870048

A Review of Impact Models for Vehicle Collision

Raymond M. Brach
Department of Aerospace & Mechanical Engineering
University of Notre Dame
R. Matthew Brach
Triodyne Inc.
Niles, IL

ABSTRACT

Automobile accident reconstruction and vehicle collision analysis techniques generally separate vehicle collisions into three different phases: pre-impact, impact and post-impact. This paper will concern itself exclusively with the modeling of the impact phase, typically defined as the time the vehicles are in contact.

Historically, two different modeling techniques have been applied to the impact of vehicles. Both of these techniques employ the impulse-momentum formulation of Newton's Second Law. The first relies exclusively on this principle coupled with friction and restitution to completely model the impact. The second method combines impulse-momentum with a relationship between crush geometry and energy loss to model the impact. Both methods ultimately produce the change in velocity, ΔV, and other pertinent information about a collision.

The concepts of impulse-momentum and energy loss as applied to vehicle collisions have been occasionally misrepresented and appear not always to be fully understood. This paper will present the application of these principles to collisions of two bodies in a plane. This relationship between the change in velocity and energy loss will be investigated. A review and numerical comparison of several impact models will be presented.

* R. Matthew Brach is currently employed by MPC Products Corporation, Skokie, IL.

VEHICLE COLLISION ANALYSIS and accident reconstruction has enjoyed a recent growth in popularity. This has been due primarily to two factors. The first factor is that since an accident can frequently be a tragic event to the people involved, lawsuits have become common. In order to analyze collisions, engineers have developed models which attempt to quantify the accident. Typically the pre-impact velocities of the vehicles involved in the accident are of particular interest. The second factor that has contributed to this field is the advent of the personal computer. Although the collision of two vehicles can almost exclusively be considered planar, the equations which accurately describe this phenomenon tend to be quite complicated and hence tedious to solve by hand. The personal computer has eliminated much of the tedium and has added convenience in solving the governing equations.

Accident reconstruction and analysis techniques generally separate all automobile collisions into three distinct phases. These phases are typically referred to as the pre-impact phase, impact phase and the post-impact phase or "spin out". The definition of the impact phase as the time the vehicles are in contact implies the definition of the pre-impact and post-impact phases; i.e, the pre-impact is that time prior to vehicle-to-vehicle contact, and the post-impact is the time after the vehicles have separated.

In typical collision analysis circumstances, the pre-impact velocities are usually the unknowns to be determined by using a model. The accident

0148-7191/87/0223-0048$02.50

reconstructionist would be interested, however, in any indications of evasive maneuvers that may have been used by the driver of either car in interpreting the results of a study. The post-impact phase is usually the starting point in the analysis of automobile collisions. From information obtained at the accident scene, eg., skidmarks, debris, final resting points of the vehicles and vehicle deformation, numerous models exist which can predict the velocity of each vehicle immediately after the separation of the two vehicles. It is not the intent of this paper to address this portion of the accident. Readers interested in this topic are referred to other sources [1, 2].*

In the scheme of accident reconstruction and analysis, it is the impact phase analysis which estimates the change in velocity of the two vehicles commonly referred to as "delta-V," or ΔV. Each vehicle has three velocity components. This implies six initial and six final for a total of twelve velocity components in the impact problem. As viewed from a classical mechanics perspective, the impact problem is to provide a means (a set of equations called an impact model) of calculating the final velocities for given initial velocities. From the point of view of accident reconstruction, the problem often is to calculate the initial velocity components given the finals (or perhaps a mixture as happens in some cases). In either case, the laws of mechanics and mathematics are quite uncompromising: six independent conditions (usually in the form of linear equations) are needed to provide a unique solution to the problem. Some investigators substitute assumptions for the equations as is discussed later in this paper. This, of course, can yield accurate results if the assumption(s) are appropriate.

Historically, two modeling techniques have been used in the analysis of vehicle impacts to estimate the change in velocity of the vehicles. The first of these two models employs the principles of impulse, momentum, friction and restitution to estimate ΔV of the vehicles involved in a collision. The second method employs concepts of impulse and momentum combined with an estimation of the energy absorbed in the collision from measurements of vehicle permanent deformation, or "crush." The second method also uses typical elastic-plastic behavior of vehicle structures in the energy loss estimation procedure. The techniques employing the estimation of energy loss from damage measurements will not be discussed in great detail. However, part of this paper includes comparisons of results from various readily available computer programs which use crush measurements.

*Numbers in brackets refer to references listed at the end of the paper.

Regardless of the method employed for the collision analysis, the ΔV is ultimately the quantity which is sought. The ΔV has traditionally been employed in two different manners. The first employs ΔV in magnitude form only to correlate injury severity of the occupants. The second manner employs the vector components of the ΔV to relate the post-impact to the pre-impact velocities. It should be pointed out that in the impact analysis approach employing impulse-momentum principals only, the components of the ΔV are vector quantities. In the approach employing crush deformation, the ΔV is treated as a scalar quantity. Its components are then determined from the angle of the Principle Direction of Force (PDOF), which must be determined from the physical deformation. This estimation can often be quite difficult due to the severity of the crush deformation.

As mentioned, two contemporary classes of impact models exist. One uses classical impulse and momentum principles almost exclusively and the other makes limited. use of impulse and momentum but combines direct measurement of crush and vehicle elastic-plastic behavior. Examples of the former are [3, 4, 5] and the latter are [6, 7, 8, 9, 22]. The use of energy loss due to crush deformation may seem to have a potential for greater accuracy since it is based in part on experimental data, principally barrier crashes. This is not necessarily true however, for several reasons:

1. The relationship between force, crush deformation and energy loss varies considerably from location to location and vehicle to vehicle. It is an almost impossible task to maintain a sensitive, comprehensive and up to date data base.

2. Those concepts from impulse and momentum currently used in these models are severely limited and insufficient to provide accurate results.

Furthermore, these limitations are totally unnecessary since full impulse-momentum models are relatively simple to use without severely restrictive assumptions. In fact, a properly formulated impulse-momentum model can easily furnish more energy loss information (as is shown later) and has greater potential than current crush deformation/energy loss methods for correlation to experimental data.

EQUATIONS FOR A PLANAR,, TWO-BODY COLLISION

Much of what follows is a review of specific aspects of impact theory pertinent to vehicle collisions. For more information see [10]. Newton's 2nd Law, $F = ma$, is commonly used in addressing the problem of the collision of two bodies in a plane. For this application, a variation of the law is typically employed. Newton's equation can be written as follows using the fact that $a = dv/dt$:

$$F = m \frac{dv}{dt}$$

or, since the mass m of the particle can typically be considered constant, we can write:

$$F = \frac{d}{dt}(mv) \qquad \text{(a)}$$

The vector mv is called the momentum of the particle, and we see that the force acting on a particle is equal to the rate of change of the momentum of the particle.

Integration of (a) provides:

$$\int_{t_1}^{t_2} F\,dt = mv_2 - mv_1 \qquad \text{(b)}$$

The integral in the above equation is called the impulse of the force F during the interval of time considered. Hence we see that the area under the force-time curve is the impulse.

In formulating the equation for the solution of the planar two-body collision, it is worthwhile to review the necessary assumptions that govern the formulation. The first assumption is that the duration of the contact of the two bodies is small, and large forces are developed between the two bodies. The short time duration also has an additional effect. That is, any changes in the position of the mass centers and changes in angular orientation of the two bodies are small. The second assumption is that only a single impact between the two bodies is being considered. If multiple contacts occur, each must be considered independent of any others. The third assumption is that the impulses from external forces such as friction forces between the tires and the ground, aerodynamic drag and drive train drag can be safely neglected. The fourth assumption concerns the resultant impulse vector. This vector has a specific point of application and direction. It is assumed that the location of this point is known. The importance of the location of this point will be elaborated on later in the paper. In some cases the direction of application is also assumed known.

Newton's Second Law as expressed in impulse-momentum form as shown in equation (b) above, does not impose any restrictions on the time duration. Time duration typical in vehicular collision are on the order of 0.1 to 0.2 seconds. Time intervals of this magnitude, coupled with the assumption of large intervehicular forces cause large accelerations, finite velocity changes and *small* displacements. These factors taken together usually cause the above assumptions to be satisfied for consideration of vehicle collisions.

Most vehicles have a significant yaw moment of inertia and cannot be modeled as particles. Yet, particle theory is presented here first because its simplicity allows for a thorough coverage of the relationship between energy loss due to intervehicular friction and restitution. This simplicity is lost later when the rigid body impact problem is considered. However, the concepts do not change substantially.

Consider Figure 1 which shows the free body diagrams of two particles. A normal-tangential coordinate system is chosen such that the line through the particle centers is the normal axis, **n**. The tangential axis, **t**, is perpendicular to the normal axis and lies in the plane defined by the initial velocities of the particles. Velocity symbols used throughout this paper will be double subscripted with the first subscript referring to the particle with the second subscript referring to the coordinate direction. Capital, or upper case symbols indicate final velocities; small, or lower case symbols indicate

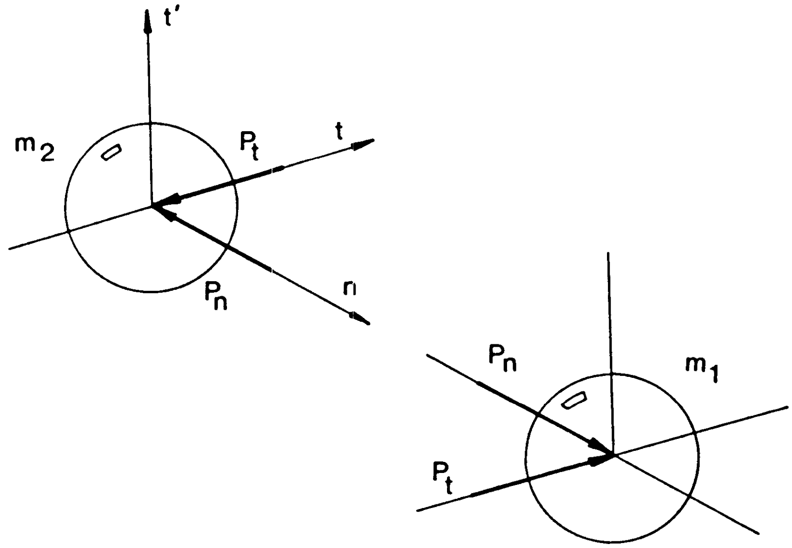

FIGURE 1, FREE BODY DIAGRAM OF TWO PARTICLES

initial velocities. For example, v_{1n} is the final velocity component of particle 1 in the normal direction; v_{2t} is the initial velocity component of particle 2 in the tangential direction.

Conservation of momentum for the system of particles along the normal axis yields:

$$m_1V_{1n} + m_2V_{2n} = m_1v_{1n} + m_2v_{2n} \qquad (1)$$

Conservation of momentum of the system of two particles in the tangential direction yields:

$$m_1V_{1t} + m_2V_{2t} = m_1v_{1t} + m_2v_{2t} \qquad (2)$$

A coefficient of restitution, e, is used to implicitly represent energy loss of a collision due to relative, normal velocity changes. This coefficient is defined as:

$$e = \frac{\text{Relative final velocity in the normal direction}}{\text{Relative initial velocity in the normal direction}}$$

where $0 \leq e \leq 1$. Using this definition, we find:

$$V_{1n} - V_{2n} = - e\,(v_{1n} - v_{2n}) \qquad (3)$$

The coefficient e can be related directly to "crush" deformation [10]. The change in normal velocities is directly related to a normal impulse, P_n, between the vehicles. This is

$$P_n = m_1(V_{1n}-v_{1n}) = -m_2(V_{2n}-v_{2n}) \qquad (4)$$

Tangential velocity changes are governed by the tangential impulse, P_t, developed during the collision. Using this tangential impulse, the normal impulse and the fact that some sliding must occur along the tangential direction, we can define an equivalent coefficient of friction, μ, as

$$\mu = P_t/P_n \qquad (5)$$

In some applications, μ may correspond directly to a coefficient of dynamic sliding friction, i.e., Coulomb friction. However, it should be noted that the above definition is not subject to any limitations and permits modeling of such diverse processes as combinations of dry friction, inelastic shear deformation of materials, etc. The tangential impulse can be obtained directly from the free body diagrams of Figure 1. Thus, we obtain:

$$P_t = m_1(V_{1t}-v_{1t}) = -m_2(V_{2t}-v_{2t}) \qquad (6)$$

Using linear combinations of equations (4) and (6), combined with equation (5), a fourth equation can be written in the form:

$$\mu m_1V_{1n} - m_1V_{1t} - \mu m_2V_{2n} + m_2V_{2t} = \mu m_1v_{1n} - m_1v_{1t} - \mu m_2v_{2n} + m_2v_{2t} \qquad (7)$$

Equations (1), (2), (3) and (7) form a set of four equations solvable for four unknown velocities. However, the question of whether sliding terminates prior to separation still remains. Relative tangential motion exists at the beginning of contact and continues as long as $|V_{2t} - V_{1t}| > 0$. If this motion ceases during the interval of contact, then the final tangential velocities are identical and we have:

$$V_{1t} - V_{2t} = 0 \qquad (8)$$

Equation (8) can be used in place of (7) when it is known that sliding terminates. For now, equation (7) and equation (8) are treated as alternatives and solutions are obtained for both cases.

It will prove enlightening for the rigid body problem to place the equations in matrix form. For the particle equations above, using equation (7), we have:

$$\begin{bmatrix} m_1 & 0 & m_2 & 0 \\ 0 & m_1 & 0 & m_2 \\ -1 & 0 & +1 & 0 \\ \mu m_1 & -m_1 & -\mu m_2 & m_2 \end{bmatrix} \begin{Bmatrix} V_{1n} \\ V_{1t} \\ V_{2n} \\ V_{2t} \end{Bmatrix} = \begin{bmatrix} m_1 & 0 & m_2 & 0 \\ 0 & m_1 & 0 & m_2 \\ e & 0 & -e & 0 \\ \mu m_1 & -m_1 & -\mu m_2 & m_2 \end{bmatrix} \begin{Bmatrix} v_{1n} \\ v_{1t} \\ v_{2n} \\ v_{2t} \end{Bmatrix} \qquad (9)$$

The solution to this set of equations can be written in several forms. The following equations seem to best display the physical symmetry of the problem:

$$V_{1n} = v_{1n} + m_2(1+e)(v_{2n}-v_{1n})/(m_1+m_2) \qquad (10)$$

$$V_{1t} = v_{1t} + \mu m_2(1+e)(v_{2n}-v_{1n})/(m_1+m_2) \qquad (11)$$

$$V_{2n} = v_{2n} - m_1(1+e)(v_{2n}-v_{1n})/(m_1+m_2) \qquad (12)$$

and

$$V_{2t} = v_{2t} - \mu m_1(1+e)(v_{2n}-v_{1n})/(m_1+m_2) \qquad (13)$$

When equation (8) is used in place of (7) the solutions for the final normal velocity components are identical to equation (10) and (12). The final tangential velocity components are:

$$V_{1t} = v_{1t} + m_2(v_{2t}-v_{1t})/(m_1+m_2) \qquad (14)$$

and

$$V_{2t} = v_{2t} - m_1(v_{2t}-v_{1t})/(m_1+m_2) \qquad (15)$$

RESTITUTION, FRICTION AND ENERGY LOSS

There is an important reason for introducing the coefficient of restitution into the collision problem as was done in the previous section. The coefficient of restitution is a convenient means of including the loss of kinetic energy due to normal deformation while maintaining linear equations. An equation containing kinetic energy explicitly would be nonlinear with respect to the unknown velocity components, and the ease of obtaining a solution is lost. Because of the importance of kinetic energy in the ultimate evaluation of the problem, the relationship between e, μ and the energy loss is presented. Using conservation of energy, we can write:

$$m_1(V_{1n}^2+V_{1t}^2)/2 + m_2(V_{2n}^2+V_{2t}^2)/2 + T_L = m_1(v_{1n}^2+v_{1t}^2)/2 + m_2(v_{2n}^2+v_{2t}^2)/2 \qquad (16)$$

where T_L represents the kinetic energy converted to other forms such as sound, light, heat, friction and/or permanent deformation. Substitution of the solutions for the final velocity components into equation (16) will give the energy loss in terms of initial velocities, particle masses and coefficients. Using the solution in which sliding does not cease prior to separation, equations (10) through (13) gives

$$T_L = \bar{m}(v_{2n}-v_{1n})^2(1+e)[(1-e)+2\mu r-(1+e)\mu^2]/2 \qquad (17)$$

where $\bar{m} = m_1 m_2/(m_1+m_2)$ and

$r = (v_{2t}-v_{1t})/(v_{2n}-v_{1n})$.

Note that equation (17) is a quadratic in the impulse ratio μ. This suggests a maximum or minimum energy loss. Analysis indicates that the energy loss, T_L, possesses a maximum with respect to μ. That is, as μ is increased from 0, the energy loss increases. At some point, $\mu = \mu_{max}$, the energy loss then decreases (and even becomes negative) for $\mu > \mu_{max}$. Intuitively, for any given collision, one expects a larger friction coefficient accompanied with sliding to dissipate more energy. This is only true to a certain point since, with enough friction, sliding will cease prior to separation. Any value of higher than that amount which causes sliding to cease, when used with equation (7), will produce an unrealistic solution with energy added to the system. This amounts to a frictional impulse "reversing" the sliding. This phenomenon will be demonstrated in later examples. Analysis of this problem has shown that for particles, the value of $\mu = \mu_{max}$ is the smallest value of μ which causes sliding to terminate prior to separation and the largest realistic value of to be used with equation (7). From equation (17) this is:

$$\mu_{max} = \frac{r}{1+e} \qquad (18)$$

The concept of a maximum coefficient μ_{max} has been corroborated recently by data by Ishikawa [22]. In [22], Ishihawa has plotted experimental values of the impulse ratio P_t/P_n determined from experimental vehicle impacts. The data points show a distinct maximum value for the oblique impact geometry. In summary, if $P_t/P_n < \mu_{max}$, sliding exists at separation and equations (10) (11), (12) and (13) present the solution to the problem. If $P_t/P_n = \mu_{max}$, sliding will cease prior to separation, and the final tangential velocities are equal and the final velocities are given by equations (10), (12), (14) and (15). Note that for colinear impacts (head-on), $v_{1t} = v_{2t} = 0$, $\mu_{max}=0$, and the solution must be independent of friction.

After having developed the solution to the impact of two particles, it is interesting to look at the work done by the impulse during the collision. To evaluate the energy, we first recall that:

$$\text{Work} = \int_{x_1}^{x_2} F dx = \int_{t_1}^{t_2} F v dt \qquad (19)$$

where $v = dx/dt$. Using the mean value theorem, equation (19) can be written as follows:

$$\text{Work} = V_{avg} \int_{t_1}^{t_2} F dt = \text{Energy} \qquad (20)$$

Where the quantity Fdt is the impulse of the force F as defined earlier in equation (a). Equation (20) is the work done by the total impulse developed during the collision. This impulse can be resolved into its normal and tangential components, P_n and P_t respectively. Thus, we can say

$$T_{Ln} = (V_{avg})_n P_n \qquad (21)$$

and

$$T_{Lt} = (V_{avg})_t P_t \qquad (22)$$

where $(V_{avg})_n$ and $(V_{avg})_t$ are the average relative velocities of the two particles in the normal direction and the tangential direction respectively. These two quantities are defined by the following:

$$(V_{avg})_n = [(v_{1n}-v_{2n})+(V_{1n}-V_{2n})]/2 \qquad (23)$$

and

$$(V_{avg})_t = [(v_{1t}-v_{2t})+(V_{1t}-V_{2t})]/2 \qquad (24)$$

Using equations (23) and (24) with equations (21) and (22) we obtain:

$$T_{Ln} = [(v_{1n}-v_{2n})+(V_{1n}-V_{2n})]P_n/2 \qquad (25)$$

and

$$T_{Lt} = [(v_{1t}-v_{2t})+(V_{1t}-V_{2t})]P_t/2 \qquad (26)$$

Impact models which use crush deformation measurements to estimate energy loss are based upon forces acting normal to the vehicle undamaged surface [13]. Since only a portion of the kinetic energy is lost in the normal mode, those models should tend to underestimate energy loss. This can result in significant errors. Table 1 lists the energy absorbed in three of the RICSAC collisions which were analyzed using an impulse-momentum model of the impact [5]. It can be seen that in each case that a substantial amount of energy is due to the tangential impulse.

TABLE 1
Examples of Energy Loss
Normal and Tangential Modes[5]

Collision	Loss, Percent of Initial System Energy Total	Normal	Tangential
RICSAC 1	52.0	19.4	32.5
RICSAC 3	34.7	34.2	0.5
RICSAC 9	28.7	15.7	13.1
RICSAC 10	31.0	15.4	15.6

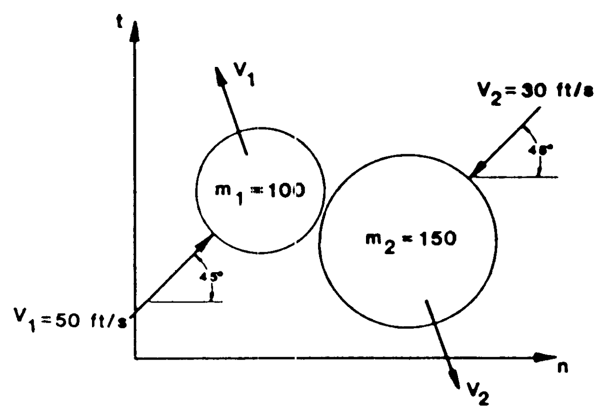

FIGURE 2, EXAMPLE
TWO PARTICLE COLLISION

To illustrate the combined influence of coefficient of restitution, the coefficient of friction, and the concept of μ_{max} on the energy absorbed in the collision, equation (17) is evaluated for several values of these coefficients. Figure 2 represents a two particle collision in which $v_1=50$ ft/s at 45° and $v_2=30$ ft/s at $-45°$. We also set $m_1=100$ and $m_2=150$ lb-s²/ft. From these values we find m=60 and r=1 and μ_{max} = 1/(1+e). Table 2 lists the amount of energy lost.

The values of T_L listed in Table 2 indicate the quadratic nature of the Eq. 17. We see that as μ is increased indiscriminately the energy loss eventually decreases; energy is added to the system during the collision, which is of course impossible. This illustrates that the choice of a value of μ when used in the solution of impact equations must be done with care and must never exceed μ_{max} for the given collision.

TABLE 2
Variations in Energy Loss for an Example Particle Impact

$e = 0,\ \mu_{max} = 1$		$e = 1,\ \mu_{max} = 0.5$	
	T_L		T_L
0	96000	0	0
1*	192000	0.5,	96000
2*	96000	1.0,	0
3	-192000	1.5	-2888000

* Physically Unrealistic Values

IMPACT OF RIGID BODIES IN A PLANE

Now that the concepts of friction, restitution and energy loss have been reviewed, a set of more general equations can be developed. The fundamental principles and assumptions made in the previous section are applicable here but several concepts require further explanation and elaboration.

When two objects moving on a plane collide, deformation takes place and forces are generated. In real collisions, the force developed between the bodies is distributed over a common contact surface. Both the forces and the contact surface change with time during the collision. However, the resultant impulse, its direction and point of application are constants. They vary neither with time nor position. Figure 3 shows free body diagrams of two vehicles involved in a collision. The resultant impulse is shown resolved into its x and y components, P_x and P_y. Since the resultant impulse is derived from surface forces, its line of action and point of application are dependent on the distribution of the force. The eventual point of application of this impulse is called the "center of impact." This point is almost always assumed to be known in collision analyses and is chosen by the analyst. It is rarely exact. If chosen with sufficient inaccuracy, the analyst fictitiously introduces a moment impulse as illustrated in Figure 3 by M. This situation is analogous to the elementary engineering mechanics procedure of replacing a single point force by an equal force at some other location and also including the appropriate moment. If the center of impact is chosen with sufficient accuracy, the resulting moment impulse would be near zero.

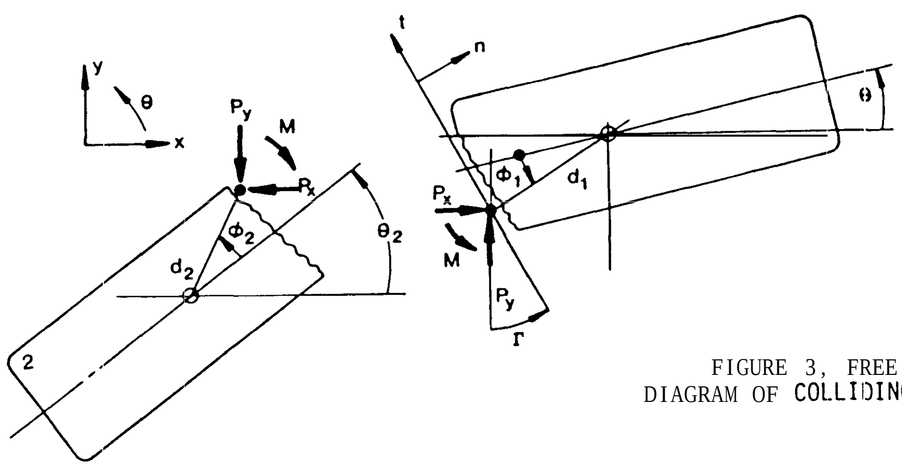

FIGURE 3, FREE BODY DIAGRAM OF COLLIDING VEHICLES

Incorporation of a moment impulse into the system is done for another reason. It is certainly possible that a physical mechanism, such as interlocking parts, can exist to transmit a moment (at least momentarily) between the vehicles. Inclusion of a moment impulse allows for the representation of such phenomena. Its inclusion leads to the definition of an angular restitution coefficient which is similar to the classical coefficient e, but has some significant differences. A major difference is that when the moment coefficient, $e_m = +1$, the moment impulse M is zero. This permits solutions where M = 0 is appropriate. Otherwise, $-1 \leq e_m \leq 0$, [10, 11].

In the particle impact problem, two unknown velocities are computed for each particle. In the rigid body impact problem, there are three unknown final velocity components for each rigid body for a total of six unknowns. Thus, V_{1x}, V_{1y}, Ω_1, V_{2x}, V_{2y}, and Ω_2 are the six unknowns, while v_{1x}, v_{1y}, ω_1, v_{2x}, v_{2y} and ω_2 are presumed known. A unique solution can be obtained with six equations. Such a complete solution has been previously presented [11, 12], along with an analytical solution [10]. The six equations modeling planar impacts are listed in this paper as an Appendix.

As mentioned previously, two different techniques have been used to model vehicle collisions: impulse-momentum and impulse-momentum-deformation energy.

In past years, numerous authors have applied the principles of impulse and momentum to vehicle collisions. One of the earliest was Emori [19, 20] who used particle models. Limpert [3] developed a rigid body model still used by many people today. In [3], Limpert initially presents a solution for the straight central impact of two vehicles. In this formulation, the concepts of conservation of linear momentum and restitution are employed to predict the final velocities of vehicles that have been involved in a central impact with known initial velocities. He then considers oblique central and straight non-central impact. In this section, he presents without derivation the final expressions for impulses and post-impact velocities of both vehicles. With careful review of these equations, several observations can be made about the assumptions made by Limpert in formulating his solution.

Although restitution of the vehicles is used in his analysis of the straight central impact, this concept is noticeably absent from his equations presented for the oblique non-central impact. This omission of restitution implies that Limpert. models the collision inelastically with a coefficient of restitution e, always equal to zero. Studies have shown [11, 12, 13] that the coefficient of restitution for most collisions is small, and the assumption that it equals zero is somewhat justified. However, these studies also indicate that for some collisions this coefficient can take on values in the range of 0.20 to 0.40. Variations this large can have a significant effect on the prediction of final velocities.

Another parameter that Limpert does not consider in his solution is the coefficient of friction between the two vehicles at the contact surface. In his presentation, he uses the assumption that during the deformation phase of the collision, the velocities of the colliding bodies will change so that at maximum deformation, both bodies will have the same (vector) velocity. This implies the assumption that the coefficient of friction, (the ratio of the tangential impulse to the normal impulse), is always large enough to stop relative motion of the vehicles prior to separation; hence $\mu = \mu_{max}$. As a consequence, Limpert's solution is not applicable to "side swipe" type collisions in which relative motion of the two vehicles at the point of impact does not go to zero prior to separation.

Limpert's model does not consider a moment between the vehicles at the collision surface. This limits the applicability of this model to collisions in which a significant moment impulse does not develop over the crush surface.

Verification of these three observations can be demonstrated using the example provided by Limpert to illustrate his solution technique for an oblique impact [Example 28-2, reference 3]. A comparison was performed using Limpert's values as input into a computer implementation of Brach's impact equations. Brach's solution allows for independent control of each of 3 coefficients, e, e_m and μ. To correspond to Limpert's assumptions, the moment coefficient, e_m, was set to +1 (zero moment impulse), the friction coefficient was set at $\mu = \mu_{max}$, (no sliding at separation), and the restitution coefficient was set to zero.

All other parameters such as distances to the center of impact, masses and velocities were identical to the values used by Limpert in his example. Table 3 presents a tabulated comparison of the two solutions. It indicates that Limpert's solution and Brach's solution yield identical results when the proper restrictions of the collision coefficients are made on the general solution.

indicates that this can be accomplished through the use of his six equations and by defining the normal impulse, I_{12}, as follows:

$$I_{12} = -(1 + 1/e)/(a - \lambda b)$$

where e is the coefficient of restitution, λ is the equivalent coefficient of friction and a and b are constants which depend upon the masses of the vehicles,

TABLE 3
Numerical Comparison between Limpert and Brach

	Limpert		Brach	
	Vehicle 1	Vehicle 2	Vehicle 1	Vehicle 2
Final x velocity	29.2	28.3	29.17	28.30
Final y velocity	16.5	58.8	16.48	58.74
Final angular velocity	6.28	-1.58	6.28	-1.58

Linear velocities in ft/sec, angular velocities in rad/sec.

Another application of the principles of impulse and momentum to vehicle collisions is presented by Macmillan [14]. In his solution of the oblique vehicular impact problem, Macmillan introduces both coefficients of friction and restitution. In his consideration of friction between the two vehicles, Macmillan also employs the definition of the friction coefficient as the ratio of the tangential impulse to the normal impulse. This allows for the modeling of the friction between the two vehicles for cases of interlocking parts as well as Coulomb friction. He does not include an impulse couple at the contact surf ace. He comments however, that it is unnecessary and, if included, would only shift the line of action of the impulse component which is normal to the impact surface. Macmillan feels that this is more conveniently done by altering the position of the point of impact. This view indicates a misunderstanding of the need for the moment impulse.

Macmillan ultimately presents a set of six equations and six unknowns as a solution to the planar impact problem. He also indicates that the "inverse" problem can be solved, namely, determining the pre-impact conditions from a given set of post-impact conditions. He their radii of gyration and crush dimensions. The motivation for obtaining an inverse solution is practical since accident reconstructions take place after the collision has occurred, and it is the speeds of the vehicles prior to the collision which are commonly of interest. However, it is not uncommon in vehicular impacts to make the assumption that the collision is completely inelastic,, a condition under which $e = 0$. This will cause problems for the inverse solution since e appears in the denominator of the equation. This situation is not unique to Macmillan's solution, and is true for all solutions which employ restitution and the solution is attempted via an inverse procedure.

To avoid this problem, small, non-zero values often can be used or the solution method can be iterated forward for differing input until the desired final velocities are obtained. The latter amounts to a trial and error procedure for matching final impact velocities. An even better approach now exists [21] which finds the pre-impact velocity components in one step for a given set of final velocities.

TABLE 4
Numerical Comparison between MacMillan and Brach

	Macmillan		Brach	
	Vehicle 1	Vehicle 2	Vehicle 1	Vehicle 2
Final x velocity	-0.90	-0.80	-0.89	-0.80
Final y velocity	-7.32	10.87	-7.31	10.91
Final angular velocity	-5.19	2.98	-5.19	2.98

Linear velocities in m/s, angular velocities in rad/s.

Macmillan presents an example problem in [14] which illustrates numerically the solution to his set of equations. A comparison was made of his solution and that using Brach's equations, with the moment coefficient set to +1 to eliminate the impulse moment. Table 4 presents a comparison of the post-impact velocities of these two techniques. A review of these results indicates that Macmillan's solution and Brach's solution yield nearly identical results when the restriction of zero moment impulse is imposed.

Newer models of vehicular collisions which employ impulse and momentum techniques are continually being developed. Recently, a model was proposed by Ronald Woolley of Collision Safety Engineering [4]. Woolley has implemented the solution of his equations on a computer and has entitled it "IMPAC" which is an acronym for Impact Momentum of a Planar Angled Collision. This acronym will be used when referring to his solution throughout this paper. IMPAC ultimately formulates a set of six equations and six unknowns which can be solved for the final three velocity components for each of the two vehicles involved. The first two equations are obtained from the conservation of linear momentum in two mutually perpendicular directions. The next two equations are obtained through a direct application of the principle of impulse and angular momentum for each of the two vehicles. The final two equations come by imposing constraints on the relative velocity of the two vehicles.

A common velocity constraint condition, as Woolley calls it in [4], imposes the condition on the collision that at some user designated point on the crush surface (i.e., the center of impact), the vehicles have the same velocity following the exchange of momentum. Woolley indicates that this is a consequence of the assumption that the collision is inelastic. The implication of an inelastic collision, however, should be that the normal velocity components of the vehicles be identical after the exchange of momentum. The tangential components of the velocities do not need to be identical for an inelastic collision. Nevertheless, the assumption that the velocity of the vehicles at the center of impulse be the same is consistent with observations for many collisions and is also employed by Limpert.

An alternative velocity constraint permitted by Woolley is the sideswipe constraint condition. This in fact, permits removal of the condition of common, final tangential velocities, and permits a relative tangential velocity called the slip velocity. It is given in the form of a percentage of the pre-impact approach velocity. This process is analogous to imposing a friction condition at the impact surface but requires the user to specify the final tangential velocity a priori. Both of these constraints, the common velocity constraint and sideswipe constraint, are combined with the assumption that only totally inelastic collisions would be considered. Hence we see that Woolley's solutions all correspond to $e = 0$.

Using data obtained in [15], a comparison with IMPAC was made between the predicted post-impact velocities for three RICSAC crashes [18]. The results are given in Table 5. Additional comparisons between the IMPAC and Brach's solution can be made by studying the energy loss for almost all of RICSAC collisions, shown in Table 6. Inspection of Tables 5 and 6 shows differences between the output of IMPAC and Brach's solution. The computed values of energy loss of Woolley and Brach agree quite closely. Where differences occur, they are due in part to the fact that Brach's model employs the moment coefficient whereas Woolley neglects this. Discrepancies may also be due to different choices for the location of the center of impact.

To make a direct comparison of the models themselves, RICSAC crashes 3, 9 and 10 were run using Brach's computer program with Woolley's dimensions for the location of the center of impact. In order to be consistent with the assumptions made in formulating the IMPAC model, e_m +1, $e = 0$, and $\mu = \mu_{max}$ were used. Table 7 illustrates that the IMPAC model and Brach's model will yield nearly identical results for the same collision if the same assumptions are made and the same data is used.

The principles of impulse and momentum do not offer a unique approach to the vehicle collision problem. A method named CRASH [6] enjoys a widespread acceptance as a model for the analysis of vehicle collisions. This is due partially to the fact that it was developed early on in this field and it was funded and used by the NHTSA of United States Department of Transporta-

Table 5
Numerical Comparison Between Brach and IMPAC
For RICSAC Collisions 3, 9,10

RICSAC Crash	Measured Vehicle 1	Measured Vehicle 2	Brach Vehicle 1	Brach Vehicle 2	IMPAC Vehicle 1	IMPAC Vehicle 2
#3						
Lin.Vel.	17.15/179.2°	23.17/170.7°	17.61/190.4°	22.40/167.0°	19.36/-176.0°	18.62/173.4°
Ang.Vel.	-0.26	0.0	-0.62	-2.57	0.44	0.02
#9						
Lin.Vel.	15.09/100.7°	26.15/112.3°	13.60/113.7°	27.96/115.0°	13.63/143.3°	28.60/108.7°
Ang.Vel.	-3.14	0.79	-3.35	1.60	-3.56	0.40
#10						
Lin.Vel.	28.63/100.2°	39.35/111.7°	25.10/112.0°	42.13/117.2°	28.6/146.1°	42.39/106.52°
Ang.Vel.	-5.24	1.26	-5.80	2.30	-6.04	1.71

Linear velocities in ft/sec, angular velocities in radians/sec.

Table 7
Numerical Comparison Between Brach and IMPAC
For Identical Center of Impact Location

RICSAC Crash	IMPAC Vehicle 1	IMPAC Vehicle 2	Brach Vehicle 1	Brach Vehicle 2
#3				
Lin.Vel.	19.36/-4.00°	18.63/6.62°	19.33/-3.95°	18.63/6.5°
Ang.Vel.	0.44	-0.02	0.41	-0.03
#9				
Lin.Vel.	13.64/36.71°	28.60/71.31°	13.44/48.77°	28.53/70.3°
Ang.Vel.	3.56	0.40	3.32	0.45
#10				
Lin.Vel.	28.60/33.87°	42.39/73.49°	27.86/39.6°	42.01/71.66°
Ang.Vel.	6.04	1.71	5.60	1.74

Linear velocities in ft/sec, angular velocities in radians/sec.

TABLE 6
Total Kinetic Energy Loss for Staged, RICSAC Collisions

RICSAC Collision	Percent of Initial Energy			
	Meas.[18]	IMPAC[15]	Brach	CRASH[13,18]
1	64.3%	57.0%	52.0%	192.0%
2		61.7%		78.8%
3	34.3%	37.6%	34.1%	6.0%
4	51.7%	38.3%	36.3%	21.7%
5	42.4%	34.3%	32.3%	13.8%
6	55.7%	53.8%	48.2%	83.0%
	55.3%	54.3%	48.8%	93.4%
8	53.6%	35.5%	36.0%	37.7%
9	38.5%	28.7%	28.8%	93.3%
10	38.7%	27.5%	31.0%	51.1%
11	94.1%	93.2%	92.2%	73.1%
12	90.7%	96.0%	93.3%	56.3%

tion. CRASH also includes a post-impact or spinout analysis.

CRASH, an acronym for Calspan Reconstruction of Accident Speeds on the Highway, was initially developed as an input program for a larger- analysis program, but soon gained popularity as a stand-alone collision model. CRASH was developed by the Calspan Corporation in the mid-1970's, and has since been updated to its latest version called CRASH3. However, the basic principles underlying the analysis have not changed.

A unique feature of CRASH is that the magnitude of the vector velocity change, "delta-V," can be computed from measurements of structural crush.The energy absorbed by the vehicles is determined by modeling the vehicle as a series of perfectly elastic springs which deform to the maximum level of crush but with no restitution or spring-back. The energy can then be calculated using the deformation measurements acquired from the accident vehicle itself. The stiffness of these springs is built into the computer program in the form of vehicle categories and is based upon experimental data. CRASH also uses some equations of impulse and momentum and the concept of a center of impact on the crush surface. It also imposes a common velocity constraint at this point in deriving the equations for the change in velocity of the vehicles. This common velocity implies that no restitution occurs and that relative slip between the vehicles ends prior to separation.

Once the energy absorbed by the vehicle due to the impact is known, the magnitude of delta-V for each vehicle can be calculated. To do this, CRASH requires that a PDOF, Principal Direction of Force, be specified (in fact, this should be the direction of the resultant linear impulse vector.) In an impulse-momentum model such as Brach's and Macmillan's, this direction is controlled by the friction coefficient, $\mu = P_t/P_n$. Crash thus uses $\mu = \mu_{max}$ to calculate the magnitude of ΔV and then allows the user to choose perhaps a different value of μ (through PDOF choice) to determine the vector values of ΔV. This is an inconsistency of which many users of CRASH are unaware. Nevertheless, Smith and Noga [16] indicate that the choice of the PDOF plays a dominant role in the accuracy of the solution. Pre-impact velocities can then be calculated if the post-impact velocities are known. These latter velocities are found by analyzing the post-impact dynamics, including the rest position, physical constraints such as pavement friction coefficients and accident scene information such as skidmarks.

Some improvements and extensions to CRASH have been made recently [23]. In addition, this version [23], has been adapted to a hand calculator.

CRASH has recently been the subject of two papers which have addressed its accuracy [16, 17]. These papers discuss in detail several topics which the authors felt can involve significant error or required additional investigation. It is not the intent of the present paper to delve into each of these topics in detail. Rather, several of the more apparent sources of possible error and inaccuracy will be discussed briefly and some additional observations will be made.

The damage basis method (use of structural crush) by CRASH for accident reconstruction analysis consists primarily of two operations: estimation of the energy absorbed by each of the vehicles during the collision and relating the crush energy to the changes in velocity of the vehicles. The basis for the model of CRASH damage energy is the assumption that the stiffness coefficients used can be established from barrier test data.

CRASH assigns these stiffness coefficients on a vehicle class basis, since this data is usually not available for individual vehicles. Aside from the obvious problem of uncertainty of the accuracy of CRASH's assigned coefficients to any particular vehicle (for all potential collision geometries), this data must be continually updated as new cars with new body structures are manufactured. This is a lengthy and costly process and hence updating of these coefficients has been performed relatively infrequently.

The accuracy of CRASH is dependent on the deformation of the vehicles since this, along with the stiffness coefficients, determines the energy which is absorbed by the vehicle during impact. This requires that the user ascertain the crush dimensions through measurement of the actual vehicle. Damage profiles are inherently highly irregular and subject to variable interpretation both in depth and length. Hence it is quite likely that two independent determinations of delta-V for the same accident will be different. Reference 18 attempts to quantify this sensitivity of CRASH to differences in the field data measurements.

The deformation profile used by CRASH to compute the energy absorbed by the vehicles is described using the damage dimensions which lie parallel to the longitudinal axis of the vehicle in the case of front or rear damage; or perpendicular to this axis in the case of side damage. This is consistent with the CRASH assumption that the residual crush provides a direct measure of the energy absorbed by the compressive forces created during the collision. Inherent in this statement is the fact that any additional work done by tangential shear forces at the impact surface provide no directly measurable damage evidence.

In many collisions, it is true that the shear forces will contribute little to the residual crush. This is certainly true for head on and rear end type collisions. However, it is quite common that the front end of the impacting vehicle in an intersection collision can exhibit substantial lateral deformation. The technique used by CRASH to quantify residual crush does not account for this deformation, and hence the energy absorbed by the vehicle due to intervehicular shear is not included in the calculation of the change in velocity.

The impulse-momentum models do not have any difficulty with energy loss due to shear and normal crush. Equations 17, 25 and 26 show how all of these are related for point mass collisions. In fact, the equation relating scalar ΔV's and energy loss for the general planar collision model can be shown to be:

$$V_1 = (1/m_1)\sqrt{\frac{2m(1+\mu^2)(1+e)qT_L}{(1-e)+2\mu r-\mu^2 r/\mu_{max}}} \qquad (27)$$

$$V_2 = m_1 V_1/m_2 \qquad (28)$$

and

$$1/q = 1 + md_a^2/I_2 + md_c^2/I_1 - \mu(md_c d_d/I_1 + md_a d_b/I_2) \qquad (29)$$

Refer to the Appendix for notation.

Though these equations are available, they are unnecessary since the ΔV values from the general planar model automatically include the shear and normal deformation energy losses.

One of the inadequacies of the CRASH formulation has been the fact that it has neglected the change in the angular velocity of the vehicles as a result of the collision. Recently however, Smith and Tsongas [13] reported that this quantity can be easily calculated once the change in linear velocity is computed. This change, if it is implemented, will allow for the evaluation of the effectiveness of CRASH to predict this change in velocity.

In [6], McHenry presents the formulation of the delta-V in terms of the vehicle parameters and the energy determined from residual crush. The result is a simplified form of Eq. 27. Little information is known as to how the simplifications affect the accuracy of CRASH.

Table 8 gives a comparison of the magnitude of the change in velocity for each vehicle predicted for four of the RICSAC crashes using the CRASH deformation model, the impulse-momentum models IMPAC and Brach, and the actual measured velocity changes. From Table 8 it can be seen that not all of the changes in velocity predicted by the models match well with the values measured at the time of the test. Woolley's and Brach's ΔV's are much closer to the measured values than those of CRASH. Among these four collisions, the largest deviation of CRASH is 19.2 ft/s, that of Woolley is 22.57 ft/s and that of Brach is 6.26 ft/s. In percentages, CRASH's largest deviation is 69%, Woolley's is 43% and Brach's is 18%.

This paper has illustrated the effectiveness of the application of the impulse-momentum models applied to vehicular collisions. Comparisons presented in this paper and in others [5, 10, 11] illustrate the accuracy with which these models can predict the changes in velocities and energy loss due to a collision.

Table 8
Comparison of Predicted Changes in Velocity
To Computed Changes in Velocity (Velocities in ft/s)

RICSAC Crash	Measured Veh. 1	Measured Veh. 2	Brach Veh. 1	Brach Veh. 2	IMPAC Veh. 1	IMPAC Veh. 2	CRASH Veh. 1	CRASH Veh. 2
#1								
Delta-V	18.49	22.84	15.11	22.65	16.73	24.96	27.13	40.63
#3								
Delta-V	13.94	23.17	14.13	22.41	11.9	18.62	4.55	7.19
#9								
Delta-V	31.93	12.06	28.50	13.12	21.62	9.94	28.01	12.91
#10								
Delta-V	52.05	19.09	45.79	22.37	29.48	14.41	32.85	15.99

Several versions of impulse-momentum models were reviewed. It is shown that the model developed by Brach is the most general and offers more versatility in the definition and use of several of the parameters used in modeling a collision. This versatility enables this model, with the proper restrictions, to duplicate the results of the models by Limpert, Macmillan and Woolley, but also allows for a more accurate modeling of accidents which do not adhere to the simplifying assumptions made in formulating these other models. In particular, it is well known that CRASH cannot be used for sideswipe collisions, a restriction not found in a general impulse-momentum model.

The deformation basis model employed by CRASH has come under some legitimate criticism [17], and it appears that some reexamination of the process in in order. This reassessment, such as [23], will only enhance CRASH's widespread popularity as a tool for evaluating vehicle collisions. The impulse-momentum techniques present an alternative to the deformation approach with added versatility accompanied with a potential for greater accuracy.

Note: The authors would like to express their appreciation to Dr. D. F. Schoeberle of the University of Illinois at Chicago for serving as advisor during the preparation of this paper.

REFERENCES

1. Ellis, J.R., Vehicle Dynamics, Business Books Ltd. London, 1969.

2. Martinez, J.E., "Computer Aided Automobile Accident Reconstruction -- Theory, Graphics, Animation," Paper No. 850250, SAE, Warrendale, PA 15096.

3. Limpert, R., "Motor Vehicle Accident Reconstruction and Cause Analysis," The Michie Company, 1978.

4. Woolley, R.L., "The 'IMPAC' Computer Program for Accident Reconstruction," Paper No. 850254, SAE, Warrendale, PA 15096.

5. Brach, R.M., "Impact Analysis of Two-Vehicle Collisions," Paper 830468, SAE, Warrendale, PA 15096.

6. McHenry, R.R., "A Comparison of Results Obtained With Different Analytical Techniques for Reconstruction of Highway Accidents," Paper No. 750893, SAE, Warrendale, PA 15096.

7. Hess, R.L., "Microcomputer Programs Useful for Aspects of Accident Reconstruction," Paper No. 800174, SAE, Warrendale, PA 15096.

8. Day, T.D. and Hargens,R.L., "Differences Between EDCRASH and CRASH3," Paper No. 850253, SAE, Warrendale, PA 15096.

9. Guenther, D.A. and Wiechel, J.F., "Damage Algorithm for Determination of Speed," Prcceedings of the 1986 ASME Conference on Computers in ME, Chicago, August, 1986.

10. Brach, R.M., "Friction, Restitution, and Energy LOSS in Planar Collisions," Journal of Applied Mechanics, Trans. of ASME, March, 1984.

11. Brach, R.M., "An Impact Coefficient for Vehicle Collision Analysis," Paper 770014, SAE,, Warrendale, PA 15096.

12. Brach, R.M., "Momentum and Energy Analysis of Automobile Collisions," Trans. Structural Impact and Crashworthiness, Elsevier Appl. Sci. Publ., 1984.

13. Smith, R. and Tsongas, N., "Crash Phase Accident Reconstruction," Paper No. 860209, SAE, Warrendale, PA 15096.

14. Macmillan, R.H., "Dynamics of Vehicle Collisions," Proceedings of the International Association for Vehicle Design, Inderscience Enterprises Ltd., 1983.

15. Personal correspondence with R.L. Woolley.

16. Smith, R.A. and Noga, J.T., "Accuracy and Sensitivity of CRASH," Paper No. 823,169, SAE, Warrendale, PA, 15096.

17. Woolley, R.L., Warner, C.Y. and Tagg, M.D., "Inaccuracies in the CRASH3 Program," Paper No. 850255, SAE, Warrendale, PA, 15096.

18. Jones, I.S. and Baum, A.S., "Research Impact for Computer Simulation of Automobile Collisions, Volume IV, Staged Collision Reconstructions," DOT HS-805040, December, 1978.

19. Emori, R.I., "Analytical Approach to Automobile Collisions," Paper 680016, SAE, Warrendale, PA 15096.

20. Emori, R.I., "Vehicle Mechanics of Intersection Collision Impact," Paper 700177, SAE, Warrendale, PA 15096.

21. Brach, Raymond M., "Least Squares Collision Reconstruction," SAE International. Congress, Detroit, February 1987.

22. Ishikawa, H., "Computer Simulation of Automobile Collision-Reconstruction of Accidents," Paper 851729, SAE, Warrendale, PA 15096.

23. Fonda, A.G., "CRASH Extended for a Handheld Computer," SAE International Congress, Detroit, February 1987.

APPENDIX - EQUATIONS OF IMPULSE/MOMENTUM MODEL

Conservation of momentum along the x axis:

$$m_2(V_{2x}-v_{2x}) + m_1(V_{1x}-v_{1x}) = 0$$

Conservation of momentum along the y axis:

$$m_2(V_{2y}-v_{2y}) + m_1(V_{1y}-v_{1y}) = 0$$

Conservation of angular momentum:

$$I_2(\Omega_2-\omega_2) + I_1(\Omega_1-\omega_1) + m_2(d_a+d_c)(V_{2x}-v_{2x}) + m_1(d_b+d_d)(V_{1y}-v_{1y})=0$$

Restitution normal to the crush line at angle Γ:

$$(V_{1y}-d_d\Omega_1-V_{2y}-d_b\Omega_2)\sin\Gamma + (V_{1x}+d_c\Omega_1-V_{2x}+d_a\Omega_2)\cos\Gamma = -e[(v_{1y}-d_d\omega_1-v_{2y}-d_b\omega_2)\sin\Gamma + (v_{1x}+d_c\omega_1-v_{2x}+d_a\omega_2)\cos\Gamma$$

Friction along the crush line at angle Γ:

$$m_1(V_{1y}-v_{1y})(\cos\Gamma-\mu\sin\Gamma) + m_2(V_{2x}-v_{2x})(\sin\Gamma+\mu\cos\Gamma) = 0$$

Moment restitution at impact surface:

$$(\Omega_2-\Omega_1)(1-e_m) = e_m[(\Omega_1-\omega_1) - m_1d_c(V_{1x}-v_{1x})/I_1 + m_1d_d(V_{1y}-v_{1y})/I_1 - (\Omega_2-\omega_2) - m_2d_a(V_{2x}-v_{2x})/I_2 + m_2d_b(V_{2y}-v_{2y})/I_2]$$

In the above:

$$d_a = d_2\sin(\theta_2+\phi_2) \quad d_b = d_2\cos(\theta_2+\phi_2)$$

$$d_c = d_1\sin(\theta_1+\phi_1) \quad d_d = d_1\cos(\theta_1+\phi_1)$$

NOTATION

e coefficient of restitution

e_m moment coefficient of restitution

d distance between mass center and impact center

I vehicle yaw inertia about its mass center

m mass of vehicle

T kinetic energy

V, v velocity

μ equivalent coefficient of friction along the impact surface

θ heading angle of vehicles relative to the x axis

Γ angle of impact surface relative to the y axis

Ω, ω angular velocity

ϕ angle between the length axis of a vehicle and a line between its center of gravity and the center of impact