SAE The Engineering
Resource For
Advancing Mobility 400 COMMONWEALTH DRIVE WARRENDALE, PA 15096

SAE Technical Paper Series

860015

Impact of Articulated Vehicles

Raymond M. Brach
University of Notre Dame
Notre Dame, IN

Reprinted from SP-660—
Operational Characteristics of
Recreational Vehicle Dynamics

International Congress and Exposition
Detroit, Michigan
February 24–28, 1986

ISSN 0148-7191

Printed in U.S.A.

860015

Impact of Articulated Vehicles

Raymond M. Brach
University of Notre Dame
Notre Dame, IN

ABSTRACT

A mathematical model is developed which permits calculation of velocity changes of vehicles involved in a collision where one or both vehicles are articulated. This includes any single vehicle pulling a trailer (such as an automobile towing a recreational vehicle) or tractor, semitrailers. The equations of the model are based upon direct application of Newton's laws of impulse and momentum. Typically made assumptions (such as the insignificance of external impulses) are discussed and analyzed. Examples of the model's application are provided including the impact of tractor, semitrailers into rigid barriers.

INTRODUCTION

A great deal has appeared in recent literature concerning the modeling of two vehicle collisions. At least two different approaches are common. One is to study the deformation due to the impact using various methods such as elastic-plastic finite element procedures and/or lumped mass, spring damper methods. These could be categorized under the realm of structural analysis techniques. Another approach is to use classical impulse, momentum and energy concepts (1,2,3,4)* to provide means of calculating velocity changes; these fall more properly under the topic of rigid body dynamics. Generally these approaches yield different information and are used for different purposes. Structural techniques are more suited for design whereas rigid body dynamics methods are better suited for accident reconstruction. As a rule, the structural methods are much more sensitive to the relative orientation and structural properties of the vehicles. The impulse-momentum methods are much more broadly applicable. Since they both model the same physical process they are complementary. In some situations, such as occupant protection, they can both provide useful information. In some cases, the impact models are imbedded within accident reconstruction models (4,5,6) which include post impact vehicle dynamic simulations as well. As with all mathematical modelling, accuracy can be a problem in at least two different ways. The first is related to the accuracy of the input information needed by the model. The second is related to the simplifying assumptions made during the derivation of the equations. The theory behind an impact dynamics model more general than others (1,2,3,4) has been presented (7) along with some applications (8,9). This impact model is more general since it includes the provision for a moment impulse between the colliding vehicles. (It also allows this moment impulse to be zero, for which the impact equations reduce to less general models).

It is the purpose of this paper to extend this more general approach to the problem of impact of articulated vehicles. This includes collisions between tractor, semitrailers, vehicles pulling trailers and collisions of these with single vehicles. It is capable of modelling barrier collisions of these vehicles as well. Much of what is done is simply to apply Newton's laws to cover the impact of pin connected rigid bodies. However, since articulated vehicles can be more massive than most other highway vehicles some additional and important effects must be treated. These effects are related to the ordinarily made assumption that the impulses of all forces other than the intervehicular collision force itself are negligible. A discussion of this

---

*Numbers in parentheses designate references at end of paper.

is presented in the first section of this paper by way of a simple example.

Since the number of equations of impact of two articulated vehicles is numerous, some economy of presentation is necessary. Not all equations will be documented; like equations with differing subscripts will not be repeated. Because the equations are numerous, and some lengthy, (as many as 23 linear algebraic equations and unknowns), solution using a digital computer is necessary. This is also omitted, although some examples and results are furnished.

In this paper, only the impact phase of vehicle collisions is covered, that is, the duration during which the vehicles are in contact. For all examples, it is assumed that no moment exists over the crush surface and that relative transverse sliding between the vehicles ends before separation (9).

ASSUMPTIONS; EXTERNAL FORCES

The planar rigid body dynamical impact model used to calculate velocity changes of vehicles (1,2,3,5,8) involves the concepts of impulse and momentum. Specifically, Newton's second law of motion (F=ma) can be integrated in full generality to provide the proper equations for calculating velocity changes. These can be applied to planar vehicle impacts provided certain assumptions are satisfied. These are:

1. the duration of the impact is short,
2. changes in both linear and angular positions of the vehicles during the impact are negligible,
3. the location of the resultant intervehicular impulse is known,
4. impulses due to external forces are small,
5. changes in the physical geometry (ostensibly due to the crush damage) is either small (negligible) or known and accounted for, and
6. out of plane effects are small.

These assumptions are usually satisfied when two vehicles (automobiles, pickup trucks, vans, etc.) collide. Because of the relatively large mass and the pinned, rigid body configurations of articulated vehicles, assumption 4 must be critically examined. This assumption is not always legitimate as shown by the following example.

Figure 1-a shows an automobile and tractor semitrailer as they would appear at the beginning of a head-on (nose-to-nose) collision. Figure 1-b shows a like configuration for two automobiles. These two hypothetical collisions will be compared under the conditions of no relative rebound (zero coefficient of restitution), no rotational velocities and crush durations of about 0.2 sec. Physical data, initial velocities, final velocities and the internal impulses are given in Table 1. These results are those of simple colinear impact theory.

Table 1. Collision Data

a, Collision of Fig. 1-a

	Vehicle 1	Vehicle 2
Mass	1800 kg (3969 lb)	27000 kg (59530 lb)
Initial Speed	15 m/s (33.6 mph)	-15 m/s (33.6 mph)
Final Speed	-13.1 m/s (-29.4 mph)	-13.1 m/s (-29.4 mph)
Intervehicular Impulse	-50.6 kN-s (-11380 lb-s)	50.6 kN-s (-11380 lb-s)

b, Collision of Fig. 1-b

	Vehicle 1	Vehicle 3
Mass	1800 kg (3969 lb)	2400 kg (5292 lb)
Initial Speed	15 m/s (33.6 mph)	-15 m/s (33.6 mph)
Final Speed	-2.14 m/s (-4.8 mph)	-2.14 m/s (-4.8 mph)
Intervehicular Impulse	-30.9 kN-s (-6937 lb-s)	30.9 kN-s (-6937 lb-s)

These solutions neglect the impulses due to all external forces during the duration of the impact. Suppose now that vehicles 2 and 3 were in a locked wheel skid throughout the impact with tire-to-pavement coefficients both 0.6. The respective frictional forces are f_2=158.9 kN (35718 lb) and f_3=14.1 kN (3175 lb). Assuming these are constant during the 0.2 second duration, their respective impulses are 31.8 kN-s (7144 lb-s) and 2.8 kN-s (635 lb-s). Note that the frictional impulse of 31.8 kN-s is about 63% of the intervehicular (crush) impulse of collision 1-a but 2.8 kN-s is only about 9% of the crush impulse for collision 1-b.

This example illustrates that impulses from sources other than the crush surface may not be negligible when analyzing heavy vehicles. Other circumstances can arise when external impulses may be significant. Consider a 90° front-to-side collision between two automobiles. Suppose the vehicle being struck on the side is pulling a trailer whose wheels are freely rolling and remain in that condition throughout the impact (that is, the trailer wheel side forces do not exceed the friction threshold). Then, in effect, the trailer's transverse velocity is constrained to be zero. As a result of these examples, a need is seen for the model to handle velocity constraints and external impulses. As will be seen in the next section, introduction of a velocity constraint requires an external impulse; velocity constraints and external impulses are related.

DERIVATION OF EQUATIONS

Figure 2 shows free body diagrams of two articulated vehicles along with an illustration of the variables and the coordinate system used. Vehicle A includes two rigid bodies 1 and 3; vehicle B includes two rigid bodies 2 and 4. These rigid body pairs are pinned together at points with impulses R and Q, respectively. Note that the symbols M, P, Q, R and S represent impulses not forces (7). Impulse P represents the intervehicular force impulse and M is the corresponding moment impulse. This vehicular arrangement requires that the impact occurs between bodies 1 and 2 of vehicles A and B. (Note that either 1 or 3 could be an auto or trailer of vehicle A, though.) No moment impulses are permitted at the pinned connections.

Newton's laws require that the impulse of all forces in each coordinate direction must equal the change in momentum in that direction. Thus

$$m_i(V_{ij} - v_{ij}) = \sum N_j \ . \qquad (1)$$

In this equation, V's and v's are mass center velocity components*, i is the rigid body number (1, 2, 3 or 4) and j represents either coordinate x or y, m is mass and N represents any of the force impulses. Two of these equations (one for j = x and one for j = y) can be written for each of the 4 rigid bodies for a total of 8. For example for body 1 in the y direction, Eq 1 would take the form

$$m_1(V_{1y} - v_{1y}) = P_y + R_y \ .$$

According to Newton's laws for rotational motion

$$I_i(\Omega_{ij} - \omega_{ij}) = \pm M + \sum r_k N_j \qquad (2)$$

Here, I is mass moment of inertia, Ω and ω are final and initial rotational velocities respectively, M is the moment impulse at the crush surface (which may or may not be zero) and the r_k's are appropriate moment arms for the force impulse components N_j. One of these equations can be written for each rigid body for a total of 4. For example, for rigid body 1,

$$\begin{aligned} I_1(\Omega_1 - \omega_1) = M &+ d_1 \sin(\theta_1 + \phi_1)P_x \\ &- d_1 \cos(\theta_1 + \phi_1)P_y \\ &- d_5R_x \sin\theta_1 + d_5R_y \cos\theta_1 \end{aligned}$$

An additional 4 equations can be written to require the final velocity components of the pin-connected rigid bodies to be equal at the pins

$$V_{ij} \pm r_k\Omega_i = V_{i+2} \pm r_{k-2}\Omega_{i+2} \qquad (3)$$

For example, the x components of the final velocities of the pin connection between bodies 1 and 3 must be equal; that is:

$$\begin{aligned} V_{1x} - d_5\Omega_1 \sin\theta_1 = V_{3x} \\ + d_3\Omega_3 \sin(\theta_3 + \phi_3) \end{aligned}$$

At this point, a total of 16 equations have been described. For an unconstrained problem, 19 unknowns exist. These are:

Final Velocity Variables

V_{1x}, V_{1y}, Ω_1, V_{2x}, V_{2y}, Ω_2, V_{3x}, V_{3y},

Ω_3, V_{4x}, V_{4y}, Ω_4

Impulse Variables

P_x, P_y, Q_x, Q_y, R_x, R_y, M

Three more equations must be found. Note in Figure 2 that an angle Γ is defined. This angle represents a hypothetical flat surface normal to which all crush deformation takes

---

*Note that throughout this paper, capital or upper case V's represent final velocity components and small or lower case v's represent initial velocity components.

place and along which all sliding takes place. With this defined, the ratio of impulses tangential to and normal to this surface, P_t and P_n respectively, can be defined as

$$P_t/P_n = \mu \qquad (4)$$

Note that μ is the ratio defined in Eq 4 and may or may not be a friction coefficient. The symbol f is reserved for the coefficient of Coulomb friction. The quantity μ is bounded, $|\mu| \leq |\mu_{max}|$, where μ_{max} is the ratio of P_t to P_n for which relative tangential motion ("sliding") ceases prior to the end of impact at separation [7]. The bound, μ_{max}, is the value of P_t/P_n found from a solution of the full set of equations but with Eq 4 replaced by $V_{1t} = V_{2t}$. This is the condition for no sliding at separation. Note also that if an actual friction coefficient f is greater than, or equal to μ_{max}, this means represents one equation. Rebound and energy loss at the crush surface can be represented through the use of the coefficient of restitution, e. Thus

$$(V_{1n} - V_{2n}) = -e(v_{1n} - v_{2n}) \qquad (5)$$

where these velocities are the components in the normal direction of each rigid body at the "point" of contact. Finally an equation can be written which uses a moment coefficient of restitution [7,9] (which controls any angular rebound). This is

$$e' M = (1 + e')(\Omega_2 - \Omega_1)\bar{I} \qquad (6)$$

where $\bar{I} = I_1 I_2/(I_1 + I_2)$. Note that $0 \leq e' \leq 1$ or $e' = -1$. If $e' = -1$, then the form of Eq 6 requires that the moment impulse M be zero. Otherwise e' relates final and initial relative angular velocities as e regulates relative normal velocities at the crush surface. For example, note the if $e' = 0$, Eq 6 requires that $\Omega_1 = \Omega_2$ which can represent the situation where bodies 1 and 2 become attached during the impact.

Any number of velocity constraints may be added as long as an appropriate, unknown external impulse is added for each. For example, if a rear wheel of a trailer, say Body 3 is up against a curb during an impact and can move only parallel to the curb with angle α_s, the velocity constraint can be written as

$$V_{sy} \cos \alpha_s - V_{sx} \sin \alpha_s = 0 \qquad (7)$$

This requires the addition of an unknown impulse with components C_x and C_y. These must be included in the summations involving N_j in Eq 1 and 2. Also, of course, the number of unknowns increases for each velocity constraint added. All known impulses (such as due to tire frictional forces) can simply be incorporated as additional N_j's. Consequently, the number of equations and unknowns is 19 plus the number of velocity constraints.

RIGID BARRIER COLLISIONS

A set of equations as described above can be used to calculate velocity changes when an articulated vehicle collides with a rigid barrier. To accomplish this, one of the vehicles, say B, is given an infinite mass and in effect becomes the barrier. For a computer solution this can be done conveniently by using values of m_2, m_4, I_2 and I_4 large enough such that the velocity changes for these rigid bodies is smaller than the number of significant figures printed out. Fig. 3 shows the configuration being considered. A range of solutions corresponding to different barrier angles and friction coefficients will be presented and discussed in order to to detect some general trends. For example, if the angle Γ is small the collision is near head on and it might be expected that the angular velocity of the tractor after impact would be positive (counterclockwise). On the other hand if the angle Γ is larger and/or the friction between the vehicle and barrier is low, the tractor might be expected to "glance-off" of the barrier and have a negative (clockwise) angular velocity at rebound. What is the cutoff condition between these two possibilities and how is the energy loss affected are two questions which will be illustrated.

Table 2 contains data corresponding to a hypothetical tractor semitrailer. Two conditions are considered, one for an empty trailer and another fully laden and the results are presented in Figure 4 and Figure 5. For the fully loaded trailer, Figure 4 shows energy loss plotted as a function of barrier angle with impulse ratio, $\mu = P_t/P_n$ as a parameter. Figure 5 corresponds to the empty trailer. Note that these figures are for hypothetical collisions only and are not based upon any experimental results. An arbitrary value of the coefficient of restitution e = 0.05 has been used. The solid rising curve is the impulse ratio for which sliding ceases prior to separation. This corresponds to those conditions for which an impulse ratio P_t/P_n, smaller in magnitude than the friction coefficient is sufficient to cause sliding to end prior to separation. This is the condition that

$$\mu = \frac{P_t}{P_n} = \mu_{max} \leq f \qquad (8)$$

When $\mu_{max} > f$ an impulse ratio greater than the friction can supply is needed to cause sliding to cease and so sliding continues throughout the impact. This is the condition corresponding to the solid horizontal curve and the rapidly decreasing dashed energy loss

Table 2

CONDITIONS CHOSEN FOR RIGID BARRIER IMPACT

Tractor Mass/Weight	9500 kg	20945 lb
Tractor Yaw Inertia	21000 kg-m^2	15478 ft-lb-s^2
Trailer Mass/Weight		
a. Loaded	20000 kg	44096 lb
b. Empty	6000 kg	13229 lb
Trailer Yaw Inertia		
a. Loaded	150000 kg-m^2	110550 ft-lb-s^2
b. Empty	48000 kg-m^2	35375 ft-lb-s^2
Initial Speed	13.41 m/s	30 mph
Initial System Energy	2.65×10^6 N-m	1.95×10^6 ft-lb
Coefficient of Restitution	0.05	
Moment Coefficient	-1.0 (No moment)	
Vehicle/Barrier Friction Coefficient, f	0.2 and 0.6	

curves in Figures 4 and 5. Thus, for example if f = 0.6 and the barrier angle is 30 degrees, sliding will cease prior to separation. For a barrier angle of 60 degrees, a friction coefficient of 1.2 is needed to stop the initial sliding, so for f = 0.6, the vehicle will slide over the barrier all through the impact. Similarly, for f = 0.2 the vehicle will continue sliding along the barrier surface throughout the collision for barrier angles of about 14 degrees or greater. These results do not differ greatly from Figure 4 to Figure 5.

The corresponding energy loss curves start at near 100% (for e = 0.05) and $\Gamma = 0$ degrees and drop rapidly once $\mu = f$ is less than μ_{max}. This clearly shows that if an objective of barrier design is to deflect the path of a vehicle rather than to cause energy dissipation (damage), it should have as low a possible friction coefficient. This follows whether or not the vehicle is articulated, small or large.

Tables 3 and 4 provide some specific results from barrier impacts for f = 0.6 and the other conditions in Table 2. The reader is encouraged to make comparisons and contrasts from these results.

TABLE 3-a

NEAR DIRECT BARRIER IMPACT (Γ = 15°)
FULLY LOADED TRAILER

Final Tractor Velocity		
1. x (Initial) Direction, CG	0.58 m/s	1.30 mph
2. y Direction, CG	0.86 m/s	1.92 mph
3. θ, Rotation	0.18 rad/s	10.3 deg/s
Final Trailer Velocity		
1. x (Initial) Direction, CG	0.58 m/s	1.30 mph
2. y Direction, CG	0.45 n/s	1.01 mph
3. θ, Rotation	-0.24 rad/s	-13.8 deg/s
Barrier Impulse		
1. Normal	403164 N-s	90639 lb-s
2. Tangential	-90270 N-s	-20295 lb-s
Kinetic Energy Loss	99.4%	99.4%

TABLE 3-b

NEAR DIRECT BARRIER IMPACT (Γ = 15°)
EMPTY TRAILER

Final Tractor Velocity		
1. x (Initial) Direction, CG	0.59 m/s	1.3 mph
2. y Direction, CG	0.81 m/s	1.8 mph
3. θ, Rotation	0.16 rad/s	9.2 deg/s
Final Trailer Velocity		
1. x (Initial) Direction, CG	0.59 m/s	1.3 mph
2. y Direction, CG	0.44 n/s	1.0 mph
3. θ, Rotation	-0.22 rad/s	-12.6 deg/s
Barrier Impulse		
1. Normal	212215 N-s	47710 lb-s
2. Tangential	-46202 N-s	-10387 lb-s
Kinetic Energy Loss	99.4%	99.4%

VEHICLE-TO-VEHICLE COLLISIONS

In this section a few examples will be presented of velocity change calculations for vehicle-to-vehicle collisions. One collision configuration will be used, that of Figure 6, which depicts a tractor semitrailer, Veh A, travelling in the +x direction. The impact occurs at the trailer wheels due to a vehicle, Veh B, travelling in the +y direction. This corresponds to a 90 degree collision which might occur at an intersection. Several cases will be considered:

1. Vehicle B is a fully loaded tractor semitrailer otherwise identical to Vehicle A (which in all cases has an empty trailer).
2. Vehicle B is an automobile and with
    a. no external impulses accounted for
    b. zero angular velocity constraint on the tractor of Vehicle A
    c. trailer wheel/road surface frictional impulse and zero angular velocity constraint on the tractor of Vehicle A.

In all cases, the initial speed of the vehicles is 13.41 m/s (30 mph) and is in the direction of each vehicle's heading (no initial turning or rotational velocities exist). The physical parameters of the tractor semitrailers, are those given in Table 2, Veh A has an empty trailer and Veh B is fully loaded. The automobile has a mass of 1500 kg (weight of 3307 lb) and a yaw inertia of 2800 kg-m^2 (2064 ft-lb-s^2). For the trailer chosen, the total static weight on the trailer wheels is 36222 N (8143 lb). When an external impulse is considered, it is due to this static weight with a tire/road friction coefficient of 0.6. The impulse of this friction force is that of a constant The semi-to-semi collision causes both of these to be much larger. Perhaps somewhat surprising are the similarities. Whether struck by an auto or another semi, the speed change (in the original direction of motion) of Vehicle A is less than 10%. Whether Veh B is an auto or semi, the system kinetic energy change differs by less than 1%. This illustrates that collision geometry can significantly influence energy loss as has been noted for automobile collisions (8).

Tables 6, 7 and 8 illustrate results of the same collision under different assumptions. The results of Table 6 are for the typical assumption that the effects of impulses of external forces over the duration of the impact can be neglected. Table 7 includes the effects of a frictional impulse between the trailer wheels and the ground. It seems apparent that the differences are quite small. Further, when the tractor of Vehicle A is constrained from rotating (presumably due to transverse static friction on the tractor's tires), as indicated in Table 8, the final velocities of the vehicles still do not differ greatly. This example shows that neglecting effects of external impulses may be a legitimate assumption for some collision geometries. However, no general rules or guidelines exist and the validity of the assumption should be checked for each collision.

CONCLUSIONS

A very high proportion of fatal crashes on highways involve heavy trucks with automobiles. Articulated recreational vehicles also are involved notably in more accidents than their relative number of occurances in traffic indicate. Velocity changes of these articulated vehicles can be calculated using classical impulse and momentum methods. The equations for doing so were presented in this paper in summary form. The equations are relatively easy to solve using digital computers; all examples in this paper were prepared using an IBM PC. All of these examples were hypothetical however, primarily due to the fact that little experimental data is available for velocity changes and the coefficients of friction and restitution encountered in practice. This is not true for automobile collisions (9). Until some instrumented, vehicle-to-vehicle collisions involving articulated vehicles are conducted, it will be necessary to estimate and/or bracket these coefficients in applications.

REFERENCES

1. Limpert, R., Motor Vehicle Accident Reconstruction and Cause Analysis, Mitchie Co., Charlottesville, VA, 1984.
2. Macmillan, R.H.,Dynamics of Vehicle Collisions, Inderscience Ent., Ltd., United Kingdom, 1983.
3. Woolley, R.L., "The 'IMPAC' Computer Program for Accident Reconstruction," Field Accidents, Data Collection, Analysis, Methodologies, and Crash Injury Reconstructions, SAE Publication P-159, Warrendale, PA, 1985.
4. NHTSA, "CRASH3 User's Guide and Technical Manual," DOT HS 805 732, Washington, DC, 1982.
5. Martinez, J.E., "Computer Aided Automobile Accident Reconstruction -Theory, Graphics, Animation," Field Accidents, Data Collection, Analysis, Methodologies, and Crash Injury Reconstructions, Warrendale, PA, 1985.

TABLE 4-a

GLANCING BARRIER IMPACT ($\Gamma = 75^\circ$)
FULLY LOADED TRAILER

Final Tractor Velocity		
1. x (Initial) Direction, CG	-13.08 m/s	-29.3 mph
2. y Direction, CG	1.40 m/s	3.1 mph
3. θ, Rotation	-0.64 rad/s	-36.7 deg/s
Final Trailer Velocity		
1. x (Initial) Direction, CG	-13.08 m/s	-29.3 mph
2. y Direction, CG	-0.19 n/s	-0.4 mph
3. θ, Rotation	0.10 rad/s	5.7 deg/s
Barrier Impulse		
1. Normal	11757 N-s	2643 lb-s
2. Tangential	-7054 N-s	-1586 lb-s
Kinetic Energy Loss	4.4%	4.4%

TABLE 4-b

GLANCING BARRIER IMPACT ($\Gamma = 75^\circ$)
EMPTY TRAILER

Final Tractor Velocity		
1. x (Initial) Direction, CG	-12.86 m/s	-28.8 mph
2. y Direction, CG	1.10 m/s	2.5 mph
3. θ, Rotation	-0.71 rad/s	-40.7 deg/s
Final Trailer Velocity		
1. x (Initial) Direction, CG	-12.86 m/s	-28.8 mph
2. y Direction, CG	-0.37 n/s	-0.8 mph
3. θ, Rotation	0.18 rad/s	10.3 deg/s
Barrier Impulse		
1. Normal	10164 N-s	2285 lb-s
2. Tangential	-6098 N-s	-1371 lb-s
Kinetic Energy Loss	7.2%	7.2%

6. Day, T.D. and Hargens, R.L., "Differences Between EDCRASH and CRASH3," Field Accidents, Data Collection, Analysis, Methodologies, and Crash Injury Reconstructions, SAE Publication P-159, Warrendale, PA, 1985.

7. Brach, R.M., "Friction, Restitution and Energy Loss in Planar Collisions," Trans. ASME, J. Appl. Mechanics, Vol. 51, March 1984, pp. 164-170.

8. Brach, R.M., "Impact Analysis of Two-Vehicle Collisions," SAE Paper 830468.

9. Brach, R.M., "Momentum and Energy Analysis of Automobile Collisions," Trans. Structural Impact and Crashworthiness, Elsevier Appl. Sci. Publ., 1984.

TABLE 5

90 Degree Impact of a Loaded Tractor Semitrailer
into the Trailer of an Unloaded Tractor Semitrailer

Initial Speeds, 13.41 m/s (30 mph)

Final Tractor Velocity, Veh A		
1. x (Initial) Direction, CG	12.22 m/s	27.3 mph
2. y Direction, CG	-0.34 m/s	-0.8 mph
3. θ, Rotation	0.48 rad/s	
Final Trailer Velocity, Veh A		
1. x (Initial) Direction, CG	12.22 m/s	27.3 mph
2. y Direction, CG	7.12 m/s	15.9 mph
3. θ, Rotation	-2.24 m/s	
Final Tractor Velocity, Veh B		
1. x Direction, CG	3.2 m/s	7.2 mph
2. y (Initial) Direction, CG	12.07 m/s	27.0 mph
3. θ, Rotation	-1.64 m/s	
Final Trailer Velocity, Veh B		
1. x Direction, CG	-0.6 m/s	-1.3 mph
2. y (Initial) Direction, CG	12.07 m/s	27.0 mph
3. θ, Rotation	0.32 m/s	
Intervehicular Impulse		
1. x (Tangential) Direction	-18395 N-s	-4136 lb-s
2. y (Normal) Direction	39453 N-s	8870 lb-s
External Impulse	0	0
Kinetic Energy Loss		
1. Vehicle A	-2.9%	
2. Vehicle B	15.6%	
3. System	9.3%	

TABLE 6

90 Degree Impact of an Automobile into the Trailer Wheels of an Unloaded Tractor Semitrailer

Initial Speeds, 13.41 m/s (30 mph)

Final Automobile Velocity		
1. x Direction, CG	3.85 m/s	8.6 mph
2. y (Initial) Direction, CG	3.89 m/s	8.7 mph
3. θ, Rotation	-4.13 rad/s	
Final Tractor Velocity		
1. x (Initial) Direction, CG	13.04 m/s	29.2 mph
2. y Direction, CG	-0.12 m/s	-0.3 mph
3. θ, Rotation	0.16 m/s	
Final Trailer Velocity		
1. x (Initial) Direction, CG	13.04 m/s	29.2 mph
2. y Direction, CG	2.56 m/s	5.7 mph
3. θ, Rotation	-0.8 m/s	
Intervehicular Impulse		
1. x (Tangential) Direction	- 5778 N-s	-1299 lb-s
2. y (Normal) Direction	14279 N-s	3210 lb-s
External Impulse	0	0
Kinetic Energy Loss		
1. Vehicle A	3.0%	
2. Vehicle B	65.7%	
3. System	8.5%	

TABLE 7

90 Degree Impact of an Automobile into the Trailer Wheels of an Unloaded Tractor Semitrailer, Trailer Wheel-to-Ground Friction Impulse Included

Initial Speeds, 13.41 m/s (30 mph)

Final Automobile Velocity		
1. x Direction, CG	3.89 m/s	8.7 mph
2. y (Initial) Direction, CG	3.23 m/s	7.2 mph
3. θ, Rotation	-4.17 rad/s	
Final Tractor Velocity		
1. x (Initial) Direction, CG	13.03 m/s	29.1 mph
2. y Direction, CG	-0.11 m/s	-0.2 mph
3. θ, Rotation	0.15 m/s	
Final Trailer Velocity		
1. x (Initial) Direction, CG	13.03 m/s	29.1 mph
2. y Direction, CG	2.17 m/s	4.9 mph
3. θ, Rotation	-0.69 m/s	
Intervehicular Impulse		
1. x (Tangential) Direction	-5837 N-s	-1312 lb-s
2. y (Normal) Direction	15276 N-s	3434 lb-s
External Impulse	-3260 N-s	733 lb-s
Kinetic Energy Loss		
1. Vehicle A	3.7%	
2. Vehicle B	67.8%	
3. System	9.3%	

TABLE 8

90 Degree Impact of an Automobile into the Trailer Wheels of an Unloaded Trailer Tractor Semitrailer, Trailer Wheel-to-Ground Impulse on Tractor Rotational Constraint Included

Initial Speeds, 13.41 m/s (30 mph)

Final Automobile Velocity		
1. x Direction, CG	3.92 m/s	8.8 mph
2. y (Initial) Direction, CG	3.19 m/s	7.1 mph
3. θ, Rotation	-4.21 rad/s	
Final Tractor Velocity		
1. x (Initial) Direction, CG	13.03 m/s	29.1 mph
2. y Direction, CG	0	0
3. θ, Rotation	0	0
Final Trailer Velocity		
1. x (Initial) Direction, CG	13.03 m/s	29.1 mph
2. y Direction, CG	2.37 m/s	5.3 mph
3. θ, Rotation	-0.59 m/s	
Intervehicular Impulse		
1. x (Tangential) Direction	-5887 N-s	-1324 lb-s
2. y (Normal) Direction	15324 N-s	3445 lb-s
External Impulse	-3260 N-s	-733 lb-s
Kinetic Energy Loss		
1. Vehicle A	3.8%	
2. Vehicle B	67.4%	
3. System	9.4%	

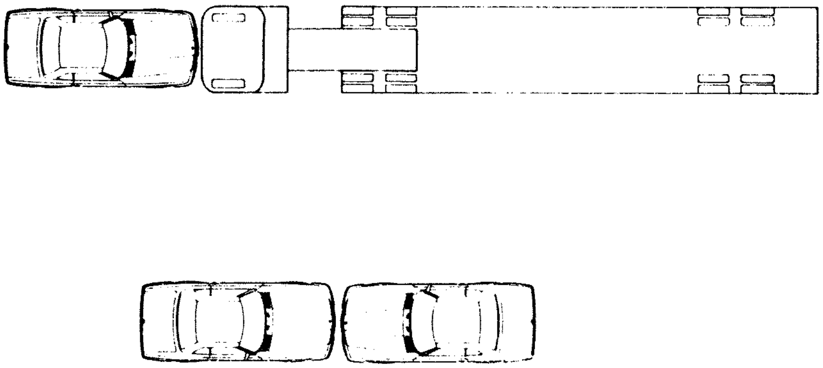

Fig. 1. Front-to-Front Impact Geometry
1-a. Automobile and Tractor Semi-Trailer
1-b. Two Single Vehicles

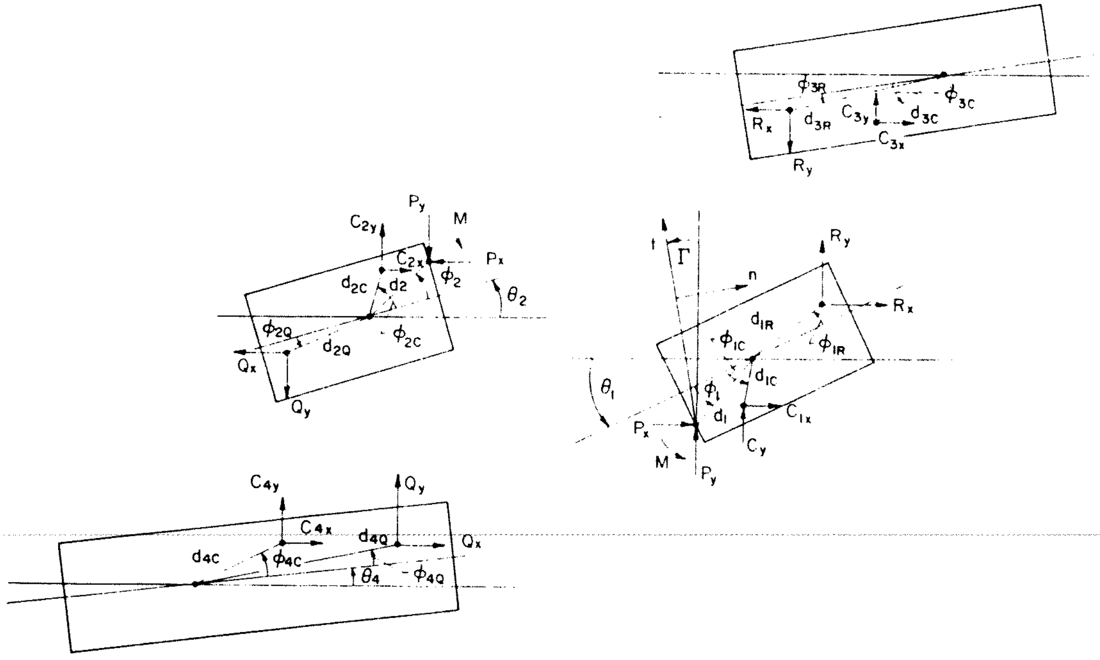

Fig. 2. Free Body Diagrams, Two Articulated Vehicles

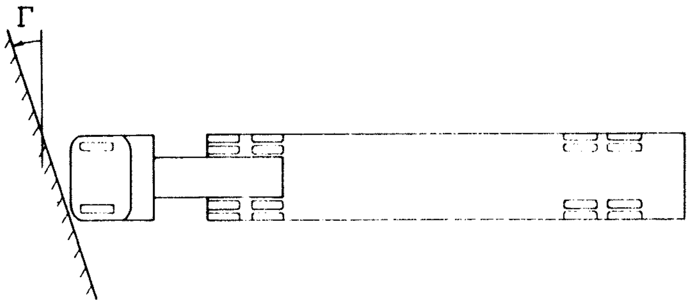

Fig. 3. Impact of an Articulated Vehicle into a Rigid Barrier

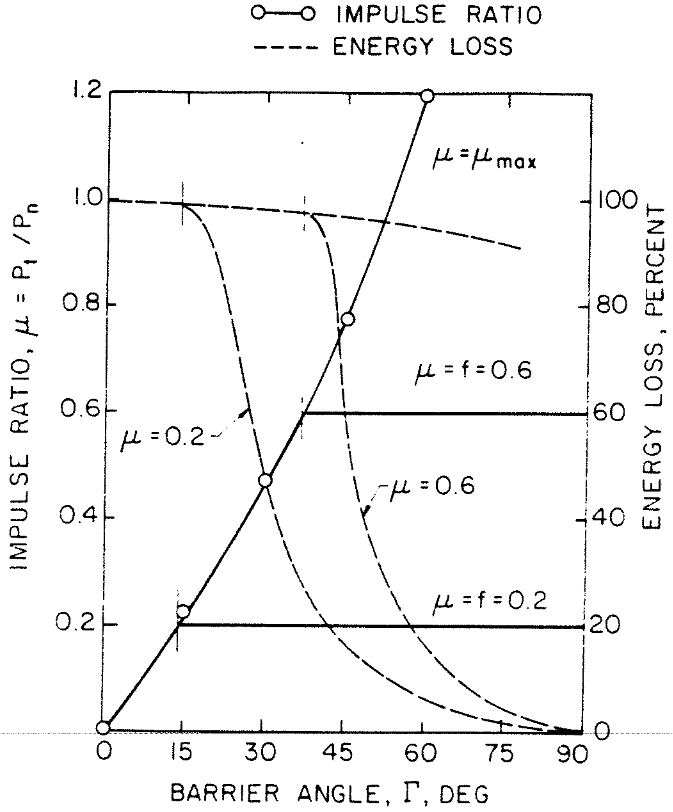

Fig. 4. 48 km/hr Inelastic (e = 0.05) Barrier Impact, Fully Loaded Trailer

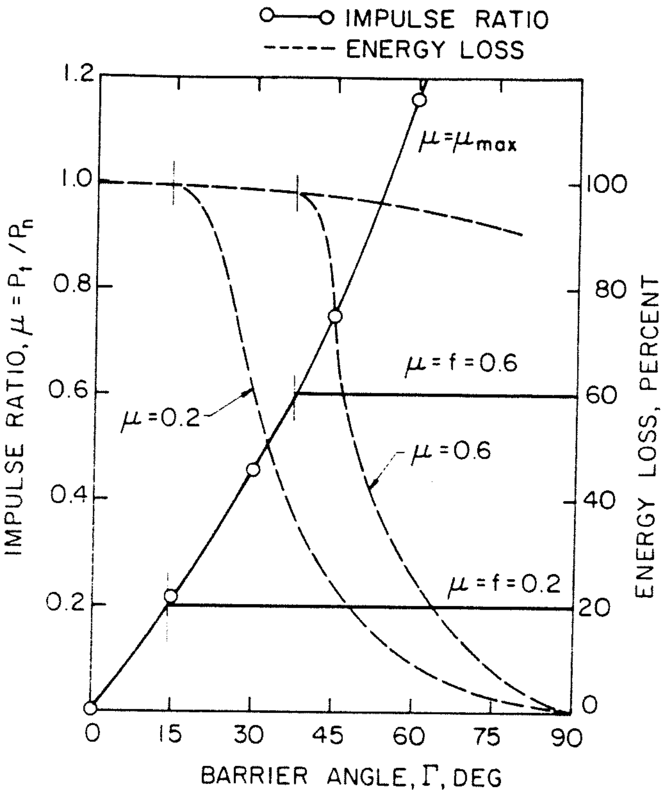

Fig. 5. 48 km/hr Inelastic (e = 0.05) Barrier Impact, Empty Trailer

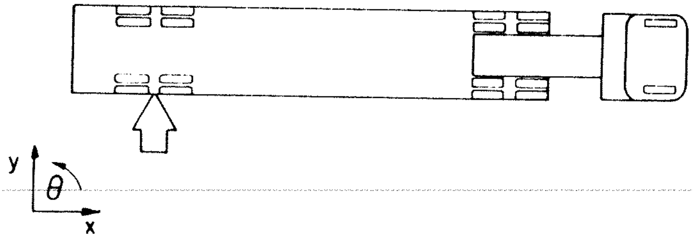

Fig. 6. Intersection Impact (90° Front-to-Side)