870429

Least Squares Collision Reconstruction

Raymond M. Brach
Aerospace & Mechanical Engineering
University of Notre Dame
Notre Dame, IN

ABSTRACT

A new method is described and illustrated which solves the planar, two vehicle collision reconstruction problem. The method, called LESCOR (LEast Square COllision Reconstruction), determines the initial velocity components when given: (1) final velocity components, (2) vehicle physical data, (3) damage geometry, (4) collision geometry and (5) the impact coefficients (restitution and friction). A novel feature is that if the impact coefficients are unknown but some of the initial velocity data is known (such as zero initial yaw rates and vehicle headings), the method will find the remaining initial velocities and the unknown coefficients. Using a six equation impact model and the method of least squares, LESCOR calculates any combination of 6 or less unknown initial velocity components and impact coefficients.

Five example collision reconstructions are presented based on RICSAC collisions and a field example. The method has provided results which range from good to excellent and is superior to trial and error methods used in the past.

ACCIDENTIAL COLLISIONS OF VEHICLES with stationary objects and other vehicles are always subject to questions such as how and why they occurred. The questions arise from the inquisitive to official investigators representing police, insurance companies, law firms and safety agencies. When conditions warrant a formal investigation an "accident reconstruction" is carried out. The major components of a reconstruction include some or all of the following:

1. Review of documented information
    a. witness statements
    b. police reports
    c. medical reports
    d. photographs
2. Gathering of physical data and information
    a. site examination and measurements
    b. vehicle examination and measurements
    c. traffic control system evaluation
    d. experimentation and simulation
3. Mannual and/or computerized calculations
    a. vehicle dynamics
    b. traffic dynamics
    c. occupant and/or pedestrian dynamics
4. Presentation of results
    a. written or oral reports
    b. physical models
    c. audio-visual simulations and/or video animation

This paper deals with the vehicle dynamics part of a reconstruction, specifically the determination of velocity changes which occur during the time interval when the vehicles are in contact. This is referred to as the collision phase in distinction to the preimpact and post impact phases. The work concentrates on a single, planar impact between two (non articulated) vehicles. If multiple impacts occur, the method can be used repeatedly to study each in a sequential fashion. If the vehicles are articulated, a different impact model must be used (11).

In a collision, just prior to contact, each vehicle has three velocity components, normal, tangential and rotational. The same number exists at separation; for 2 vehicles this leads to a total of six initial velocity components and six final. Various methods exist for relating the initial and final velocities of two vehicles colliding in a plane (1,2,3,4,5,6). A review of some of these is available (7). All use the concepts of impulse and momentum from Newton's

TABLE 1
DATA CATEGORIES
FOR LESCOR,
LEast Squares COllision Reconstruction

DATA	STATUS
1. Three initial velocity components for each vehicle.	At least 2 of the 6 must be unknowns to be determined.* Known values are simply specified.
2. Three final velocity components for each vehicle.	All 6 known; estimated from prior information.
3. Impact coefficients: e, coeff of restitution e_m, moment coeff mu, coeff of friction	May be known or unknown.* If unknown, they are determined; if known, they must be specified.
4. Vehicles physical properties; weights, inertias and dimensions.	All quantities known.
5. Configuration of vehicles during contact; relative angles and damaged contact surfaces.	All quantities known.

---

* The total number of unknowns cannot exceed 6.

laws of physics; some also take into account energy lost through physical deformations (4,5). A mathematical model consisting of 6 linear equations (6) which relates the initial and final velocity components is used here as the basis of the collision reconstruction method.

It is presumed that a reconstruction of the post impact phase of the vehicle dynamics, witness statements, scenario evaluation, etc. has provided values of the 6 velocity components at separation, ie, the final impact velocities. It is further presumed that the 6 initial velocity components are to be computed. Other information is needed and also presumed known. This includes all of the vehicle parameters such as dimensions, weight, inertia, etc. It also includes the orientation of the vehicles throughout contact and the damage geometry.

The method to be presented uses the well known method of least squares to find a combination of unknown initial velocities and impact coefficients such that the 6 impact equations are satisfied and the specified final velocity components are closely matched (in a least squares sense). Results are provided from a computerized implementation of the least-square procedure. Throughout this paper, the method being developed will be referred to by the term LESCOR, an acronym from the paper's title.

IMPACT MECHANICS AND COEFFICIENTS

All real collisions are accompanied by a loss of kinetic energy. Vehicle collisions are no exception with typical values ranging at least from 25% to 95% (8). Most is lost through metal deformation, friction, and vibrational energy. Impact models represent energy loss through the use of cofficients and/or velocity constraints. The impact model used here has 3 coefficients, the coefficient of restitution e, an equivalent friction coefficient, μ, and a moment, or rotational, restitution coefficient, e_m. The coefficient of restitution is the classical coefficient encountered in elementary dynamics texts and is used to model energy loss due to material deformation in a mode normal, or perpendicular, to the crush surface. The equivalent friction coefficient is in reality the ratio of the tangential to normal impulse components which develop between the vehicles. The tangential impulse (parallel to the crush surface) is typically attributed to and referred to as friction, though shear deformation is probably equally significant. Fortunately, the model provides correct results for the proper value of μ, regardless of the origin or nature of the tangential forces. The third coefficient, e_m, is a restitution coefficient governing rotational effects (6). Except for some special cases (for example when the two vehicles "attach" and rotate with a common final angular velocity), a value of 1 for this coefficient seems to be appropriate (although more study of this coefficient is being carried out). A value of $e_m = 1$ means that no moment impulse is developed between the vehicles during the collision and that the center of impact is known exactly; otherwise, $-1 < e_m < 0$.

The model equations, their derivation and a description of the notation is given in previous paper (6). Note a sign error in eq. 24 of Ref 6; a "+" should replace the minus immediately to the right of the equal sign.

FORMULATION OF THE LEAST SQUARES PROBLEM

Table 1 illustrates the categories of data necessary to carry out a collision reconstruction. Other than the known data (items 4 and 5) the six final velocity component values provide a starting point for LESCOR. The 3 coefficients, e, e_m and μ, enter next and would be found from experimental data (8), through experience and/or chosen for scenario evaluation. From these nine quantities, a set of 6 or fewer is selected as unknown. (This is discussed more, in the example applications later in this paper). Collectively, this group of data may or may not satisfy conservation of momentum, friction laws, etc. That is, a set of 6 initial velocities combined with the coefficients chosen may not exist for which the impact model will give the final velocity component values exactly as specified. On the other hand, some nearby set of values may. It is this nearby set of final velocities which is provided by LESCOR along with the corresponding unknown initial velocities and coefficients.

Suppose, however, a set of initial velocities and impact coefficients does exist which corresponds to an exact solution of the impact equations for the given set of final velocities. It is possible in many cases to simply solve the impact equations "backward", for these initial velocities. This is what can be called an inverse solution. This would be preferable to the least squares method to be developed here. But an inverse solution is not always possible. Mathematically, one does not exist when the coefficient of restitution, e=0. Secondly, in practice we do not always know what the appropriate impact coefficient values are for a given collision. Consequently, the least squares approach (or other, such as trial and error) is required.

For LESCOR the specified final velocity values are treated as estimates and a sum of squares is defined as

$$Q = \sum_{i=1}^{6} \sum_{j=1}^{n_i} w_i (V_i - V_{ij})^2 = \sum_{i=1}^{6} \sum_{j=1}^{n_i} \Phi_{ij}^2 \qquad (1)$$

V_i represents the ith of 6 final velocity components which satisfy the impact equations, V_{ij} is one of n_i estimates of V_i (multiple estimates are permitted, though typically n_i=1). A weighting factor w_i is used.

As seen in the list of notation the velocity variables are coded. For example $V_1 = V_{1x}$, $V_2 = V_{1y}$ and $V_5 = \Omega_1$. Since the units of translational and rotational velocities differ, the weighting factors w_i are used to appropriately bring each term of Q to a common dimensional basis. A convenient scheme is to choose weights such that each term of Q represents an expression of kinetic energy. Thus for translational velocities w_i should be the equivalent of mass and for rotational velocities, w_i should have a value representative of a moment of inertia. The same can be accomplished by letting w_i=1 for translational and w_i=I/m for rotational velocities. I/m for many automobiles is of the order of magnitude of 25; this is used currently by LESCOR.

The approach followed by LESCOR is to find the unknown initial velocity components and unknown impact coefficients and a set of final velocities which minimize Q and for which all of the data satisfies Newton's laws, namely, the 6 impact equations. Problem complexity requires the use of a digital computer. A computer program was written in IBM PC BASIC. It is outlined in Fig. 1 with a brief description as follows.

The impact equations are used to solve for a starting set of intial velocities for the given final velocities and all other data. The coefficients of restitution, e and e_m, are adjusted so the starting initial velocities can be calculated with an inverse solution of the impact equations. A well known iterative algorithm exists for solving least squares problems (9). Known as Gauss' formula, it can be written for the kth iteration as

$$u_{k+1} = u_k + (JJ')^{-1} J\Phi \qquad (2)$$

where u_{k+1} is a vector of new values of the unknowns, u_k is the current set of values, J is a matrix of derivatives and Φ is a vector of values defined in Eq. 1. The derivatives are found numerically by repeated use of the impact model. With new values of unknowns, the impact equations can be solved for a new set of final velocities; then Eq. 1 provides a new value of Q, the sum of squares. If the process converges, the new set of values of the unknowns yields a smaller value of Q. The process is repeated until Q becomes as close to zero as possible for the particular collision being analyzed.

EXAMPLE CALCULATIONS AND RESULTS

The National Highway Traffic Safety Administration conducted a series of staged collisions (10) which are referred to by the acronym RICSAC. Data from two of these are used to demonstrate LESCOR. Of course all of the initial and final velocites of these collisions are known by measurement. The coefficients are known by previous analysis (8). To demonstrate LESCOR solutions, various combinations of values of initial velocites and coefficients will be intentionally treated as unknown. The values found by LESCOR will then be compared to the already known experimental values.

An additional example is presented using a sideswipe type collision encountered in practice. Although the true initial velocities are not known, the LESCOR solution is compared to an earlier, independent trial-and-error solution.

TABLE 2
DATA FOR TWO EXAMPLE RICSAC COLLISIONS

RICSAC #9

```
RICSAC #9; All Units in ft-lb-sec System


    Restitution Coeff, E = 0.400       Moment Coeff, Em =  1.000
    Friction Coeff, Mu = 0.501         Crush Angle, Gamma=   0.0
    Friction Coeff, MUo =  0.501 (No Sliding at Separation)

*         VEHICLE 1                *              VEHICLE 2              *

Mass =  70.1  Inertia =  976.0         Mass = 152.2  Inertia = 3953.3
Phi =   6.0      Theta =    0.0        Phi = -29.7        Theta =   90.0
D1 =  4.80                                                  D2 =   5.60

v(x) = -31.09    v(n) = -31.09         v(x) =    0.00     v(n) =    0.00
v(y) =   0.00    v(t) =   0.00         v(y) =   31.09     v(t) =   31.09
w =   0.00                                                   w =    0.00

V(x) =  -5.37    V(n) =  -5.37         V(x) = -11.85      V(n) = -11.85
V(y) =  12.89    V(t) =  12.89         V(y) =  25.15      V(t) =  25.15
W =  -3.49                                                   W =    1.58
                         Final  Velocities

Delta V = 28.77                                      Delta V = 13.25

   Kinetic Energy                                Kinetic Energy
  Initial      Final                           Initial      Final
0.3387E+05  0.6829E+04       <trans'1>       0.7354E+05  0.5882E+05
0.0000E+00  0.5946E+04       <rotan'1>       0.0000E+00  0.4961E+04
0.3387E+05  0.1277E+05        <total>        0.7354E+05  0.6378E+05

Initial System Energy = 1.0741E+05         Crush Energy Loss =  15.7%
System Energy Loss (MUo) =  28.7%     Frictional Energy Loss =  13.1%
Moment Crush Energy Loss =   0.0%         System Energy Loss =  28.7%
*                                 *                                 *


   Impulse, P(x) = 1802.7              Normal, P(n) = 1802.7
   Impulse, P(y) =  903.1          Tangential, P(t) =  903.1
   Resul't Impulse, P = 2016.3     Moment Impulse, M =    0.0
```

RICSAC #4

```
RICSAC #4; All Units in ft-lb-sec System


    Restitution Coeff, E = 0.045         Moment Coeff, Em = -0.517
    Friction Coeff, Mu =-0.042           Crush Angle, Gamma= -10.0
    Friction Coeff, MUo = -0.042 (No Sliding at Separation)

*         VEHICLE 1                *              VEHICLE 2              *

Mass = 154.7  Inertia = 4017.9          Mass =  99.1  Inertia = 2031.8
Phi = -18.2      Theta =    0.0         Phi = 171.7       Theta = 170.0
D1 =  8.02                                                  D2 =   6.94

v(x) = -56.76    v(n) = -55.90          v(x) =    0.00     v(n) =   0.00
v(y) =   0.00    v(t) =  -9.86          v(y) =    0.00     v(t) =   0.00
w =   0.00                                                    w =   0.00

V(x) = -35.14    V(n) = -33.78          V(x) = -33.74      V(n) = -34.52
V(y) =  -4.76    V(t) = -10.79          V(y) =   7.43      V(t) =   1.45
W =  -0.95                                                    W =  -0.69
                            Final  Velocities

Delta V = 22.14                                          Delta V = 34.55

   Kinetic Energy                                   Kinetic Energy
  Initial      Final                              Initial      Final
0.2492E+06  0.9724E+05         <trans'1>       0.0000E+00  0.5916E+05
0.0000E+00  0.1811E+04         <rotan'1>       0.0000E+00  0.4773E+03
0.2492E+06  0.9905E+05          <total>        0.0000E+00  0.5964E+05

Initial System Energy = 2.4917E+05          Crush Energy Loss =  36.7%
System Energy Loss (MUo) =  36.3%      Frictional Energy Loss =  -0.3%
Moment Crush Energy Loss =   0.2%          System Energy Loss =  36.3%
*                                  *                                *


   Impulse, P(x) = 3344.6               Normal, P(n) = 3421.6
   Impulse, P(y) = -736.2           Tangential, P(t) = -144.2
   Resul't Impulse, P = 3424.6      Moment Impulse, M = 1045.2
```

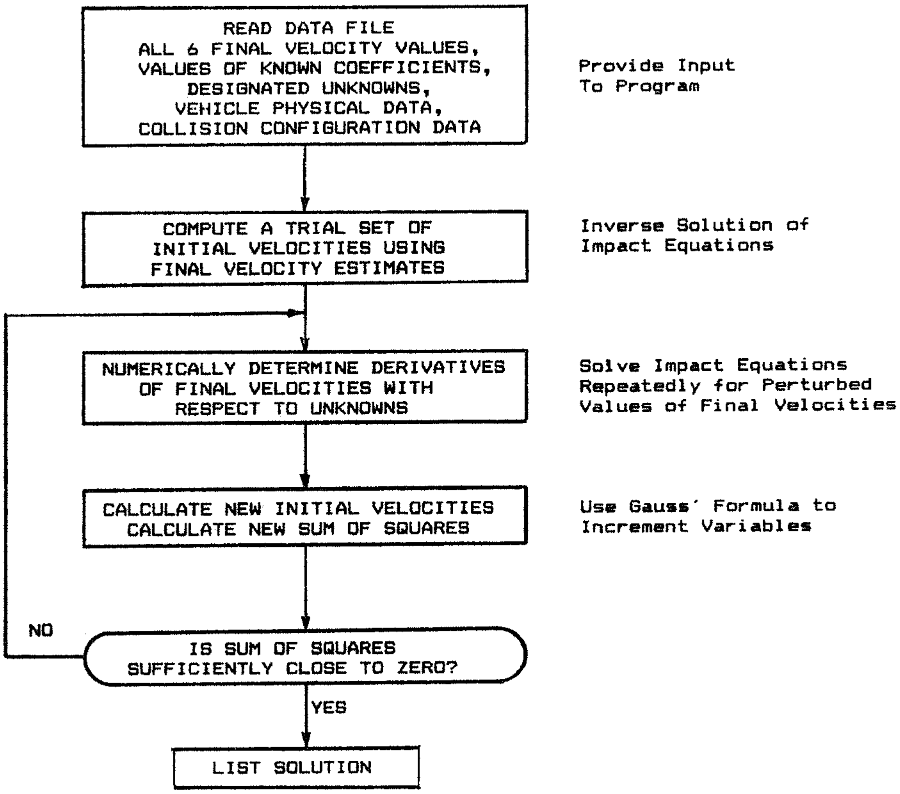

FIGURE 1
FLOW CHART FOR LESCOR
COMPUTER PROGRAM

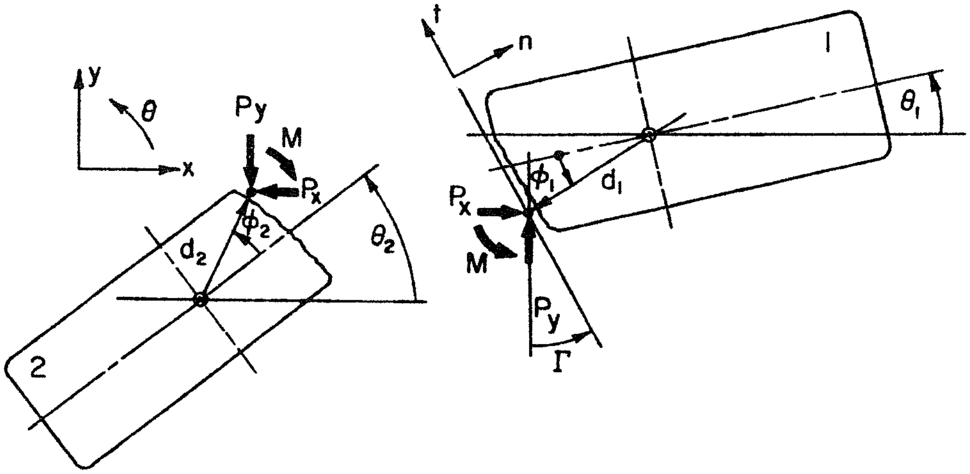

FIGURE 2
COORDINATES AND DIMENSIONS,
IMPACT MODEL

All example collisions will be discussed relative to the coordinate system and variables displayed in Fig. 2. Although the values of the variables in this figure are arbitrary, one can "visualize" that it shows free body diagrams of a near "head-on" collision with vehicle 1 travelling intially from above right to left (negative x and y velocity components) and vehicle 2 travelling from below left toward the upper right (both initial velocity components are positive). Heading angles θ_1, and θ_2 and impact point angles ϕ_1, and ϕ_2 are referred to the zero positions shown in the figure. The normal and tangential axes n,t, located relative to the x, y axes by the angle Γ, define the normal or crush direction and the tangential or friction direction. These must be established from the damage patterns of the vehicles. Table 2 shows the data from an analysis (8) of RICSAC Collisions 4 and 9. RICSAC 9 is a 90° front-to-side intersection collision of a Honda (vehicle 1) and a Ford Torino (vehicle 2). RICSAC 4 is a 10° front-to rear collision of the same types of vehicles but numbered in reverse, Torino (vehicle 1) and Honda (vehicle 2). Two variations of each collision are analyzed.

The RICSAC collisions will be used in the following way. The vehicles' measured final velocities provide input to LESCOR, as well as all of the vehicle and collision information. Two combinations of unknown initial velocities and impact coefficients will be chosen for each collision. The results of the reconstruction will then be compared to the true values.

RICSAC NO. 9 - Table 3 lists the conditions chosen for scenario 9A which represents a typical reconstruction of an intersection collision. The vehicles' forward speeds are presumed unknown as are the friction and restitution coefficents. It is further assumed that no moment impulse exists over the collision surface, ie, e_m is known to be +1. After 5 iterations, the sum of squares is reduced to 1.1×10^{-3} and the initial velocities, final velocities and coefficients found by LESCOR are those listed in Table 3. Comparison of Table 2 and 3 shows that all velocities and coefficients are found almost exactly.

Table 3 also lists the conditions for scenerio, 9B. Here, an analyst is presumed to be uncertain if either car was or was not spinning prior to the collision (that is $\omega_1 \neq 0$ and $\omega_2 \neq 0$, necessarily). However the analyst's experience with this type of collisions indiciates that e = 0.4 and that relative sliding of the two vehicles ceases prior to their separation. The coefficient μ necessary to bring this about is not known, but the condition of no sliding can be imposed by LESCOR when requested. In this scenario, 5 unknowns exists, v_{1x}, v_{2y}, ω_1, ω_2 and μ. Again, no moment impulse is assumed over the crush surface. Comparison of Tables 2 and 3 show that the least square collision reconstruction again provides almost exact results including a value of μ=.501. It correctly determines that the initial angular velocities ω_1 and ω_2 were both zero.

RICSAC No. 4 - Table 4 lists the results for Scenario 4A. In this example it is assumed to be known that the Torino struck the Honda at a 10° angle from behind. The initial forward speeds are unknown but the angular velocities are known to be zero and the directions of travel are known. Again no moment impulse is assumed and the restitution coefficient e and the impulse ratio μ are treated as unknown. Table 4 shows the results of the LESCOR reconstruction after 3 iterations. It indicates an initial Torino speed of 54.59 ft/sec compared to 56.76 ft/sec. (Table 2). Vehicle 2, the Honda which was actually at rest, is given an initial velocity by LESCOR of 4.04 ft/sec in the same direction of vehicle 1. The values of e and μ from LESCOR are both zero whereas they should be 0.045 and 0.042 respectively. Other differences exist, but in general the results appear acceptable.

In scenario 4B, the forward speeds are treated as the only unknown velocity components. A moment impulse is permitted over the crush surface with the corresponding coefficient e_m

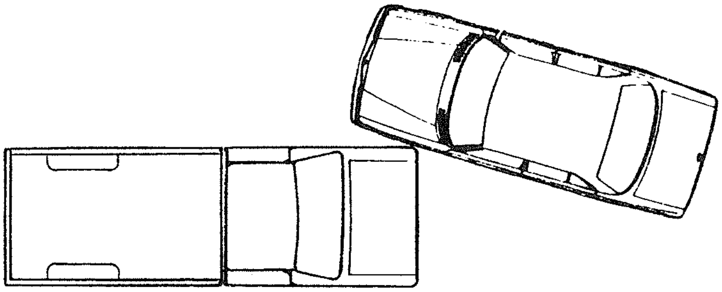

FIGURE 3
VEHICLE ORIENTATION,
SIDESWIPE COLLISION

TABLE 3
LESCOR RESULTS FOR RICSAC #9

SCENARIO 9A

```
Results After 5 Iterations

Final Sum of Squares:      1.1059D-03

Coefficients E  Em & Mu:   0.400    1.000    0.502

v1x, v1y, w1:       -31.07      0.00       0.00
                                                 Initial Velocities
v2x, v2y, w2:         0.00     31.08       0.00

V1x, V1y, W1:        -5.37     12.90      -3.50
                                                 Final Velocities
V2x, V2y, W2:       -11.84     25.14       1.58

Kinetic Energy (Init, Final & Loss): 1.07D05  7.65D04  28.7%

LESCOR 7/86 RMB
```

SCENARIO 9B

```
Results After 1 Iteration

Final Sum of Squares:      3.2930D-04

Coefficients E  Em & Mu:   0.400    1.000    0.501   (MUmax)

v1x, v1y, w1:       -31.08      0.00       0.00
                                                 Initial Velocities
v2x, v2y, w2:         0.00     31.09      -0.00

V1x, V1y, W1:        -5.36     12.89      -3.49
                                                 Final Velocities
V2x, V2y, W2:       -11.85     25.15       1.58

Kinetic Energy (Init, Final & Loss): 1.07D05  7.65D04  28.7%

LESCOR 7/86 RMB
```

TABLE 4
LESCOR RESULTS FOR RICSAC #4

SCENARIO 4A

```
Results After 2 Iterations

Final Sum of Squares:      1.5830D+01

Coefficients E  Em & Mu:   0.000    1.000    0.000

v1x, v1y, w1:       -54.59      0.00       0.00
                                                 Initial Velocities
v2x, v2y, w2:        -4.04      0.00       0.00

V1x, V1y, W1:       -36.34     -3.22      -0.82
                                                 Final Velocities
V2x, V2y, W2:       -32.51      5.02      -1.41

Kinetic Energy (Init, Final & Loss): 2.3127D05  1.60D05  30.8%

LESCOR 7/86 RMB
```

SCENARIO 4B

```
Results After 2 Iterations

Final Sum of Squares:      2.8990D+00

Coefficients E  Em & Mu:   0.045    0.000   -0.045   (MUmax)

v1x, v1y, w1:       -56.76      0.00       0.00
                                                 Initial Velocities
v2x, v2y, w2:         0.00      0.00       0.00

V1x, V1y, W1:       -35.15     -4.82      -0.84
                                                 Final Velocities
V2x, V2y, W2:       -33.72      7.52      -0.84

Kinetic Energy (Init, Final & Loss): 2.49D05  1.59D05  36.3%

LESCOR 7/86 RMB
```

unknown. In addition e is unknown and μ is unknown but transverse sliding of the vehicles over each other ceases prior to separation. This constitutes a reconstruction with 5 unknowns. Under these conditions, Table 4 shows that the LESCOR results are better than secenario 4A for the forward velocities. It provides the exact results for both forward speeds. The values of e and μ are very close to the RICSAC analysis. The fitting of e_m was not good however; LESCOR gave zero (both vehicles with the same final velocity) but should have been -0.517.

SIDESWIPE COLLISION, FIELD EXAMPLE - The last example is one taken from an actual collision. As such, the "true" initial and final velocities, collision orientation, etc. are unknown and no judgment can be made as to how good the reconstruction is. The utility of this example is that an earlier reconstruction was done by trial and error and provided a set of initial velocities which matched the final velocity components. Application of LESCOR (not available when the problem was solved earlier) yielded a quite different solution. This points out that different analysts can end up with different solutions to the same reconstruction problem; the least squares solution is always unique, however.

Fig. 3 shows the reconstructed orientation of the two vehicles at the time of impact initiation. Table 5 shows the full collision analysis arrived at by trial and error. The least squares reconstruction will assume no moment impulse (e_m = +1) and that the vehicle headings are as shown in Fig. 3. The unknowns are the forward velocity of each vehicle, the angular velocities of each vehicle, the coefficient of restitution and the coefficient of friction, 6 in all. An assumption is not made that sliding terminates prior to separation since this is not appropriate for a sideswipe collision. Table 6 lists the LESCOR initial velocites and coefficients which correspond to the final velocities from Table 5. A noticible difference exists. The new forward velocity of vehicle 1 is approximately 4.0 (v_{1x} = 3.85, v_{1y} = 1.03) ft/s, whereas earlier it was 15.0 (v_{1x} = -15.0, v_{1y} = 2.0) ft/s. For vehicle 2, instead of the earlier 70.0 (v_{2x} = 70.0, v_{2y} = 0.0) ft/s the LESCOR solution yielded 59.86 (v_{2x} = 59.86, v_{2y}=0.0) ft/s. These changes are likely due to the LESCOR value of μ= -.554 compared to the trial-and-error choice of -1.500. The value of e found in the least square solution is the same as the value selected for use in the earlier, trial-and-error reconstruction.

TABLE 5
TRIAL-AND-ERROR RECONSTRUCTION
A SIDESWIPE COLLISION

```
Sideswipe Type Collision, Field Example


   Restitution Coeff, E = 0.050       Moment Coeff, Em =  1.000
   Friction Coeff, Mu =-1.500         Crush Angle, Gamma=  75.0
   Friction Coeff, MUo = -2.440 (No Sliding at Separation)


*          VEHICLE 1                *             VEHICLE 2           *

Mass = 127.3  Inertia = 3310.0         Mass = 140.0  Inertia = 3400.0
Phi = 115.0       Theta = -15.0        Phi =  20.0       Theta =   0.0
D1 =  2.50                                                  D2 =  6.50

v(x) = -15.00     v(n) =  -1.95        v(x) =  70.00     v(n) =  18.12
v(y) =   2.00     v(t) =  15.01        v(y) =   0.00     v(t) = -67.61
w =  -0.05                                                  w =  -0.10

V(x) =   2.24     V(n) =   8.15        V(x) =  54.32     V(n) =   8.94
V(y) =   7.83     V(t) =  -0.14        V(y) =  -5.30     V(t) = -53.84
W =   1.68                                                  W =   0.00
                             Final  Velocities

Delta V = 18.20                                        Delta V = 16.55

   Kinetic Energy                                  Kinetic Energy
  Initial      Final                             Initial      Final
0.1458E+05  0.4226E+04       <trans'l>         0.3430E+06  0.2085E+06
0.4137E+01  0.4672E+04       <rotan'l>         0.1700E+02  0.2780E-02
0.1458E+05  0.8898E+04        <total>          0.3430E+06  0.2085E+06

Initial System Energy = 3.5760E+05        Crush Energy Loss =   3.3%
System Energy Loss (MUo) =  56.9%    Frictional Energy Loss =  35.9%
Moment Crush Energy Loss =   0.0%        System Energy Loss =  39.2%
*                                   *                                 *



   Impulse, P(x) = 2195.1                Normal, P(n) = 1285.4
   Impulse, P(y) =  742.6            Tangential, P(t) =-1928.1
   Resul't Impulse, P = 2317.3      Moment Impulse, M =    0.0
```

TABLE 6
LESCOR RESULTS FOR A SIDESWIPE COLLISION

```
Results After 3 Iterations


Final Sum of Squares:      3.1352D-03

Coefficients E  Em & Mu:    0.050    1.000  -0.554

v1x, v1y, w1:        -3.85      1.03      0.94
                                                Initial Velocities
v2x, v2y, w2:        59.86      0.00      0.89


V1x, V1y, W1:         2.25      7.87      1.68
                                               Final Velocities
V2x, V2y, W2:        54.31     -5.28      0.01


Kinetic Energy (Init, Final & Loss):   2.5464D+05  2.1736D+05  14.6%

LESCOR 7/86 RMB
```

COMMENTS AND CONCLUSIONS

Over 15 sample solutions of LESCOR were run, also based upon RICSAC collisions other than those presented in this paper. Only one did not compare well; that example had 6 unknowns. All other cases converged accurately and rapidly (5 or fewer iterations).

Gauss' iteration procedure was chosen after first trying a steepest descent method. The latter was very slow, sometimes requiring 20 or more minutes to converge on an IBM PCAT. Three to five iterations with Gauss' method takes only a few minutes on an IBM PC. A starting solution is required for each reconstruction. An algorithm was developed using most of the input data and using an inverse solution of the impact equations. Since final velocities must all be specified, the starting, inverse solution algorithm chooses coefficients which guarantee a reasonable inverse solution. This procedure could probably be improved.

There are several reasons why the least square collision reconstruction approach is an important tool for accident reconstruction purposes.

1. The impact model is general enough to be used for all types of collision configurations (head-on, sideswipe, etc).
2. Typically it is unnecessary to choose or estimate appropriate values of the impact coefficients. However, if known, the coefficients can be specified.
3. Though all judgement is not eliminated, the results of the least square reconstruction are much less subjective than trial-and-error methods.
4. The least square method is much quicker than trial and error methods.
5. It makes efficient use of typically known preimpact vehicle motion (no yaw spin, known headings, etc.)
6. LESCOR appears to yield quite accurate results.

NOTATION

d_1, d_2	distance between mass center and crush center
e	coefficient of restitution
e_m	moment coefficient of restitution
I_1, I_2	vehicle yaw inertia about its mass center
J	Jacobian matrix; matrix of derivatives of final velocities with respect to the unknowns
m_1, m_2	mass of vehicles
n_i	number of estimates available for the ith final velocity component
Q	total sum of squares
V	final velocity component
u	unknown variables in reconstruction problem
v	initial velocity component
w_i	weighting factor to dimensionalize the sum of squares uniformly
x, y	coordinates: x,y are fixed at the scene
n, t	coordinates; n,t are normal and tangential to crush surface
μ	ratio of tangential to normal impulses (equivalent coefficient of friction)
θ	heading angle of vehicles relative to the x axis
Γ	angle of impact surface relative to the y axis
Ω	final angular velocity
ω	initial angular velocity
Φ_i	weighted deviation between final velocity estimate and least square estimate
ϕ	angle between the length axis of vehicle and a line between its center of gravity and the center of impact

NOTE: To maintain a single subscript notation in Q, the following coding has been used

$V_1=V_{1x}$, $V_2=V_{1y}$, $V_3=V_{2x}$, $V_4=V_{2y}$, $V_5=\Omega$, and $V_6=\Omega_2$

REFERENCES

1. Limpert, R., Motor Vehicle Accident Reconstruction and Cause Analysis, The Michie Co., 1978.
2. Macmillan, R.H., Dynamics of Vehicle Collisions, Inderscience, U.K., 1983.
3. Woolley, R., "The IMPAC Computer Program for Accident Reconstruction", SAE 850254, Feb. 1985.
4. Ishikawa, H.,"Computer Simulation of Automobile Collision - Reconstruction of Accidents", SAE 851729, Feb. 1985.
5. Smith, R.A., and Tsongas, N.G., "Crash Phase Accident Reconstruction", SAE 860209, Feb. 1986.
6. Brach, Raymond M., "An Impact Moment Coefficient for Vehicle Analysis", Paper No. 770014, SAE International Congress, Detroit, 1977.
7. Brach, Raymond, M., and Brach, R. Matthew, "A Review of ΔV Impact Models for Vehicle Collision Analysis", 1987 SAE International Congress, Detroit, 1987.
8. Brach, Raymond M., "Impact Analysis of Two-Vehicle Collisions", Paper No. 830468, SAE International Congress, Detroit, 1983.
9. Beightler, Phillips and Wilde, Foundations of Optimization, Prentice-Hall, NJ, 1979.
10. Jones, I.S. and Baum, A.S., "Research Input for Computer Simulation of Automobile Collisions, Volume IV: Staged Collision Reconstructions", DOT HS-805 040, Final Report, December, 1978.
11. Brach, Raymond M., "Impact of Articulated Vehicles", Paper No. 8600015, SAE International Congress, Detroit, 1986.