for an accident reconstruction is presented. All of the usual assumptions are made which allow an impact analysis to apply to vehicular collisions. It is assumed that the impact durations are relatively short and that the magnitudes of all impact forces are high, relative to all other forces acting on the system such as tractive, suspension and steering forces.

NOTATION

e	coefficient of restitution
e_m	impact moment coefficient
d	distance between two points in a vehicle
I	moment inertia of vehicle about its center of gravity
M	impulse of the moment developed between two impacting vehicles
m	mass of vehicle
P	impulse of the force developed between two impacting vehicles
R	radius of a curve
V	velocity component of a vehicle following impact
v	velocity component of a vehicle before impact
μ	equivalent coefficient of friction along the impact surface
θ	heading angle of vehicles relative to the x axis
Γ	angle of impact surface relative to the y axis
Ω	angular velocity of a vehicle following impact
ω	angular velocity of a vehicle before impact
ϕ	angle between the length axis of a vehicle and a line between its center of gravity and the center of impact

Subscripts:

a	corresponds to vehicle A
b	corresponds to vehicle B
x	component along the x axis
y	component along the y axis
1, 2, 3, 4	corresponds to distances defined in Equations 15 through 18

IMPACT OF TWO RIGID BODIES

Figure 1 shows two rigid bodies, A and B, moving in circular paths of radius R. For

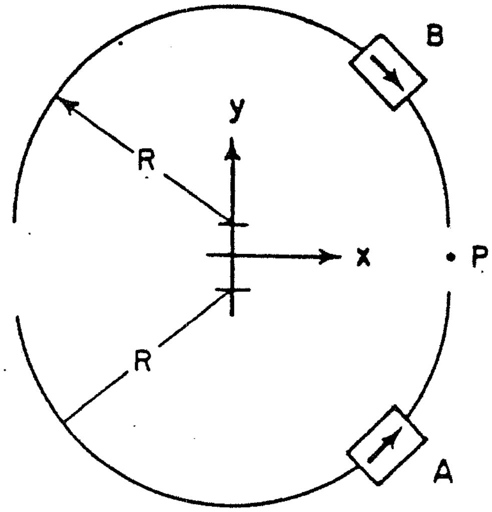

Fig. 1 - Example impact of two rigid bodies

simplicity, assume that both bodies have the same linear and angular velocities and the same mass and moment of inertia. They approach each other and impact at point P. Let the tangential velocity of A at P before impact* be $v_{ay} = v$. The velocity of B will be $v_{by} = -v$. Similarly the angular velocities will be $\omega_a = v/R$ and $\omega_b = -v/R$ respectively. The special case of a perfectly inelastic collision is considered.

The problem of finding the unknown, final velocities for bodies A and B will be solved two ways in this section. The first solution assumes central impact with a uniform surface force distribution over the contact surface; this solution is incorrect. The second solution allows nonsymmetric surface forces to develop which is represented by central-impact forces along with a moment. Fig. 2a is the free body diagram corresponding to central impact (that is where the resultant impulses lie along the line of centers of bodies A and B). Fig. 2b is the proper free body diagram for the impact of Fig. 1. Using Fig. 2a, the equations relating impulse to change in momentum are:

$$m_a(V_{ay} - v_{ay}) = -P_y \quad (1)$$

$$m_b(V_{by} - v_{by}) = P_y \quad (2)$$

$$I_a(\Omega_a - \omega_a) = 0 \quad (3)$$

$$I_b(\Omega_b - \omega_b) = 0 \quad (4)$$

*The notation used is such that before impact all velocities will have lower-case symbols and following impact, all velocities will have upper case symbols.

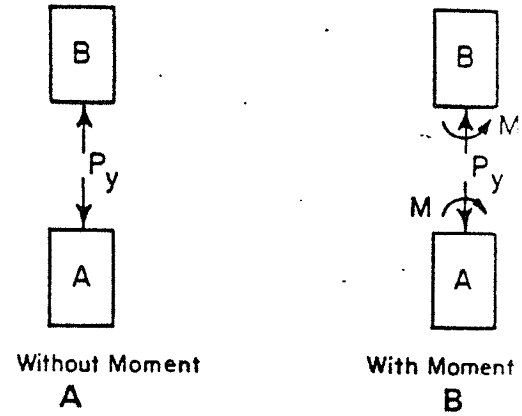

Fig. 2 - Free body diagrams of impacting bodies

For a non-elastic collision, the velocities are related by the coefficient of restitution. This equation is

$$(V_{by} - V_{ay}) = -e(v_{by} - v_{ay})$$

These equations can be solved without difficulty. For the situation depicted in Figure 1 and for a perfectly inelastic collision, e = 0, the velocities following collision are $V_{ay} = V_{by} = 0$ and $\Omega_a = -\Omega_b = v/R$. Equations 1 through 5 yield the unrealistic and incorrect result that for a perfectly inelastic impact, the two bodies rotate through each other around point P and have a residual, non-zero kinetic energy.

Now consider the free body diagrams shown in Figure 2b. Here a moment is permitted to develop between the two bodies at impact. The equations of impulse and momentum given by Eq. 1 and Eq. 2 are still valid. Using Fig. 2b, Eq. 3 and 4 becomes:

$$I_a(\Omega_a - \omega_a) = -M$$

$$I_b(\Omega_b - \omega_b) = M$$

Eliminating M gives

$$I_a(\Omega_a - \omega_a) + I_b(\Omega_b - \omega_b) = 0$$

Equation 5 is still valid for the rebounding velocities in the y direction, but another equation is needed since one equation, 6, replaces two equations, 3 and 4. This is furnished by introducing a coefficient, e_m, associated with the moment and angular velocities. This can be written as

$$(\Omega_b - \Omega_a) = e_m(\omega_b - \omega_a) \quad (7a)$$

or

$$(\Omega_b - \Omega_a)(1 - e_m) \quad (7b)$$

$$= -e_m[(\Omega_b - \omega_b) - (\Omega_a - \omega_a)]$$

Equations 1, 2, 5, 6 and 7a or 7b form a set of equations whose solution gives the final velocities and P_y if the initial velocities and coefficients are known. For the previous example of a perfectly inelastic collision, where now e and e_m are zero. There is no residual kinetic energy.

Note that e_m is not a coefficient of restitution although its negative, $-e_m$, is. This can be seen by comparing Eq. 7a with Eq. 5. Also note that Equation 7 can be written as

$$(\Omega_b - \Omega_a)(1 - e_m) = -e_m M(I_a + I_b)/I_a I_b \qquad (8)$$

This equation was obtained from Eq. 7 by using the angular impulse-momentum relationships derived from the free body diagrams in Figure 2b. It can be seem that when $e_m = 0$, Eq. 8 (and therefore Eq. 7) requires that the final angular velocities Ω_a and Ω_b be equal. This means that following impact, the bodies rotate together. However, when $e_m = 1$, Eq. 8 reduces to the condition that $M = 0$ which is true for central impact. Finally, $e_m = -1$, it can be seen by Eq. 7a that this corresponds to a perfectly elastic angular rebound.

Although two different and independent coefficients e and e_m are used, their choice of values in any given problem is not completely arbitrary and will depend upon the circumstances. Thus in the example above $e_m = 1$ cannot be used because it is not a central impact and the moment M cannot be assumed to be zero. Discretion must be used in choosing e and e_m; no general guidelines are presented here.

EQUATION OF IMPACT OF TWO VEHICLES

Now that the concept of the impact moment coefficient has been introduced, a set of more general equations are presented. These equations correspond to the impact of two vehicles on a flat surface with arbitrary initial velocities and orientations. All forces other than impact forces are neglected. Figure 3 shows the free body diagrams of the two vehicles. A and B, during impact. The equations of impulse and momentum for each vehicle along the x, y and θ coordinates are, respectively:

$$m_a(V_{ax} - v_{ax}) = P_x \qquad (9)$$

$$m_a(V_{ay} - v_{ay}) = P_y \qquad (10)$$

$$I_a(\Omega_a - \omega_a) = P_x d_3 - P_y d_4 + M \qquad (11)$$

$$m_b(V_{bx} - v_{bx}) = -P_x \qquad (12)$$

$$m_b(V_{by} - v_{by}) = -P_y \qquad (13)$$

and

$$I_b(\Omega_b - \omega_b) = P_x d_1 - P_y d_2 - M \qquad (14)$$

where

$$d_1 = d_b \sin(\theta_b + \phi_b) \qquad (15)$$

$$d_2 = d_b \cos(\theta_b + \phi_b) \qquad (16)$$

$$d_3 = d_a \sin(\theta_a + \phi_a) \qquad (17)$$

and

$$d_4 = d_a \cos(\theta_a + \phi_a) \qquad (18)$$

Before presenting the remaining equations, the impulses, P_x, P_y and M are eliminated to reduce the number of unknowns and equations. When this is done Eq. 9 through 14 becomes:

$$m_b(V_{bx} - v_{bx}) + m_a(V_{ax} - v_{ax}) = 0 \qquad (19)$$

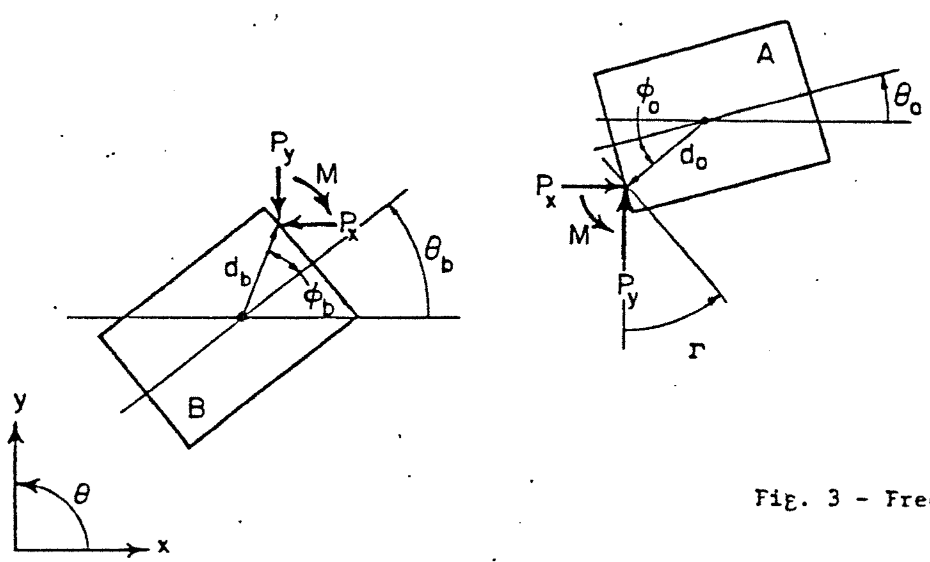

Fig. 3 - Free body diagrams of impacting vehicles

$$m_b(V_{by} - v_{by}) + m_a(V_{ay} - v_{ay}) = 0 \qquad (20)$$

and

$$I_b(\Omega_b - \omega_b) + I_a(\Omega_a - \omega_a) + m_b(d_1 + d_3)(V_{bx} - v_{bx}) + m_a(d_2 + d_4)(V_{ay} - v_{ay}) = 0 \qquad (21)$$

These are the familiar equations of conservation of momentum. Introduction of the coefficient of restitution, e, which relates the final to initial normal velocities requires the definition of the direction normal to the impact. In Figure 3 this is designated by the angle Γ. The velocities of both vehicles normal to the line designated by the angle Γ are then related by:

$$(V_{ay} - d_4\Omega_a - V_{by} - d_2\Omega_b)\sin\Gamma + (V_{ax} + d_3\Omega_a - V_{bx} + d_1\Omega_b)\cos\Gamma = -e[(v_{ay} - d_4\omega_a - v_{by} - d_2\omega_b)\sin\Gamma + (v_{ax} + d_3\omega_a - v_{bx} + d_1\omega_b)\cos\Gamma] \qquad (22)$$

During impact a force can develop along the line given by Γ. In classical impact theory this is generally considered to be a frictional force and a coefficient, μ is used. (In actual collisions of vehicles, this force can develop through friction and through momentary interlocking of deformed parts (5). Consequently, μ is an equivalent friction coefficient.) The coefficient, μ, relates the impulse along Γ to the impulse normal to Γ. This is:

$$P_y\cos\Gamma - P_x\sin\Gamma = \mu(P_y\sin\Gamma + P_x\cos\Gamma)$$

Equations 9, 10, 12 and 13 can be used to eliminate the impulse components from this equation; this gives

$$m_a(V_{ay} - v_{ay})(\cos\Gamma - \mu\sin\Gamma) + m_b(V_{bx} - v_{bx})(\sin\Gamma + \mu\cos\Gamma) = 0 \qquad (23)$$

Suitable care must be taken in application of Eq. 23. In some cases when sliding occurs, a theoretically maximum value of μ exists above which a reversal of relative velocities along Γ can take place (and a corresponding reversal of the sign of the impulse during impact) and unrealistic results occur. See (8).

The last equation needed is the one involving the impact moment coefficient which relates the angular velocities before and after impact. Following the form of Eq. 7b, this expression is:

$$(\Omega_b - \Omega_a)(1 - e_m) = -e_m[(\Omega_a - \omega_a) - (P_x d_3 - P_y d_4)/I_a - (\Omega_b - \omega_b) + (P_x d_1 - P_y d_2)/I_b]$$

When the components of the impulse, P, are eliminated, this equation becomes:

$$(\Omega_b - \Omega_a)(1 - e_m) = -e_m[(\Omega_a - \omega_a) - m_a d_3(V_{ax} - v_{ax})/I_a + m_a d_4(V_{ay} - v_{ay})/I_a - (\Omega_b - \omega_b) - m_b d_1(V_{bx} - v_{bx})/I_b + m_b d_2(V_{by} - v_{by})/I_b] \qquad (24)$$

Equation 24 is a generalized form of Eq. 7b and requires e_m to have the same properties as in the previous, simpler example. Thus when $e_m = 0$, Eq. 24 reduces to $\Omega_b = \Omega_a$, the perfectly inelastic collision. By using Eq. 11 and 14, when $e_m = 1$, it can be seen that Eq. 24 is the condition that the moment M be zero; $e_m = -1$ is perfectly elastic.

From a classical point of view, Eq. 19 through 24 form a set of six linear equations in the variables V_{ax}, V_{ay}, V_{bx}, V_{by}, Ω_a and Ω_b. If all of the coefficients, vehicle parameters, impact geometry and initial velocities are known, the unknown final velocities can be obtained. The value of this set of equations is not in using them in this manner however. These equations relate six velocity components and three coefficients to the physical properties of the vehicles and the impact geometry. When analyzing an actual collision, these equations can be used to solve for various combinations of six of these quantities assuming all the others are known. In the next section an example is presented which uses known quantities from a collision to estimate the initial speed of one of the vehicles.

EXAMPLE COLLISION ANALYSIS

When analyzing the collision of two vehicles or the impact of a vehicle with a barrier or structure, it is common to encounter the question: how fast was a vehicle traveling just prior to the impact? Methods of obtaining reliable estimates are being developed. Some of these methods involve fairly sophisticated computer simulations (3). The method used with the example in this section considers the pre-impact motion of the vehicles and post-impact motion separately from the impact itself.

It must be pointed out that it is not the purpose of the following example to show that omission of the moment from the impact equations can lead to incorrect results. This should be obvious from the simple example presented earlier. Whenever two vehicles are mechanically interconnected during impact a moment develops and the angular velocities are affected. In a general simulation, correct changes in angular velocities will not result without including e_m and allowing it to take on a value between -1 and +1. Rather this example is being presented to illustrate how this new coefficient can be treated in an accident reconstruction situation where no a priori value of e_m is available. This is the usual situation where the analyst cannot deduce the value of e_m from the accident events.

Figure 4 shows a top view of the sequence of events of an actual collision. The positions of vehicles A and B following the collision were measured with reasonable accuracy at the scene. Vehicle A is known to have been traveling in its lane with a speed of about 45 mph with little or no evasive maneuvers. Vehicle B lost directional control, skidded across the center line leaving skid marks along its path. The primary damage to B indicated it collided with A about as shown in Figure 4, along its driver's side. This correlated with left-front damage to A. The impact point and the distances traveled after

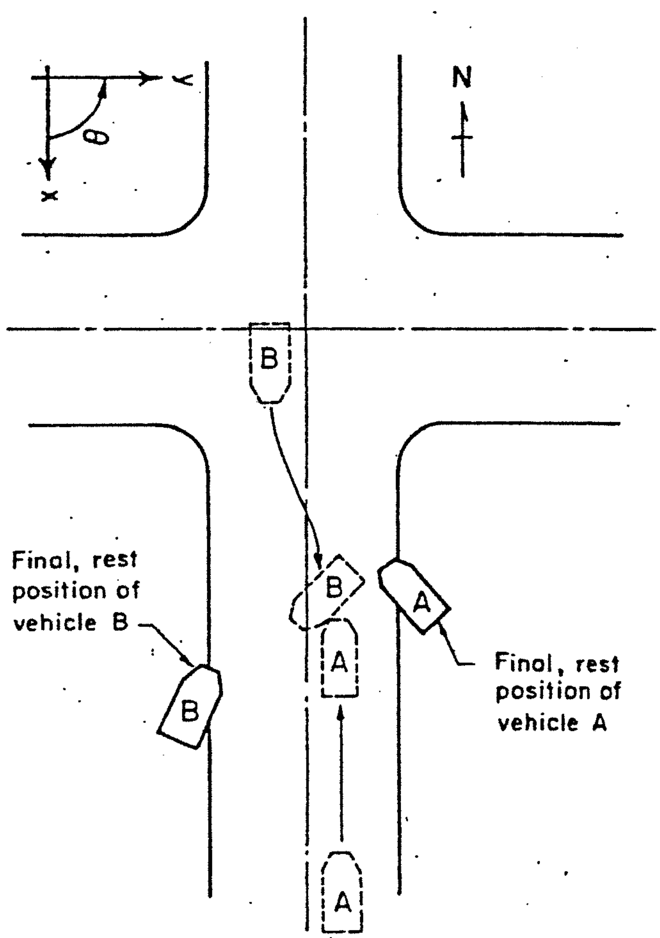

Fig. 4 - Sequence of positions of colliding vehicles

impact were estimated from photographs and measurements. The problem here is to estimate the initial speed of vehicle B prior to impact.

This will require the use of the impact equations derived earlier, but not until all of the known and unknown variables are sorted out. Specifically, what are needed are the vehicles' velocities immediately after impact but before moving to their final positions of rest. Ellis (10) gives the equations of motion of a vehicle for various tractive and frictional forces between the wheels and the ground. Because both vehicles had to have considerable yaw rotation to reach their final positions and from photographs of the scene it is a reasonable assumption that each vehicle skidded to a stop following impact. Using speed-dependent coefficients of friction (11) for both the road and off-road surfaces, equations similar to Ellis' were programmed for a digital computer and used to find the velocities vehicles A and B needed, following impact, to travel the post-impact distances measured at the scene. Various combinations of values of coefficients of friction for the road and berm were used. Table 1 shows the velocities from the computer program of vehicle A following impact in order to end up where it did, for various combinations of friction coefficients. Table II shows the corresponding values of velocities for vehicle B. These two tables show a relative insensitivity of the velocities to friction and so the first entry in each table will be used as the final velocities for the impact.

The initial velocity of vehicle A is assumed as 45 mph in the negative x direction. The remaining unknown velocity components are v_{bx}, v_{by} and ω_b, the desired values for B. Equation 19 through 24, six equations, furnish the capability of solving for six unknowns. Since e, e_m and μ are not known, these six equations will be used to solve for these coefficients and the unknown initial velocity components of vehicle B. Thus, in this particular example at least, the coefficients need not be measured or estimated but can be computed.

Equations 19 through 24 are no longer linear equations if the coefficients are treated as unknowns, as is the case here. Fortunately, the equations can be solved sequentially in closed form. Table III shows the input to the impact equations for this example. The solution of the equations gave the results in Table IV. The impact equations give a total speed of 19.3m/sec (63.2 ft/sec) for vehicle B. Also of interest, the coefficient of restitution, e, is 0.1; this small value correlates well with the fact that in the actual accident the cars were badly damaged and also due to the fact that after this relatively high speed impact, the cars were only about 25 feet away from each other. The coefficient of friction, μ turned out to

Table 1 - Velocities of Vehicle A Following Impact

Friction Coefficients

Road		Berm 0.400	0.425	0.450
0.50	V_{ax}	-7.0 (-23.)	-7.0 (-23.)	-7.0 (-23.)
	V_{ay}	2.4 (8.)	2.7 (9.)	2.7 (9.)
	Ω_a	.61 (35.)	.70 (40.)	.70 (40.)
0.55	V_{ax}	-7.3 (-24.)	-7.3 (-24.)	-7.3 (-24.)
	V_{ay}	2.4 (8.)	2.7 (9.)	2.7 (9.)
	Ω_a	.61 (35.)	.61 (35.)	.70 (40.)
0.60	V_{ax}	-7.6 (-25.)	-7.6 (-25.)	-7.6 (-25.)
	V_{ay}	2.7 (9.)	2.7 (9.)	2.7 (9.)
	Ω_a	.61 (35.)	.61 (35.)	.61 (35.)

Units: SI; (ft/sec, degrees/sec)

Table 2 - Velocities of Vehicle B Following Impact

Friction Coefficients

Road		Berm 0.400	0.425	0.450
0.50	V_{bx}	5.2 (17.)	5.2 (17.)	5.2 (17.)
	V_{by}	-6.1 (-20.)	-6.1 (-20.)	-6.1 (-20.)
	Ω_b	9.9 (565.)	9.9 (565.)	9.9 (565.)
0.55	V_{bx}	5.5 (18.)	5.5 (18.)	5.5 (18.)
	V_{by}	-6.4 (-21.)	-6.4 (21.)	-6.4 (-21.)
	Ω_b	10.3 (590.)	10.4 (595.)	10.4 (595.)
0.60	V_{bx}	5.8 (19.)	5.8 (19.)	5.8 (19.)
	V_{by}	-6.7 (-22.)	-6.7 (-22.)	-6.7 (-22.)
	Ω_b	10.7 (615.)	10.7 (615.)	10.7 (615.)

Units: SI; (ft/sec, degrees/sec)

Table 3 - Input Velocity Values For Impact Equations

Variable	S.I.	U.S.
V_{ax}	-7.0 m/sec	-23 ft/sec
V_{ay}	2.4 m/sec	8 ft/sec
Ω_a	0.6 rad/sec	35 deg/sec
v_{ax}	-20.1 m/sec	-66 ft/sec
v_{ay}	0.0 m/sec	0 ft/sec
ω_a	0.0 rad/sec	0 ft/sec
V_{bx}	5.2 m/sec	17 ft/sec
V_{by}	-6.1 m/sec	-20 ft/sec
Ω_b	9.9 rad/sec	565 deg/sec

Table 4 - Solution of Impact Equations

Variable	S.I.	U.S.
v_{bx}	18.9 m/sec	62 ft/sec
v_{by}	-3.7 m/sec	-12 ft/sec
ω_a	-0.3 rad/sec	-15 deg/sec
μ	0.35	
e	0.10	
e_m	0.70	

be 0.35 (values of this coefficient for various impacts generally range from 0.4 to 0.6 (5)). Finally, the impact moment coefficient, e_m, has a value of 0.70 for this impact. It is relatively high; but this seems reasonable because the vehicles had high relative rotational velocities following the impact.

CONCLUSIONS

A new coefficient, the impact moment coefficient has been introduced into the study of collisions of vehicles. This coefficient is analagous to the classical coefficient of restitution but differs in two respects. Its range of realistic values is between -1 and 1. When it is 0, the vehicles have zero relative angular velocity following impact but when it is -1, the angular impact is elastic. When the coefficient is +1, no moment is transmitted at impact which can be considered to be the case of direct central impact. The moment coefficient represents the extent to which a moment has developed across the surface of deformation between the two vehicles.

Because the impact moment coefficient depends upon the type and extent of the damaged surface which develops during an impact (and which obviously changes during the collision), the amount of mechanical interaction over the deformed surface and the location of the center of impact, it would probably be difficult to measure during experiments. Nevertheless it may prove fruitful to attempt experimental measurements.

The value of e_m may be difficult to establish a priori in analyses of actual collisions. However, when other information concerning the impact is known, e_m may be treated as an unknown as was done in the example. The use of this coefficient brings an existing physical phenomenon into the impact equations generally neglected in the past. This should make collision analysis through the use of impulse-momentum techniques more realistic and accurate.

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