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Crush Energy and Planar Impact Mechanics for Accident Reconstruction

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ABSTRACT

The algorithm used in the third version of the Calspan Reconstruction of Accident Speeds on the Highway (CRASH3) and planar impact mechanics are both used to calculate energy loss and velocity changes of vehicle collisions. They (intentionally) solve the vehicle collision problem using completely different approaches, however, they should produce comparable results. One of the differences is that CRASH3 uses a correction factor for estimating the collision energy loss due to tangential effects whereas planar impact mechanics uses a common velocity condition in the tangential direction. In this paper, a comparison is made between how CRASH3 computes the energy loss of a collision and how this same energy loss is determined by planar impact mechanics. The main factors that control energy loss as calculated by CRASH3 are the determination of the PDOF (principal direction of force), definition of a common impact point of the two vehicles, the common normal velocity condition and the tangential correction factor. In the planar impact mechanics solution, the controlling factors are the definition of a crush surface, definition of a common impact point, common velocity conditions and the values of normal and tangential coefficients. Experimental collisions (RICSAC, Research Input for Computer Simulation of Automobile Collisions) are used to provide a basis for comparison. A method is proposed that exploits the features of both methods for vehicle accident reconstructions.

INTRODUCTION

The CRASH3 method (McHenry, 1975; Anon., 1981) is widely used for vehicle accident reconstruction. Among the information CRASH3 needs in order to provide estimates for the crush energy loss in the collision are measurements of the deformation of both vehicles and experimentally determined crush stiffness coefficients. This energy loss is used together with equations of impact to determine velocity changes, ΔV's. In conjunction with impact and rest position information, CRASH3 can also estimate initial velocities. This aspect of CRASH3 is not discussed in this paper. Commercial versions of the CRASH3 method are available in the original formulation (Day & Hargens, 1987) and a modified formulation (Fonda, 1987).

CRASH3 uses a tangential correction factor in the calculation of energy loss and the ΔV's. This correction factor is directly related to a quantity called the PDOF, the principal direction of force, estimated by the user of CRASH3 based on inspection of the damaged vehicles.[1] CRASH3 assumes that a *common velocity conditions* holds but only the condition normal to the surface. The common tangential velocity condition is satisfied through the energy correction factor.

An alternative for analyzing vehicle collisions is to use planar impact mechanics[2] (PIM). PIM uses two coefficients: the coefficient of restitution and a tangential coefficient. In place of user input, the direction of the

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1. In this paper, the PDOF (or Direction Of Principal Force, DOPF) and the direction of the impulse, DOI, are considered to be the same quantity.

2. By *planar impact mechanics* is meant the direct application of Newton's laws using impulse and momentum.

impulse, DOI, and the total energy loss are determined exactly within the limitations of planar mechanics (see Weaver and Brach, 1995). PIM does not automatically assume common velocity conditions, although it allows their use. It is the objective of this paper to investigate how well the CRASH3 method calculates the ΔV's and the energy loss of collisions, in comparison to PIM.

SUMMARY OF PLANAR IMPACT MECHANICS

Newton’s equations of motion in the form of impulse and momentum have been and continue to be used to provide an analysis technique for vehicle collisions (Brach, 1983 and 1991; Ishikawa, 1985; Steffan and Moser, 1996; Woolley, 1987). Planar models only are discussed here, since a full rigorous set of three dimensional equations does not exist[3]. Complete planar equations are not presented in this paper (see Brach, 1983), but rather features of existing equations are covered as they relate to collision energy loss and to damage crush energy.

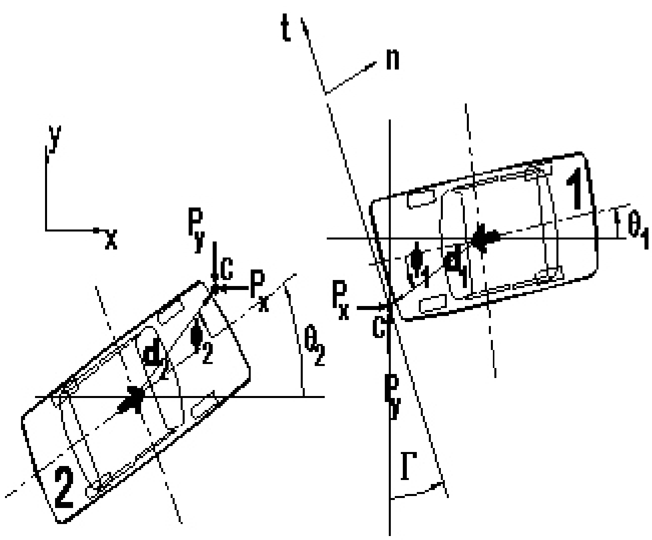

Figure 1. Free body diagrams of two colliding vehicles with 2 coordinate systems associated with the crush plane, n-t, and with the collision site, x-y.

Figure 1 shows free body diagrams of two colliding vehicles. To use PIM (planar impact mechanics), it is necessary to establish an n-t coordinate system where the t axis is parallel to a hypothetical, flat crush (or contact) plane common to both vehicles and n is normal to that plane (surface). This crush plane is chosen using the judgement of the analyst to represent a nominal or average flat surface of deformation. The n-t coordinate system is related to the x-y scene coordinate system by the angle Γ. A point on the contact surface, C, is the point of application of the resultant vector impulse, $P = (P_x, P_y)$, generated during the collision. Ishikawa (1994) discusses a procedure for choosing C. The process of choosing point C requires judgement on the part of the analyst to choose the point of application of the resultant impulse. In effect, point C is an average over time and position of the resultant intervehicle forces. A sensitivity study of the effects of choices of different crush surfaces and locations of point C is presented elsewhere (Brach, 1987a).

A coefficient of restitution, e, is used and defined in PIM as the negative ratio of the final to initial relative normal velocity components at C. All vehicle collision models based on impulse and momentum use this coefficient (or an equivalent coefficient based on a ratio of impulses). But when modeling the tangential contact process, almost all models differ:

- Woolley (1987) uses a common velocity condition[4] in which the relative normal and tangential velocity components of the vehicles at point C are zero. He also optionally permits the relative tangential velocity change to be specified for the analysis of sideswipe collisions;
- Ishikawa uses a tangential coefficient of restitution;
- For two dimensional collisions, Steffan and Moser allow the choice between a tangential common velocity condition and a friction coefficient between the tangential and normal impulse components; and
- Brach (1983, 1991) formulates the PIM problem by defining the ratio of the tangential and normal impulses to be a coefficient, $\mu = P_t/P_n$. The value of this coefficient is chosen to represent the proper tangential condition that exists for a collision, such as a tangential common velocity condition, or an equivalent friction value for sideswipes.

3. Among other problems in going from 2 to 3 dimensions, ignoring impulses of some forces becomes questionable, the coupling of restitution and intervehicle friction coefficients can be unresolvable and the use of an intervehicle coefficient of friction disallows an algebraic solution and forces the use of numerical integration, etc.

4. The term *common velocity condition(s)* is used in the analysis of vehicle collisions both here by others. It can mean that one or both of the normal and tangential relative velocity components of the vehicles at point C are zero at the end of the collision.

Calculation of ΔV: An advantage of the impulse ratio as a coefficient it can be interchanged as either a velocity constraint or as an equivalent coefficient of intervehicle friction over the crush plane, if known. It is shown by Brach and Brach, (1987) that the momentum change, $m_i \Delta v_i$, of each vehicle can be written as[5]

$$|m_i \Delta v_i| = \sqrt{\frac{2\bar{m}(1+\mu)(1+e)\,q\,E_L}{(1-e)+2\mu r-\dfrac{\mu^2 r}{\mu_c}}} \qquad (1)$$

In Eq 1, $\bar{m} = m_1 m_2/(m_1 + m_2)$, E_L is the collision energy loss, $r = (v_{2t} - v_{1t})/(v_{2n} - v_{1n})$, $\mu_c = r/(1 + e)$, q is found from

$$\frac{1}{q} = 1 + \frac{\bar{m} d_a^2}{m_2 k_2^2} + \frac{\bar{m} d_c^2}{m_1 k_1^2} + \mu\left(\frac{\bar{m} d_c d_d}{m_2 k_1^2} + \frac{\bar{m} d_a d_b}{m_2 k_2^2}\right) \qquad (2)$$

In Eq 2, k_1 and k_2 are the centroidal yaw radii of gyration, $d_a = d_2 \sin(\theta_2 + \phi_2)$, $d_b = d_2 \sin(\theta_2 + \phi_2)$, $d_c = d_1 \sin(\theta_1 + \phi_1)$ and $d_d = d_1 \sin(\theta_1 + \phi_1)$. In Eq 1, μ_c is the critical value of μ, corresponding to its unique value when the final relative tangential velocity of the vehicles is zero (common tangential velocity condition).

Energy Loss: The total energy loss of the collision is given by

$$E_L = \frac{1}{2}\bar{m}(v_{2n} - v_{1n})^2 (1+e)[(1-e) + 2\mu r - (1+e)\mu^2] \qquad (3)$$

A pertinent and convenient feature of the above equations is that for given vehicles and collision configuration, substitution of appropriate values of e and μ directly provides collision energy loss and the ΔV's of the vehicles. (These are similar, but more general, than the equations used by CRASH3.) Using e = 0 and $\mu = \mu_c$ satisfies the normal and tangential common velocity conditions, respectively. Each can be used independently.

5. The equations given here are for zero initial angular velocities for both vehicles. For more general equations see Brach (1991).

Energy is not a vector and does not have directional components, but it can be related to the work of the normal and tangential impulses. For example, the normal impulse, P_n, acts internal to the collision but is an external action to each vehicle. The work of P_n on each vehicle can be viewed as the energy loss associated with the normal direction. The work of the tangential process (sliding, entanglement of parts, plastic deformation, etc.) can be viewed similarly. A relationship for computing the work of an impulse is given by a theorem of Thompson (Lord Kelvin) and Tait (1903), stating that the work of any impulse, P_x, is equal to the product of the impulse with the average velocity along the line of action of the impulse. In symbols this means that the work W_P of an impulse P_x is

$$W_P = \frac{1}{2} P_x (v_x + V_x) \qquad (4)$$

where V_x is the final velocity and v_x is the initial velocity. Using this with P_n and P_t and the respective *relative* velocity components at point C from the solution of the PIM equations gives their work. Since P_n and P_t are the only impulses, their combined work must equal the energy loss, E_L, of the collision. The CRASH3 method is formulated to provide the energy loss associated with crush normal to the damaged surface (Anon., 1981), so the work of P_n corresponds to the crush energy calculated by the CRASH3 algorithm. The work of P_t corresponds to the tangential energy loss of the collision. The CRASH3 method uses a correction factor to estimate the energy loss associated with the tangential crush/friction process. A comparison of the work of P_n with the crush energy work of the CRASH3 method and the comparison of the work of P_t with the energy correction factor of the CRASH3 method forms a primary goal of this paper.

ANALYSIS OF THE CRASH3 DAMAGE ALGORITHM

The CRASH3 program contains two methods from which the change of velocity of two automobiles involved in an collision can be determined. Method one uses measurements of the vehicle deformation to estimate the energy absorbed by the vehicles from which the change in the vehicle velocities can be computed. Method one is frequently referred to as the DAMAGE approach. Method two is a trajectory based approach, using the laws of conservation of momentum. A review of the DAMAGE approach consistent with the focus of this study is presented.

The damage based approach to determination of the change of velocity is well documented. One of the first articles that presents this topic (Campbell, 1974), and the one on which the CRASH3 algorithm is based, postulates that the crush energy of a vehicle can be approximated by integrating over the damage width. This technique uses a linear relationship between the residual crush deformation and the velocity. A detailed derivation of the algorithm is presented elsewhere (Anon., 1981 and Day, 1987). Only the results of the derivation are used.

The calculation required to compute the change in velocity, ΔV, from the vehicle deformation requires two steps. Step one calculates the energy absorbed by each vehicle based on the residual deformation. The equation used to calculate the absorbed energy is:

$$E = \frac{L}{5}\Big[\frac{A}{2}(C_1 + 2C_2 + 2C_3 + 2C_4 + 2C_5 + C_6) + \frac{B}{6}(C_1^2 + 2C_2^2 + 2C_3^2 + 2C_4^2 + +2C_5^2 + C_6^2 + C_1C_2 + C_2C_3 + C_3C_4 + C_4C_5 + C_5C_6) + 5G\Big] \quad (5)$$

In Eq 5,

E = Energy absorbed by the vehicle
C_i = Residual crush of the vehicle measured normal to the undeformed surface at position i along the length
L = Width dimension of the damage region
A, B, G = Empirical coefficients of unit width properties obtained from crash test data.

The velocity change for each vehicle is determined using the following equations in CRASH3:

$$\Delta V_1 = \sqrt{\frac{2\gamma_1(E_1 + E_2)}{M_1\left(1 + \frac{\gamma_1 M_1}{\gamma_2 M_2}\right)}} \quad (6)$$

$$\Delta V_2 = \sqrt{\frac{2\gamma_2(E_1 + E_2)}{M_2\left(1 + \frac{\gamma_2 M_2}{\gamma_1 M_1}\right)}} \quad (7)$$

In Eq 6 and 7,

M_1 and M_2 = Masses of vehicles 1 and 2 respectively
γ_1 and γ_2 = $\gamma_i = \frac{k_i^2}{k_i^2 + h_i^2}$
k_1 and k_2 = radii of gyration where $I_i = M_i k_i^2$
h_1 and h_2 = perpendicular distance from the line of action of the PDOF to the CG of the vehicle

For central collisions, $\gamma_1 = 1$ and $\gamma_2 = 1$.

An observation can now be made regarding the two methods under consideration. Recall that in the previous section for PIM, a hypothetical flat crush surface common to both vehicles (establishing the orientation of the n-t coordinate system) is defined as part of the formulation of the problem. Inspection of the residual deformation of the vehicles is used to define the orientation of this surface. The direction of the impulse is determined as part of the results of the PIM. In contrast, in the DAMAGE approach, the direction of the impulse, referred to as the Principle Direction of Force (PDOF) or the Direction of Impulse (DOI), must be visually estimated from damage and is part of the input to the program. The sensitivity of the CRASH3 program to the PDOF has been discussed previously (Smith and Noga, 1982; Woolley, et. al., 1985).

The energy absorbed by each vehicle computed using Eq 5 is based on the measurements made of the residual crush, C_i, in the direction perpendicular to the undeformed surface of the vehicle. In the DAMAGE algorithm, the crush energy is assumed to be caused by the intervehicular force (impulse), the direction of which is defined by the PDOF (DOI). Since the direction of this force typically is not perpendicular to the surface of the vehicle, a correction factor is introduced to account for the work of the tangential component of the force (impulse). Figure 2 shows the geometry of the resultant force (impulse). The PDOF is established by the angle α from the perpendicular to the residual deformation shown by the irregular shape.

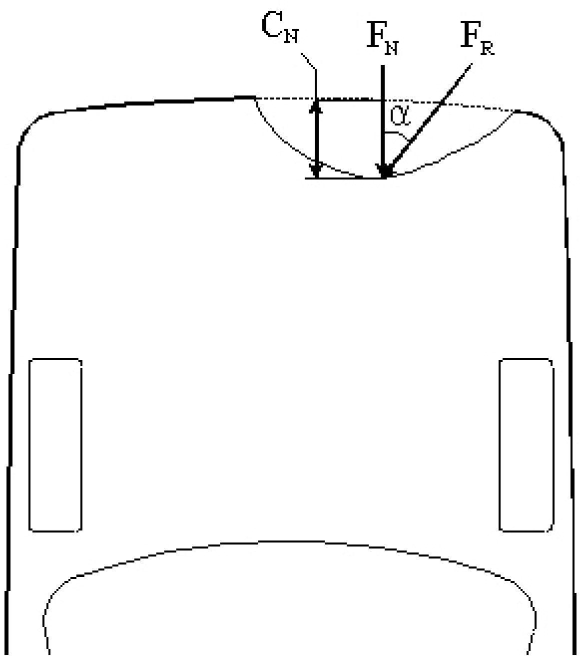

Figure 2

From Fig 2 it can be seen that $F_R = F_N/\cos\alpha$ and $C_R = C_N/\cos\alpha$ where C_R and F_R are collinear. The work done in the direction of the resultant force is established in the CRASH literature to be:

$$\int_0^{C_R} F_R \, dC_R = \int_0^{C_R} \frac{F_N}{\cos^2\alpha} \, dC_N = (1 + \tan^2\alpha) \int_0^{C_N} F_N \, dC_N \tag{8}$$

The final integral in Eq 8 is the work of the normal force and impulse. The factor $(1 + \tan^2\alpha)$ is the correction factor for the energy. In the DAMAGE algorithm approach, the angle α is defined in the local coordinate system of each vehicle. Therefore, each vehicle has its own correction factor. In the PIM approach to this problem, the coordinate system is common between the two vehicles by definition, and therefore the angle between the normal to the crush surface and the DOI is the same between the two vehicles. The two methods also differ in how the total energy loss is computed. In the PIM approach, which has a common crush surface between the two vehicles, the work done by the impulse components produces a single value for the energy absorbed in the collision. In the DAMAGE approach to the problem, each vehicle has a correction factor and the energy absorbed in the tangential direction is determined by subjective assessment of each vehicle's crush surface and PDOF.

The influence of the correction factor on the DAMAGE algorithm prediction of the energy absorbed by the vehicles must not be underestimated. For example, if $\alpha = 30°$, (which is not unusual for typical 90° front to side accident geometry), the correction factor, $(1 + \tan^2\alpha) = 1.25$, thus increasing the estimate of the damage energy by 25%. If $\alpha = 40°$, (which is not unusual for typical 60° front to side accident geometry), the correction factor, $(1 + \tan^2\alpha) = 1.7$, a significant increase in the estimated energy loss. Most DAMAGE algorithms set a limit of 45° for the angle α, thereby imposing an upper limit for the correction factor to 2, which is a 100% increase in the (normal) energy absorbed by the vehicle. This limit on α is reasonable; the maximum values of 2 is exceeded only occasionally.

An alternative correction factor has been proposed for the DAMAGE algorithm that incorporates a tangential friction force (McHenry and McHenry, 1986). This approach does not appear to have found widespread use.

ANALYSIS OF THE RICSAC COLLISIONS

This section contains a comparison between the PIM and the DAMAGE algorithms. The comparison was performed by analyzing the ΔV's predicted by the two methods with the actual ΔV's from the RICSAC collisions. Particular attention is paid in this comparison to the predictions of the two methods for the ΔV associated with the normal impulse. This is accomplished by comparing the ΔV from the DAMAGE algorithm, computed without using the tangential correction factor, to the ΔV computed with the PIM algorithm, using only the work done by the normal impulse. In addition, the full ΔV's of the two algorithms are compared.

This comparison was hampered by a lack of information from the DAMAGE analysis of the RICSAC collisions. Much of the detailed information related to the CRASH3 DAMAGE analysis of the RICSAC collisions, such as the PDOF, the values of tangential correction factor α, and the magnitudes of the distances from the vehicle CG to the line of action of the impulse for each vehicle, the

h_1 and h_2, could not be found in the literature.[6] Therefore, the values of these required parameters were computed using the DOI as predicted by the PIM solutions (Brach, 1983). This reference (Brach, 1983) presents the PIM analysis of each of the RICSAC collisions in complete detail including the locations of the point of application of the resultant impulse, point C. Use of information from the PIM solution to generate input to the CRASH3 DAMAGE analysis creates a situation that will likely produce better comparative results than if an uncorrelated subjective assessment of the vehicle deformation was used to determine the PDOF's.

Table 2 contains the results for comparison, categorized according to collision geometry. The upper group (row of numbers) in each collision geometry contains the solutions associated with the normal direction only. The legend for Table 2 refers to these as the Crush ΔV_i (the work of the normal impulse from PIM) and the Uncorrected ΔV_i (CRASH3 results uncorrected for tangential effects). The second, middle, group contains resultant or total ΔV's (Total ΔV_i and Corrected ΔV_i). As expected, in all cases, the introduction of the tangential effects (the work of the tangential impulse in PIM and the correction factor in CRASH3) increases the energy loss and the ΔV's (see Eq 6 and 7). Other comparative trends are mixed. For the 60° Front to Side collisions, RICSAC #1, #6 and #7, the PIM solution consistently predicts ΔV's lower than the DAMAGE algorithm. With one or two minor exceptions, the opposite is true for all the others. There seems to be no way to determine from these collisions which set of normal ΔV's is "better" or more accurate, but they certainly are highly correlated.

The data show that both solution algorithms display variability relative to the measured results, with no uniform or consistent trends. The question of which method produces more accurate results is explored next.

ACCURACY AND SENSITIVITY

This section covers two topics. The first topic is a comparison of the ΔV's calculated using planar impact mechanics and those calculated using CRASH3 to the measured values from the RICSAC tests. The second topic is a sensitivity analysis that assesses the changes that occur in the CRASH3 solutions as the angle of the PDOF changes relative to a nominal value.

The last row within each collision geometry group in Table 2 contains the experimental values for the RICSAC collisions. Table 1 lists rms differences between the computed and experimentally measured ΔV's listed in Table 2. The planar impact mechanics results are significantly more accurate than those of CRASH3. This is particularly true for the last 3 collision geometries.

In all published examples of CRASH3 pertaining to RICSAC, including the original RICSAC reports, the values used for the PDOF, the moment arms h_1 and h_2, and the angles α_1 and α_2 are not documented. Consequently, it was necessary to use the PDOF calculated from the PIM solutions in all of the examples covered in this paper. Yet, in practice, the PDOF chosen by the analyst is likely to be significantly different than the value resulting from PIM analysis for the same accident.

A study was done to investigate the sensitivity of the ΔV's calculated by the CRASH3 DAMAGE to changes in the direction of the PDOF. This was done by using the PDOF computed by PIM analysis as the nominal, changing this angle by ΔPDOF, computing the moment arms h_1 and h_2, the angles α_1, and α_2 and then computing the new ΔV's. This was done for RICSAC collision geometries of 60° Front to Side (1, 6, 7) and 10° Front to Rear (3, 4, 5). Two values either side of the nominal were used in both cases with a ΔPDOF = 10° for RICSAC 1, 6, and 7 and ΔPDOF = 5° for RICSAC 3, 4, 5. Table 3 contains the numerical results of the study. In both cases, positive ΔPDOF is defined as a counterclockwise change in the PDOF. The last column of Table 3 lists the rms deviations from the nominal values for a given ΔPDOF. This gives an indication of how differences in the PDOF can affect the ΔV's when using the CRASH3 method for two of the collision geometries.

6. Information such as vehicle masses, deformation profiles, etc. were taken from Jones and Baum (1978).

Table 1
Differences* between Calculated and Measured ΔV Values for RICSAC Collisions (in ft/s)

Collision Geometry	PIM Solution	CRASH3 Algorithm
60°, Front-to-Side	2.73	5.46
90°, Front-to-Side	3.72	15.22
10°, Front-to-Front	1.22	12.57
10°, Front-to-Rear	2.77	12.20

* Values are root-mean-square values. That is, they are the square root of the average value of the squares of the differences between the calculated ΔV for each vehicle and the corresponding experimental ΔV value for each Collision Geometry category.

USING PIM AND CRASH3 IN COMBINATION, AN EXAMPLE RECONSTRUCTION

This section demonstrates a way in which PIM and CRASH3 DAMAGE can be used together. Figure 3 shows the orientation and position of an angled intersection collision between a 1994 GMC Jimmy and a 1995 Nissan Sentra. The Nissan was accelerating from a stop into cross traffic. Vehicle characteristics, results of the crush analysis and the planar impact analysis are presented in an appendix.

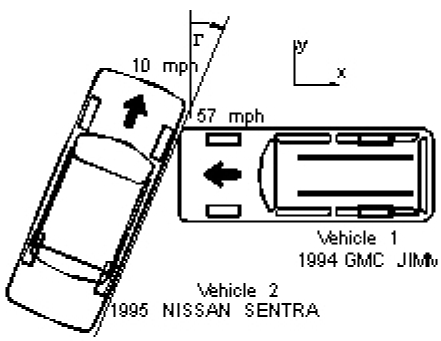

Figure 3. Example collision

Measurements of the crush were made for both vehicles. Crush stiffness coefficients were taken from the open literature and a crush analysis was performed according to the CRASH3 algorithm. The resulting energy losses of vehicles 1 and 2 are 1.02 x 10^5 N-m (7.53 x 10^4 ft-lb) and 3.87 x 10^4 N-m (2.85 x 10^4 ft-lb) giving a total energy loss of 1.41 x 10^5 (1.04 x 10^5). The planar impact mechanics solution was then run for an assumed initial speed of Veh 2 of 10 mph and by changing the initial speed of Veh 1 until the crush (normal) energy loss equaled 1.41 x 10^5 N-m (1.04 x 10^5 ft-lb). This happened at v_1 = 22.9 m/s (75 ft/s). The PIM solution was for a tangential impulse ratio of $\mu = \mu_c = 0.371$ (common tangential velocity condition). At this value, the total energy loss of the collision is 1.88 x 10^5 N-m (1.38 x 10^5 ft-lb). This gives velocity changes of Δv_1 = 43.6 km/h (27.1 mph) and Δv_2 = 47.3 km/h (29.4 mph).

Had the CRASH3 method been used (with the DOI from the PIM solution, not estimated visually from the vehicles), the corresponding corrected crush energy losses would be 1.16 x 10^5 N-m (8.57 x 10^4 ft-lb) for Veh 1 and 7.31 x 10^4 N-m (5.39 x 10^4 ft-lb) for Veh 2. The total corrected energy loss from the CRASH3 damage analysis is 1.89 x 10^5 N-m (1.40 x 10 ft-lb) which is 13% below the total energy loss of the PIM solution. The corresponding CRASH3 velocity changes are Δv_1 = 35.1 km/h (21.8 mph) and Δv_2 = 38.1 km/h (23.6 mph), significantly lower than the PIM solution. It appears that this difference is due primarily to the value of e = 0.3 used for the PIM solution, whereas e = 0 is assumed by CRASH3.

The above procedure is summarized in the following manner:

1. calculate the energy loss using a damage only CRASH3 analysis; and
2. vary the conditions of a PIM solution (with an appropriate coefficient of restitution e and for $\mu = \mu_c$) until the (normal) crush energy loss equals that of the CRASH3 analysis.

The results of the PIM solution then provide the reconstructed velocities. Note that the initial velocities of Veh 1 are found from the damage energy and the known initial velocities of Veh 2.

DISCUSSION AND CONCLUSIONS

The salient differences between the CRASH3 and PIM solutions are:

- •CRASH3 limits the tangential correction factor to a value of 2, regardless of the appropriate value, whereas PIM uses the exact tangential impulse required to cause the common tangential velocity condition;
- •CRASH3 requires that the PDOF be estimated visually from examination of vehicle damage prior to making calculations, whereas PIM calculates the DOI based on the definition of the crush plane angle, Γ, and the common velocity conditions;
- •CRASH3 uses 2 values of the energy correction angle α, one for each vehicle whereas PIM needs no correction factors; and

- CRASH3 uses the common velocity condition only for the velocity component normal to the crush surface. PIM allows arbitrary values of the coefficients e and μ; this permits the use of values of e greater than zero, when appropriate (see Brach, 1983) and the tangential common velocity for nonsideswipe collisions.

One of the differences between CRASH3 and planar impact mechanics that arose many times above is that for CRASH3 the angle α between the PDOF (DOI) and each vehicle must be estimated. For the PIM solution, a common damage surface must be established. The collision orientations of the vehicle must be estimated for both methods. For PIM, the damaged surfaces are physically present after a collision and remain visible. On the other hand, establishing the direction of an (unobservable) impulse for CRASH3 can challenge even the most experienced investigator. Overall, this gives an advantage to planar impact mechanics.

It was shown that the CRASH3 energy loss of a collision can be combined with planar impact mechanics for accident reconstruction. Matching the normal crush energy from the CRASH3 algorithm with the work of the normal impulse for a given collision, results in a full reconstruction with improved accuracy. This approach allows the use of crush measurements with the versatility of arbitrary values of the coefficient of restitution and the tangential impulse ratio.

On the basis of comparisons with RICSAC experiments, collision analyses carried out using planar impact mechanics appear to provide significantly more accurate values of ΔV than the DAMAGE routine of CRASH3.

REFERENCES

Anon., 1981, *CRASH3 User's Guide and Technical Manual*, NHTSA, DOT Report HS 805 732, February.

Brach, R. M., 1983a, "Identification of Vehicle and Collision Impact Parameters From Crash Tests". Presented at the Design and Production Engineering Technical Conference, Dearborn, MI, Sept. 11-14, 1983. ASME paper 83-DET-13

Brach, R. M., 1983b, "Impact Analysis of Two-Vehicle Collisions", Paper 830468, SAE, Warrendale, PA, 15096.

Brach, R. M., 1987a, "Least Squares Collision Reconstruction", Paper 870429, SAE, Warrendale, PA, 15096.

Brach, R. M., 1987b, "Energy Loss in Vehicle Collisions", Paper 871993, Transactions, SAE, Warrendale, PA, 15096.

Brach, Raymond. M., and R. Matthew Brach, 1987, "A Review of Impact Models for Vehicle Collisions", Paper 870048, SAE, Warrendale, PA, 15096.

Brach, R. M., 1991, *Mechanical Impact Dynamics*, John Wiley & Sons, NY.

Campbell, Kenneth, 1974, "Energy Basis for Collision Severity", Paper 740565, SAE, Warrendale, PA, 15096.

Day, T. D. and R. L. Hargens, 1987, "An Overview of the Way EDCRASH Computes Delta-V", Paper 870045, SAE, Warrendale, PA, 15096.

Fonda, A. G., 1987, "CRASH Extended for Desk and Handheld Computers", Paper 870044, SAE, Warrendale, PA.

Jones, Ian S. and A. S. Baum, "Research Input for Computer Simulation of Automobile Collisions, Vol. IV: Staged Collision Reconstructions". DOT HS 805 040, December 1978.

Ishikawa, H., 1994, "Impact Center and Restitution Coefficients for Accident Reconstruction", Paper 940564, SAE, Warrendale, PA, 15096.

Ishikawa, H., 1985, "Computer Simulation of Automobile Collision - Reconstruction of Accidents", Paper 851729, SAE, Warrendale, PA, 15096.

Lord Kelvin and P. Tait, 1903, Treatise on Natural Philosophy, Cambridge University Press, Cambridge, UK.

McHenry, R. R., 1975, "The CRASH Program - A Simplified Collision Reconstruction Program", Motor Vehicle Collision Investigation Symposium, Calspan, Buffalo, NY.

McHenry, Raymond R. & Brian G. McHenry, 1986, "A Revised Damage Analysis Procedure for the CRASH Computer Program, Paper 861894, SAE, Warrendale, PA, 15096.

McHenry, Raymond R. & Brian G. McHenry, 1997, “Effect of Restitution in the Application of Crush Coefficients”, SAE Paper 970960, SAE, Warrendale, PA, 15096.

McHenry, Brian G. & Raymond R. McHenry, “RICSAC-97 - A Reevaluation of the Reference Set of Full Scale Crash Tests”, 1997, SAE Paper 970961, SAE, Warrendale, PA, 15096.

Smith, Russell A. & J. Thomas Noga, 1982, "Accuracy and Sensitivity of Crash", DOT HS 806 152, NHTSA, Washington, D.C., March 1982.

Steffan, H. and A. Moser, 1996, The Collision and Trajectory Models of PC-CRASH”, SAE Paper 960886, SAE, Warrendale, PA, 15096.

Tumbas, N. S., and R. A. Smith, 1988, “Measurement Protocol for Quantifying Vehicle Damage From an Energy Basis Point of View”, Paper 880072, SAE, Warrendale, PA, 15096.

Weaver, J. and R. M. Brach, 1995, "Limitations of Momentum Methods for Accident Reconstruction", *Forensic Accident Investigation*, Michie Publishers.

Woolley, Ronald L., Charles Y. Warner, and Melanie D. Tagg, "Inaccuracies in the CRASH3 Program". Paper 850255, SAE, Warrendale, PA, 15096.

Woolley, R. L., 1987, The “IMPAC” Program for Collision Analysis”, SAE Paper 870046, SAE, Warrendale, PA, 15096.

	RICSAC #1		**RICSAC #6**		**RICSAC #7**	
	PIM	CRASH	PIM	CRASH	PIM	CRASH
60° Front to Side	9.2 13.9	14.2 21.2	9.7 15.9	14.4 23.6	14.5 20.6	20.4 29.0
	15.1 22.7	17.7 26.4	15.2 24.8	18.1 29.7	22.2 31.4	25.0 35.5
	18.5 22.8		13.5 21.6		17.9 30.2	
	RICSAC #8		**RICSAC #9**		**RICSAC #10**	
	PIM	CRASH	PIM	CRASH	PIM	CRASH
90° Front to Side	14.3 13.7	10.8 10.2	16.2 7.4	16.6 7.6	24.6 12.0	18.0 8.8

	18.3 17.4	11.7 11.1	28.8 13.3	18.5 8.5	45.8 22.4	20.9 10.2
	22.9 16.2		31.9 12.1		52.0 19.1	
	RICSAC#11		**RICSAC #12**			
10° Front to Front	PIM	CRASH	PIM	CRASH		
	35.8 22.4	34.6 22.1	52.8 36.6	38.1 26.4		
	35.7 22.4	35.1 22.5	58.3 40.4	38.5 26.7		
	35.8 23.1		60.3 39.2			
	RICSAC #3		**RICSAC #4**		**RICSAC #5**	
10° Front to Rear	PIM	CRASH	PIM	CRASH	PIM	CRASH
	11.3 17.9	5.5 8.7	21.2 33.2	15.1 23.6	19.5 35.4	11.7 21.2
	14.0 22.2	5.6 8.9	22.1 34.5	15.4 24.0	20.4 37.1	11.8 21.4
	13.9 23.2		27.5 32.6		23.9 37.3	

LEGEND	PIM	CRASH
Collision Geometry	Crush ΔV_1 Crush ΔV_2	Uncorrected ΔV_1 Uncorrected ΔV_2
	Total ΔV_1 Total ΔV_2	Corrected ΔV_1 Corrected ΔV_2
	Measured ΔV_1 Measured ΔV_2	

Table 2. Comparison of ΔV's from PIM and CRASH3 DAMAGE Algorithm for RICSAC Collisions (All numbers in ft/s)

	RICSAC 1		RICSAC 6		RICSAC 7		rms Deviations from Nominal
	ΔV_1	ΔV_2	ΔV_1	ΔV_2	ΔV_1	ΔV_2	
PDOF +20°	24.09	35.92	23.57	38.64	31.29	44.36	7.74
PDOF +10°	19.78	29.49	19.87	32.58	26.99	38.28	2.49
Nominal	17.71	26.40	18.09	29.66	25.03	35.49	0.00
PDOF -10°	16.63	24.80	17.26	28.29	24.14	34.22	1.21
PDOF -20°	16.01	23.87	16.83	27.59	23.78	33.72	1.82

	RICSAC 3		RICSAC 4		RICSAC 5		rms Deviations from Nominal
	ΔV_1	ΔV_2	ΔV_1	ΔV_2	ΔV_1	ΔV_2	
PDOF +10°	5.15	8.15	14.57	22.74	10.89	19.78	1.03
PDOF +5°	5.41	8.57	14.80	23.11	11.35	20.62	0.58
Nominal	5.64	8.93	15.36	23.97	17.77	21.39	0.00
PDOF -5°	5.80	9.19	15.78	24.63	12.11	22.01	0.45
PDOF +10°	5.89	9.34	16.05	25.05	12.34	22.43	0.74

Table 3. Results of Sensitivity Analysis of Changes in PDOF Direction (All numbers in ft/s)

APPENDIX: This appendix summarizes the vehicle characteristics, crush analysis and planar impact solutions corresponding to the Section, *Using PIM and CRASH3 in Combination, An Example Reconstruction*.

Physical properties and dimensions of the vehicles:

Vehicle 1: 1994 GMC Jimmy, 4-dr utility vehicle
weight: 3696 lb
yaw moment of inertia, 2437 ft-lb-s^2
wheelbase: 8.9 ft

Vehicle 2: 1995 Nissan Altima, 4-dr sedan
weight: 3413 lb
yaw moment of inertia, 2165 ft-lb-s^2
wheelbase: 8.6 ft

Damage Crush Analysis:

	C_1	C_2	C_3	C_4	C_5	C_6
Veh 1	21.3	20.3	13.8	11.3	5.3	4.8

	C_1	C_2	C_3	C_4	C_5	C_6
Veh 2	3.0	15.8	13.5	6.7	3.3	5.0

Crush Width:
Veh 1, 1.65 m (65 in)
Crush Stiffness Coefficients:
Veh 1: A = 587.4 N/cm, B = 63.3 N/cm^2
$d_0 = 35\ \sqrt{lb}$, $d_1 = 115\ \sqrt{lb}/ft$
Crush Energy: Veh 1, 1.02 x 10^5 N-m (7.54 x 10^4 ft-lb)
Total, 1.41 x 10^5 N-m, (1.04 x 10^5 ft-lb)

Crush Width:
Veh 2, 2.13 m (84 in)
Crush Stiffness Coefficients:
Veh 2: A = 339.3 N/cm, B = 28.8 N/cm^2
$d_0 = 30\ \sqrt{lb}$, $d_1 = 77.5\ \sqrt{lb}/ft$
Crush Energy: Veh 2, 3.87 x 10^4 N-m (2.85 x 10^4 ft-lb)

ΔV: Veh 1, 8.41 m/s (27.6 ft/s, 18.8 mph)
All uncorrected

ΔV: Veh 2, 9.91 m/s (29.9 ft/s, 20.4 mph)
All uncorrected

Planar Impact Mechanics Solution:

coefficient of restitution: e = 0.3
impulse ratio: $\mu = \mu_c = 0.371$
crush plane angle: $\Gamma = -23^\circ$

Veh 1: $d_1 = 6.3$ ft, $\theta_1 = 0^\circ$, $_1\phi = -22.4^\circ$,
$v_x = -75.0$ ft/s, $v_y = 0.0$ ft/s, $\omega_1 = 0$
$V_x = -35.3$ ft/s, $V_y = -1.8$ ft/s, $\Omega_1 = -4.0$ rad/s
$\Delta v_1 = 12.12$ m/s (27.1 mph)

Veh 1: $d_2 = 3.8$ ft, $\theta_2 = 67^\circ$, $\phi_2 = -39.0^\circ$,
$v_x = -5.7$ ft/s, $v_y = 13.5$ ft/s, $\omega_2 = 0$
$V_x = -37.3$ ft/s, $V_y = -15.5$ ft/s, $\Omega_1 = 4.1$ rad/s
$\Delta v_2 = 13.13$ m/s (29.4 mph)

initial system energy: 4.53 x 10^5 N-m (3.34 x 10^5 ft-lb)
total energy loss: 1.87 x 10^5 N-m (1.38 x 10^5 ft-lb), 41.4%
crush energy loss: 1.41 x 10^5 N-m (1.04 x 10^5 ft-lb), 31.0%

normal impulse: $P_n = 19045$ N (4281.7 lb-s)
tangential impulse: $P_t = 1064$ N (1588.2 ft-s)
resultant impulse: 20313 N (4566.7 ft-s)